Noncommercial Joint-Stock Company

ALMATY UNIVERSITY OF POWER ENGENEERING AND TELECOMMUNICATIONS

Automation and Control Department

MODELING AND IDENTIFICATION OF CONTROL OBJECTS

Abstracts of lectures for the specialty 5B070200 - Automation and control

Almaty 2018

Prepared by: L. K. Ibrayeva. Modeling and Identification of Control Objects. Abstracts of lectures for specialty 5B070200 – Automation and Control. – Almaty: AUPET, 2018. – 66 p.

Abstracts of lectures / Lecture notes have been prepared in accordance with the working program of the discipline "Modeling and Identification of Control Objects" for undergraduate students majoring in 5B070200 – Automation and Control. The issue contains 15 lectures.

The abstracts of lectures are intended to help students in studying theoretical material and for exam preparation, to be used in practical and laboratory classes.

Ill. - 6, bibliography – 5.

Reviewer: Dr.Tech.Sc, prof. Kaim T.T.

Printed according to the plan for publications of NC JSC "Almaty university of power engineering and telecommunications" for 2018.

Introduction

In the design, development and creation of complex objects ~~the~~ knowledge about the quantitative and qualitative characteristics of the considered objects is required. It is often impossible to carry out a practical test of certain laws inherent in complex objects for a number of reasons. In addition, it would require high material costs and time-consuming. In this regard, the study of the properties and laws of the considered complex objects on the basis of modeling methods is of great importance.

The purpose of teaching the discipline "Modeling and Identification of Control Objects" is to form the necessary students' knowledge in the field of modeling control objects, which will contribute to further use of the developed models in the design of control systems.

A distinctive feature of the Control Systems design process is its combination in time with the development and manufacture of technological units. This means that the only possibility of obtaining information about the properties of not yet created technical system is an *analytical description* of the processes typical for the elements of such a system. Specific characteristics of the studied object are the basis of such analytical methods. That is, analytical methods take into account the structure and all physical and chemical features of the processes occurring in the Control Objects.

In many cases, the model adopted in the design differs significantly from the real object; it considerably reduces the effectiveness of the developed control system. In this context in control theory one of the approaches appeared, is associated with the model creation based on the observations of object's input and output variables. This approach is known as system *identification*. In this approach its own principles and methods are developed.

Knowledge of the material of the discipline "Modeling and identification of Control Objects" for future specialists in automation, whose activities are related to the design, modernization and maintenance of automatic control systems, in our opinion is mandatory.

The working program of the discipline "Modeling and Identification of Control Objects" includes a large amount of theoretical and practical material. Limited classroom hours do not allow to provide students with comprehensive necessary information, so most of the material is studied in the framework of independent work.

The proposed Abstracts of Lectures are drawn up in accordance with the working program of the discipline and contain 15 themes.

The publication is only a brief summary of lectures and may not contain all the necessary information. Other sources should be used for successful and comprehensive mastering of the material.

1 Lecture №1. The concept of Control Object modeling. The types of modeling

The content of the lecture: main definitions of Control Object modeling.

The goal of the lecture: to learn the main definitions and types of Control Objects models.

1.1 The place of models in the structure of Control System

Control objects are complex objects. For automation system design it is necessary to know the control object's properties, that is its static and dynamic characteristics and disturbances that influence on the specified mode. Modeling methods are the most convenient description methods of the Control Object's properties.

Model is some object, material or virtual, certain properties of which fully or partially match the properties of the origin, i.e. a simplified system that reflects the individual, limited in the right direction, characteristics of the studied process.

The substitution of one object by another one with the aim of obtaining information about the important properties of the original object using the object models is called *modeling*.

In the automatic Control Theory the Control Object is a device, machine or process, state and behavior which is characterized by certain physical quantities. These values can be measured, that is, it is in principle measurable, direct or indirect. In the general case, the object can be described by some physical quantities that cannot be measured. The Control Object is able to accept external impacts and respond to them by changing the output values. Description of the Control Object is the expression of relationships between the object response, as a function of time and the input signals. Technological objects as usually have multiple input and multiple output variables, which, depending on control objectives, can be controlling, controlled or disturbing.

Object control means fulfilling of commands, implementation of which ensures a change of object state according to pre-specified requirements and constraints. Any control system in the simple form consists of two main elements: the controlled object (in which you want to control the process) and the controller (carrying out the control functions of this process).

The simplified structural scheme is presented in figure.1.1. Controller *C* gets the information about the control goal - *x(t)* signal which changes in time *t* and forms the control influence *m(t)* on the control object *CO* by a way that the controlled value *y(t)* changes in accordance with the change of *x(t);* thus the control goal *y(t)=x(t)* could be achieved. In the schema *l(t)* are the disturbing influences, which can be any in the number (including the non-controlled ones).

Such a control system can function in reality if only there is one meaningful accordance between the change *y(t)* and the caused change *m(t)*. This accordance is reflected in the *mathematical model* of the object that is supposed to be known

beforehand and can be used for the determination of the controller's functioning algorithm (control algorithm). The mathematical description of the object must be known for any moment of time in high accuracy. It means that it is necessary to have the main dependencies expressed in the way of equations, which are inherent to this object and which characterize the static and dynamical connections between its incoming and outcoming values.

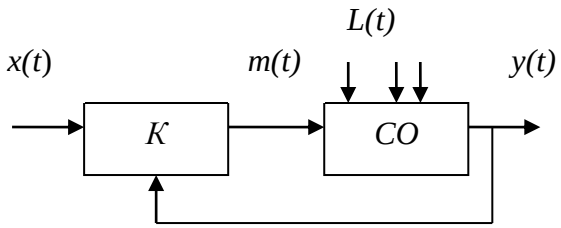

Figure 1.1 – The structural scheme of Control System

The practical meaning of modeling is:

- models are more convenient for the study than the original objects. Besides some objects can be studied only at the models;
- modeling allows to identify more valuable factors of the studied object or a phenomenon, that's why it is an instrument for more deep studying of reality.

Not a single model can be exhaustively complete. It is always limited and must be consistent with the purposes of modeling, reflecting as much source object properties and in such fullness as needed for a particular study. However, the resulting model should reflect the laws acting in the real object with the precision determined by the requirements of the control problem. The control quality depends on it.

1.2 Types of modeling

Model is the material or mentally imagined object that replaces the real object in the process of study; and at the same time it preserves some typical features which are important for this research. The method of research that is based on the developing and using of models are called *modeling.*

The models can be divided into two groups: *material* and *ideal*, and, respectively, there can be distinguished *subject* and *abstract* modeling.

The main types of the subject modeling are the *physical* modeling and *analog* modeling.

The first group includes such methods in which research is conducted on the basis of a model that reproduces the basic geometric, physical, dynamic and functional characteristics of the original. The main types of subject modeling are physical and analog modeling. Physical modeling is the modeling, when the real object is replaced by the enlarged or reduced copy, allowing studying with the subsequent transfer of the processes and phenomena properties from the model to the original on the basis of *similarity* theory.

This method of modeling is widely used in the technique when designing of different kinds of technical systems. For example, a study of aircraft based experiments in the wind tunnel.

The disadvantage of physical models is that when you change the parameters of the process or when creating a new object, you must create a new model, that is typically associated with high costs of time and money. Besides the complex objects models cost is relatively high. That is, these models are not universal.

The *analog* simulation is based on replacing the original object by the object of another physical nature but with similar behavior. For example, vibration and resonance can be studied with the help of mechanical systems, or using electrical circuits. At analog modeling it is important to see the needed features in the substitute object and interpret them correctly.

In both types of the material modeling the model is the material reflection of the initial object and is connected to it by its characteristics; the process of researching consists in the subject experiment.

The *ideal* modeling differs from subject modeling principally and is based at the ideal, mental analogue. It is theoretical in nature.

There are two types of ideal modeling: *intuitive* and *symbolic* modeling.

Intuitive modeling is modeling based on the intuitive idea about the object under study which either can not be formalized or does not need it. That is, the model is not formed, and instead, some not definitely fixed mental representation of the real fact, serves as a basis for reasoning and decision making. Thus, any argument that does not use the formal model can be considered as intuitive modeling when the thinking individual has some idea about the object of study that can be interpreted as the model of the reality.

The *symbolic simulation* is the modeling that uses signs and symbols as the model: schemas, graphics, drawings, texts in different languages including formal, mathematical formulas and theories. The obligatory participant of the symbolic modeling is the interpreter of the symbolic model (mainly a person). Drawings, texts and formulas themselves have no meaning, they have the meaning when the individual understands them and uses them in everyday activity.

The most important type of symbolic modeling is the *mathematical modeling*, when the object studying is fulfilled by means of the model formed in the language of mathematics with use of specific mathematical methods. By abstracting from the physical nature of the object, the mathematics studies the ideal objects. Mathematical modeling is based on the limited number of fundamental laws of nature and principle of similarity which means that the phenomena of diverse physical nature can be described by similar mathematical dependences.

Mathematical modeling is the formalized system description using mathematical relations and algorithms. Any mathematical expression which consists of physical quantities can be used as a mathematical model of the process.

Unlike physical modeling mathematical model allows to study only the parameters of the original which have a mathematical description and linked by

mathematical relations in the equations that are related either to the mathematical model or to the original. Meanwhile the physics of the studied process is not kept. The modeling here is based on the ability of the same equations to describe different phenomena and to identify the different functional connections of the individual aspects of the object behavior without a full description of its behavior.

The advantages of the mathematical model:

a) ability to quickly fulfill a series of experiments on the mathematical model with the purpose of finding an optimal technological mode or maximally valid forecast at the minimal costs of time and material resources;

b) ability of the model to specify the operating conditions, impossible in reality, to verify the optimal modes;

c) a mathematical model using the developed techniques allows you to quickly find the optimal modes of the technological process.

The transfer of the results obtained in the process of creation and study of models to the original is based on the fact that the model, in a certain sense, reproduces some features of the object which are of interest to the researcher.

The most important kind of the mathematical modeling is the *computer* simulation. A computer model is a software implementation of the mathematical model, complemented by various utilities. This is just another form of abstract models, which, however, can be interpreted not only by mathematicians and programmers, but the technical device – processor.

This special kind of models, combining abstract and physical traits, has a unique set of useful properties. Therefore, in the present, under simulation is always understood computer modeling.

2 Lecture №2. Mathematical modeling. Classification of the models

The content of the lecture: main terms in mathematical modeling of Control Objects; classification of objects and models.

The goal of the lecture: to learn main classes of models of Control Objects

2.1 Main terms in mathematical modeling of Control Objects

A mathematical model is an orderly combination of such parts as: components, variables, parameters, operators (or functional dependencies), constraints.

The components of the model are its composite parts forming a system when they are united according to certain rules. The components can be indivisible structures ("elements" of the model) or composite parts which are the "subsystems".

Typically inputs and outputs of the system are called *variables*; the other values are called *parameters*. These assumptions are conditional. It is impossible to answer without additional agreements where there are the variables and where there are the parameters. As an example of such an agreement, the class of functions may be accepted.

The variables of two types are usually differentiated: the *exogenous (input*) and *endogenous (output)* ones. The inputs of the system are generated outside the system under study and are the results of external reasons. The outputs appear in the system as a result of exogenous variables influence. Dividing of variables into output variables and input variables is also not absolute. This applies to a specific system. It is necessary to keep in mind the specific characteristic of the whole studied system.

The main components of the model are the *operators* or *functional dependencies*. Usually they describe a relation between the time variation of the output variable *y(t),* with a corresponding change of the input variable *x(t)* or between variables and parameters *(p)* which depend on them.

A set of actions to be applied to the given input function *x(t)* to determine the appropriate output function *y(t)* is called the *operator* of the system. Symbolically the accordance between input function variable *x(t)* and output function *y(t)* is written in the following way

$$y(t) = A\{x(t)\},$$

where A is the object operator.

There are different ways to define the object operator.

The last components of the models are the *constraints*. In the simplest case, the constraints include the range of variation of the vector of arguments of the model: $x \in D_x$. The model parameters can also be set at a certain permitted set $p \in D_p$

Most often it is considered that the modeling system has no effect on the environment. The question of the admissibility of neglecting the external environment needs to be justified.

Creating universal model that will meet various aspects of its application is practically impossible. The construction of the model begins with the definition of the *class* to which the object belongs.

On the basis of the temporal and spatial characteristics all variety of the Control Objects can be divided into the following classes: deterministic and stochastic; stationary and non-stationary; static and dynamic; linear and nonlinear; objects, whose variables vary in space and objects without the spatial variation of the variables.

As mathematical models are a reflection of the corresponding objects, the same classes are typical for them.

2.2 Classification of the models

Depending on the nature of the studied processes in the system, all models can be divided into the following classes.

Deterministic models reflect the deterministic processes, i.e. the processes in which the absence of any random influences is assumed.

Stochastic models represent the probabilistic processes and events, i.e., variables of the model are time-varying random processes. These processes can be

stationary and non-stationary. In non-stationary case, the probabilistic characteristics of the process are a function of time.

Stationary and non-stationary models. The models are called stationary if the operator of the model and its parameters do not change over time.

If the model parameters change over time, the model should be called the *non stationary parametrically*. The operator of the model can be dependable on time.

The model operator may also be time-varying. This is the most common type of non-stationarity. Both stationary and non-stationary systems can be linear and nonlinear.

Stationary models are described by the differential equations with the constant coefficients. The coefficients of the differential equations of non-stationary models are the functions of time.

Static and dynamic models. The basis for such separation of types of models are the features of the motion of the studied object as a material system.

Speaking about the models from the control point of view, it should be noted that the concept of space, usually understood in a geometric sense and related to mechanical systems, it becomes narrow for a wide class of technological processes. Moreover, for many of the control objects it is important not just mutual movement of the elements, but change of their internal state. Therefore, unlike geometric, we consider the *state space* of the object and its model. Then the "position" of the object or a prediction of this "position" by the model is estimated using *state coordinate y* related to output variables. Elements of the vector *y* are usually controlled technological parameters (flow, pressure, temperature, humidity, viscosity, etc.

The composition of the vector *y* elements for the object itself may be wider than for the model of this object, as the modeling requires the study of only part of the real system properties. Therefore, speaking about the vector *y(t)*, we will relate it to the model, and it is served for predicting the behavior of the object. Thus, movement of the control object in the state space and in time is estimated with the vector process *y(t).*

The model of the system is called *static* if the state of the system does not change, i.e. the system is in equilibrium, but the movement is related to the static state of the object which is in equilibrium.

Mathematical description of static models does not include time as a variable and consists of algebraic or differential equations (in the case of objects with distributed variables). Static models are usually nonlinear. They accurately reflect the state of equilibrium caused by the object transition from one mode to another.

The *dynamic* model reflects the changing object state in time. The mathematical description of such models necessarily includes a derivative in time. Dynamic models use differential equations. Exact solutions of these equations are known only for some class of differential equations. More often it is necessary to use the numerical methods which are approximate.

For control purposes, the dynamic model is represented by the transfer function.

Linear and nonlinear models. The model is called *non linear*, if there are the non linear operations in the model operator expression, in other case it is called *linear*.

The interest to linear models primarily is explained by the fact that their behavior is described by linear differentiated equation for which the general and relatively simple solution methods are worked out. Besides, the following two properties of these models are important:

1) input influences of real objects can be presented in the way of weighted sum of respectively selected typical elementary influences of the same form. That is why in order to calculate the linear object response on any input influence it is necessary to have the response of this object on these typical influences.

In other words, the behavior of linear object at arbitrary input influences can be described not only with the help of differential equations but also with the help of characteristics that determines its response for typical influence (this response is called the *dynamic characteristics*).

The advantages of the systems mathematical description with the help of the dynamic characteristics becomes especially notable at the creation of the mathematical models of complex objects, derivation of differential equations of which is usually a very difficult problem. At the same time the dynamic characteristics can be received by fulfilling of relatively simple experiments on the object;

2) in a linear stochastic system that is under the influence of random non-controlled disturbances, the effect of these disturbances influence on the output variable can be taken into account in the form of additive random noise imposed directly on the deterministic component of the output variable.

Accordingly, the description of the object behavior in this case can be obtained through the ordinary apparatus of linear differential equations (or adequate apparatus of dynamic characteristics). Of course, in this case, it is necessary to have additional information about the probabilistic characteristics of random noise.

If the disturbance is accessible for inspection, then to obtain the oscillogram of the disturbance changing, it is sufficient to register the object output with zero input effects.

In other case it is often assumed that random disturbance has normal Gaussian distribution. The theoretical reason of the role of the normal probability is the *central theorem* (in probability theory).

It is not always possible to describe the behavior of the object by linear equation. That is why the approximation of non linear dependencies by linear expressions in the specific range of the arguments is used. The procedure for replacing the real functional dependence of the output variable from the input variable by approximate linear dependence is called the *linearization.* Linearization is usually fulfilled by decomposing nonlinearities in a Taylor series.

The models with concentrated (lumped) and distributed variables. If it is possible to neglect the object state coordinates spatial changing, the model is the object with *concentrated variables*.

If the basic variables of the object change both in time and in space (or just in space), the object model is a model with *distributed variables*.

In the object with the concentrated parameters in the steady state the controlled variable have the same values throughout the volume of the object. In the transition mode in any point of the object the character of the controlled values changing is the same at the same time. Such models are described by the ordinary differentiated equations.

In the objects with distributed variables control value varies at different points of the object (pressure of the substance during transport in the pipeline, the product temperature in heat exchangers along its length, etc.). These objects are described by differential equations in partial derivatives or ordinary differential equations in the case of stationary processes with one spatial coordinate.

The continuous and discrete in time models. Models describing the state of the objects with respect to time as a continuous argument are called *continuous* (in time). The variables of the continuous model can take arbitrary meanings at any point of time in a given changing range.

Discrete models are used to describe processes that are assumed to be discrete. In such models the variables are quantized by level and by time. Quantization by level corresponds to fixing discrete levels of variables at arbitrary points in time; the quantization by time corresponds to fixing discrete time points at which levels of the model variables can take arbitrary values. For dynamic control objects quantization in time is more commonly used.

Discrete-continuous models are used for the cases when the object has both discrete and continuous processes

The full name of the model may include a combination of the listed characteristics.

3 Lecture №3. Main operators of Control Objects models

The content of the lecture: main stages of the modeling process; operators of Control Objects models.

The goal of the lecture: to learn the main stages of modeling and the operators of Control Objects models.

3.1 The stages of modeling

In general case the process of modeling consists of the following stages:

1) The *description* of the modeling object.

At this modeling stage the system and its components are determined. For this purpose we study the structure of phenomena that constitute the actual process. In the result of this studying meaningful description of the process appears, in

which it is required, where possible, to clearly represent all the necessary dependencies.

From this description the *formulation* of the applied problems follows. The problem statement defines the objectives of the simulation, the list of desired quantities, and the required accuracy. And the statement may not have a strict mathematical formulation.

A meaningful description is the basis for creating a *formalized schema* that is intermediate between meaningful description and mathematical model. It is not always developed, and when direct transition from meaningful description to a mathematical model is impossible due to the complexity of the studied process. The information presentation can be also verbal, but there needs to be a precise mathematical formulation of the studying problem, the process characteristics, system parameters, dependencies between characteristics and parameters.

2) *The model selection*. At this stage the equations of a mathematical model are formed and necessary assumptions are raised.

Transformation of the formalized scheme into a mathematical model is carried out by mathematical methods without the inflow of additional information. At this stage all relationships are written in an *analytical form*, logical restrictions in the form of *inequalities*, the analytical form is given to all information.

When constructing the mathematical description the equations of different types are used: algebraic (stationary mode), ordinary differential equations (dynamic objects), and differential equations in partial derivatives are used for the mathematical description of the distributed objects dynamics. If the process has both deterministic and stochastic properties, we use the integral-differential equations.

3) *The analysis of the model*. The model equations are solved for the desired output variables. All actions are performed on the model and aimed directly to get information about this object for defining its development laws. An important advantage of modeling is having the opportunity to repeat many of the phenomena for different initial conditions and different character of their changes in time.

4) *The results interpretation.* At this stage, the question about transferring the obtained in the mathematical model values on the real object under study is considered. The researcher is interested in the qualities of the object that was replaced by the model. The possibility of the transferring such knowledge exists due to the presence of certain accordance and relationships between the model and the original process. These relationships are defined during the modeling process.

If necessary, a repeated systems analysis or synthesis is done.

The success of application of the mathematical modeling depends on the following: how successfully was the model built, the adequacy, the degree of studying the model, the conveniences of operating it.

Application of computers in mathematical modeling gives the opportunity to investigate, at any conditions, variation parameters and external factors indicators, including those not being implemented in natural experiments. So, the researcher

has the opportunity of obtaining answers to many questions arising at the stage of development and design of objects without using other, more complex methods.

3.2 Main operators of Control Objects models

The Control Object is able to accept external influences and respond to them by changing values of output variables. In Control Theory object description is expression of the relationship between object response, as a function of time, and input effects. As previously mentioned, this relationship in the general case can be represented by the following operator:

$$y(t) = A\{x(t)\}.$$

There are various ways of the Control Object model operator determination.

The most general form of operator representation is its determination by the differential equations system describing the object behavior. We consider only objects with lumped parameters, which are described by ordinary differential equations.

The order of the differential equations system describing the object model is not directly determined by the number of inputs and outputs, and depends on the operators that transform the input signals into the output signals.

The most universal model based on differential equations is described by the expression:

$$\sum_{i=0}^{p} a_i \cdot y^{(i)} = \sum_{j=0}^{l} b_j \cdot x^{(j)}, \qquad (3.1)$$

where p is the order of the model ($p > l$); $a_p=1;$

a_i and b_j are the constant coefficients (model parameters);

$y^{(i)} = \frac{d^i y}{dt^i}$, $x^{(i)} = \frac{d^j x}{dt^j}$ - the derivatives of output and input signals, respectively.

In order to describe the dynamics of the objects that are characterized by discrete values of input and output signals, i.e. which functioning is presented for the discrete time $t_k=kT$ (in this case T is the sampling interval), instead of differential equations the difference equation can be used. Denoting the discrete values of input and output signals, respectively:

$$x_{k-j} = x(k-j), y_{k-j} = y(k-j),$$

we write the difference equation (the analogue of differential equation) as

$$y_\kappa + a_1 \cdot y_{k-1} + ... + a_p \cdot y_{k-p} = b_1 \cdot x_k + b_2 \cdot x_{k-1} + b_3 \cdot x_{k-2} + ... + b_l \cdot x_{k-l+1}.$$

In the analysis of stochastic systems encountered in various fields of science and technology, the source data for the analysis are the realization of a random process generated by this system. Obtained in the form of graphs or oscillograms, realizations of a random process are processed and presented in the *time series*. Time series contains the ordinates of the realization of a random process taken at

discrete and equally spaced points in time. Consequently, we make conclusion about the properties of the original continuous system by the results of digital processing signals (time series) generated by the system. As a result, digital parametric stochastic models of auto-regression and moving average are widely spread. These models are quite simple and typically include a small number of parameters that need to be evaluated by observations. They can be used to study time series and determining the statistical characteristics of these series.

Below are a few common discrete object models for the time domain, taking into account the effect of noise observations (discrete time moments marked by the same symbol *t*, and continuous time, *t = 0, 1, 2...).*

The easiest description is considered *the autoregressive model (AR-model)*:

$$A(z)\, y(t) = e(t),$$

where

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{na} z^{-na}$$

and the operator $z^{-1} = e^{-pT}$ is a delay operator.

Here and below *e(t)* is discrete white noise (magnitude of error *e(k)* reflects the presence of measurement error and imprecision of model parameters estimates).

More complex – *ARX-model (AutoRegressive with eXternal input)*

$$A(z)\, y(t) = B(z)\, u(t) + e(t)$$

or in expanded form:

$$y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-n) = b_1 u(t) + b_2 u(t-1) + \ldots + b_{nb} u(t-m) + e(t),$$

where

$$B(z) = b_1 + b_2 z^{-1} + \ldots + b_{nb} z^{-nb+1}.$$

ARMAX-model has the form:

$$A(z)\, y(t) = B(z)\, u(t - nk) + C(z)\, e(t)$$

where *nk* is delay,

$$C(z) = 1 + c_1 z^{-1} + c_2 z^{-2} + \ldots + b_{nc} z^{-nc}.$$

All these equations are linear difference equations of the control object.

More fully the Control Object is described in terms of *state space*. The object state is a set of variables y_i, fully defining its position at this point in time. State variables (*phase coordinates*) form the vector of state variables, control and disturbance vectors form the control and the disturbance vectors. Many of these vectors constitute the state space (phase space), the space of controls and disturbances.

It should be noted that between the input, output and state vectors there is a fundamental difference. If all components of input and output vectors are specific physical quantities, the elements of the state vector can be some abstract variables, the physical nature of which is not always defined. As state variables of the object, usually chosen *n* coordinates (output signal *y(t)* and its *n-1* derivatives).

For the linear system we can write the equation (3.1) in the form of a differential equations system. By entering the designation:

$$y_1 = y,\ y_2 = y^{(1)},\ y_3 = y^{(2)},\ ...,\ y_k = y^{(k-1)},$$

after some transformations we have vector form of the model operator (3.1):

$$\frac{dY(t)}{dt} = A \cdot Y(t) + B \cdot X'(t),$$

where

$Y = (y_1, ..., y_p)$ –the state vector,

$X' = (x, x^{(1)}, ..., x^{(l)})$ – the disturbances vector,

A – square matrix of coefficients,

B – rectangular matrix of coefficients.

This system presents the equations for the state variables where *n* coordinates are selected as the variable of object state (output signal *y(t)* and *n-1* its derivatives).

So, the differential, difference and state space equations are the main (initial) operators of control objects' models in time domain.

Applying the Laplace transform to linear differential equations with zero initial conditions, we receive the *transfer functions* of the object. The transfer function for linear and linearized systems is another form of their mathematical models representation (in frequency domain).

There are the dynamic characteristics for control objects.

The transition (response on a step signal) and *impulse response* (response on the impulse signal) *functions* are the object dynamic characteristics. Knowing these functions, it is possible to obtain the transfer function of the object (we will consider such methods later). The dynamic characteristics of the control object can also be identified using the *frequency response* (amplitude and phase characteristics).

In contrast to methods based on solution of differential equations, the step response, impulse response and frequency response methods are not only calculated, but experimental. Between these dynamic characteristics of linear or linearized control object there is a definite relationships, so there is made no difference between them. And, as we said, they can be used to obtain object's transfer function and, therefore, the model equation (in differential equations or some others). Therefore, for control objects the dynamic characteristics can be also considered as object models.

3.3 Linearization of equations

The final goal of modeling the processes dynamics is the use of models in control systems for determining the dynamic characteristics, therefore, in any way, it is necessary to find a solution to the equations. That is why the differential and difference equations that form the mathematical model of the control object are derived based on various simplifying assumptions. Linear differential equations are solved relatively easily. However, it is not always possible to describe the behavior

of an object by a linear equation. That is why the approximation of nonlinear relations in a given range of arguments of linear equations is used. In other words, in a given range of input arguments the nonlinear equations are replaced by linear, they are *linearized*. In the linear model input and output signals are easily described using the transfer functions. This idealization significantly simplifies the process of modeling. For example, *linearization* of differential and difference equations leads to *linear dynamic* models, the mathematical apparatus of which is most fully developed. As a result of solutions of these equations the characteristics of the transition process, which depends on time and parameters of the object are received.

Nonlinear elements can often be linearized in assuming that signals deviations from their stationary mode are small values.

Let some element be characterized by input $x_1(t), x_2(t), ..., x_m(t)$, output *y(t)* values and the external action *f(t)*. The dynamic equation of the element can be written in common view:

$$F(x_1, x_2, ..., x_m, x_1^{'}, x_2^{'}..., x_m^{'}, ..., x_1^{(l)}, x_2^{(l)}..., x_m^{(l)}, y, y^{'}, ..., y^{(n)}) = \varphi(f, f^{'}, ..., f^{(l)}). \quad (3.2)$$

Suppose that the steady state is the following:

$$x_1 = x_1^0, x_2 = x_2^0, ..., x_n = x_n^0, y = y_0, f = f_0.$$

Then the steady state equation according to (3.21) will be

$$F(x_1^0, x_2^0, ..., x_m^0, 0, 0..., 0, y, 0, ..., 0) = \varphi(f^0, 0^{'}, ..., 0). \quad (3.3)$$

The basis of linearization of the nonlinear equations is the assumption that in the investigated dynamic process variables $x_1(t), x_2(t), ..., x_m(t), y(t)$ are changed so that their deviations from the steady state values $(x_1^0, x_2^0, ..., x_m^0, y_0)$ are sufficiently small. Let us denote these deviations through $\Delta x_1, \Delta x_2, ..., \Delta x_m, \Delta x_1^{'}, ..., \Delta x_m^{'}, \Delta y, \Delta y^{'}, ...$

Then in a dynamic process we have:

$$x_1(t) = x_1^0 + \Delta x_1(t), x_1^{'} = \Delta x_1^{'}, x_2(t) = x_2^0 + \Delta x_2(t), x_2^{'} = \Delta x_2^{'}, ...,$$

$$y(t) = y^0 + \Delta y(t), y^{'}(t) = \Delta y^{'}, ...).$$

The condition of sufficient smallness of dynamic variables deviations from some steady state values for the automatic control system is usually performed. This is required by the idea of a closed-loop automatic system.

External influence *f(t)* does not depend on the automatic system operation, its change can be arbitrary, and thus the right side of equation (3.2) is not usually linearized (though in some cases it can be also linearized).

The linearization is usually carried out by decomposing non-linear dependencies in a Taylor series in a neighborhood of the initial stationary mode with saving only the linear parts of the decomposition and subsequent subtraction of the static equations. Using this procedure, the model equations are obtained not with respect to its variables, but with respect to deviations of variables from the initial steady state.

The obtained differential equation like equation (3.21) describes the same dynamic process. The difference of this equation from the original:

- unknown time functions in linearized equation are not the same value $x_1(t), x_2(t), ..., x_m(t), y(t)$, and their deviations $\Delta x_1, \Delta x_2, ..., \Delta x_m, \Delta y$ from some steady state values $(x_1^0, x_2^0, ..., x_m^0, y_0)$;

- the resulting equation is linear with respect to deviations of variables $\Delta x_1, \Delta x_2, ..., \Delta x_m, \Delta x_1^{'}, ..., \Delta x_m^{'}, \Delta y, \Delta y^{'}, ...$;

- this equation is more approximate, because in the linerazing process the higher order members has been discarded.

Such an equation is called a differential equation in deviations.

We must always remember that a linear model of an object obtained by the linearization procedure is valid only for small deviations from the initial steady state.

4 Lecture №4. General principles of modeling of control objects

The contents of the lecture: two approaches to the construction of control objects models; main equations of the processes dynamics.

The goal of the lecture: to learn main laws used in the analytical method of the control objects modeling.

4.1 Two approaches to modeling of control objects.

Control Systems life cycle includes several periods, such as design, exploitation and modernization. A distinctive feature of Control Systems design process is its combination in time with the development and manufacture of technological units. This means that the only possibility of obtaining information about the properties of a technical system (which is not yet created) is an analytical description of the processes typical for elements of such a system. The involvement of theoretical propositions of physics (and sometimes chemistry) in the application to the specific characteristics of the studied object is the basis of such analytical methods.

The process of exploitation of the control system imposes its own conditions on the mathematical models. They are needed to get current (operational) information:

- about uncontrolled by measuring devices technological process coordinates;

- about the properties of some parts of technological processes changing in time under the influence of various factors.

Thus, there are two fundamentally different approaches to creating mathematical models of Control Objects.

The first approach is based on the choice of models using the main physical and chemical laws that determine the studied process. Such models are called *analytical models* of the process. That is, the fundamental laws of matter and energy

conservation are used for derivation of the model equations. The equations are derived on the basis of theoretical analysis of physical and chemical processes occurring in the object. The analytical method of derivation of the mathematical model which is identical (matching) with the characteristics of the investigated object are applicable when the physical and chemical processes occurring in the object are well studied.

An analytically derived equation of one object is applicable to describe the properties of other similar objects; it is the advantage of this method.

Disadvantage of the analytical method of generating the equations is the difficulty of analyzing and solving equations, the large complexity of deriving numerical values of the mathematical description parameters.

The second approach is based on the concept of "black box", i.e. it is postulated that the internal structure of the object is unknown, and should not interest the researcher. All information is obtained only as the result of the object observations in a passive or active experiment. In active methods the object under investigation is subjected to special external influences that cause changes in the output value. These changes are recorded and the obtained data are approximated by mathematical expressions. Passive techniques use information obtained as a result of normal operation of the object without special external influences on it. This information is processed by statistical methods. The models obtained by this way are called *empirical (experimental).*

The advantage of experimental methods is their simplicity and low complexity in a sufficiently precise description of the object properties in a narrow range of coordinates changes.

The main disadvantage of experimental methods is the inability to establish functional relationships between numerical parameters of equations and structural characteristics of the object, the process indicators and physical and chemical laws of studied processes. In addition, these models cannot be extrapolated to other similar objects

4.2 Main equations of the dynamics

Analytical models are cognitive models. The essential feature of these models is a reflection of the phenomena mechanism in the structure of the model operator, i.e. all causal relations to the object.

The construction of any mathematical model starts from physical description of the modeling object. Here are the elementary processes occurring in the object modeling, which should be reflected in the model with formulation of the main assumptions taken in their description. "Elementary" does not mean "simple", but only that these processes are components of more complex processes. Usually, "elementary" refers to the process relating to a specific class of phenomena, e.g. mass transfer, heat transfer, etc. Usually the following "elementary" processes are taken into account: the phases flow, mass transfer between phases, heat transfer, phase changes, chemical reactions.

The list of elementary processes taken into account defines the set of phenomena that describe the object to include in the mathematical model.

The main technology of the heat power industry is based on physical processes, whose elementary components are:

- mechanical processes (mechanical processing of hard materials; for example, crushing of solid fuel);
- hydrodynamic processes (transport of liquid and gas);
- thermal processes (heating and cooling);
- mass transfer processes (evaporation and condensation).

Patterns of occurrence of all these processes are closely connected with the conditions of working fluid motion in which they occur and determined by the laws of hydro-dynamics.

In addition, they are based (except the first one) on the elementary processes of matter and energy transporting between separate parts of the system. The laws of this transferring are studied in thermodynamics. (The studying of the mechanical processes is based on the laws of the theory of elasticity and solid mechanics). That is the general theoretical basis for modeling the most part of technological processes is hydro- and thermodynamics.

The state of the system is usually expressed by means of the laws of thermodynamics. The first law of thermodynamics has a unique mathematical and physical formulation: the change in time of the substance in an elementary volume is equal to the sum of the inflow and outflow of the substance through its surface. This law formulates the indestructibility of the matter and its motion and is written through the set of the laws of mass, energy and momentum conservation.

The mass conservation law. This is the basic law of classical mechanics: the mass of any part of material system which is in motion does not depend on time and is constant.

The motion conservation law: the speed of changing the *motion* of any part of a material system which is in movement equals the sum of external force.

The energy conservation law: when there is an infinitely small heat supply to an isolated system and making by this the system infinitely small work δA, the energy of the system changes by the amount $de = \delta Q - \delta A$.

All these laws have a mathematical representation (integral and differential equations).

These equations must be added by another one expressing the second law of thermodynamics, which describes the entropy increasing in the system in which the matter or energy moving occurs. They are viewed as laws *of mass and energy transferring.*

The system of equations must be supplemented by the *state equation.*

All mentioned equations completely describe the behavior of a dynamic system at any point in time. An important feature of the mathematical description that contains the ordinary differential equation is the need to specify the initial conditions. For partial differential equations along with the initial it is also

necessary to specify boundary conditions, which are functions of time in the general case.

4.3 The simplification of dynamic equations

The systems of dynamic equations, as a rule, are essentially nonlinear, and analytical solution of them in general form is impossible. That is why depending on the specific goals the simplifications aimed for the exception of certain relations are fulfilled. But the essential features of the process should be kept.

The most simple (in terms of mathematical solution) is the static problem. Time and coordinates derivatives are equal to 0, and the differential equations system is reduced to the algebraic system (stationary modes of objects with the concentrated parameters).

Next in complexity is the stationary problem. For the mathematical description of non-stationary modes of objects with the concentrated parameters, as well as stationary modes of objects with the distributed parameters for one spatial coordinate are typically used ordinary differential equations. In the first case, the independent coordinate is time, and in the second case - the spatial coordinate.

One of the ways to simplify the system of equations is reducing the number of space coordinates. After reduction of the space dimensionality, the system will be open loop, therefore it is necessary to use algebraic dependencies, which reflect the real three-dimensionality of the flow.

In many practical cases, the laws of real flow motion are found on the basis of experimental data. In the experimental studying some factors are identified, which reflect the real flow structure, that is, the distributions of velocity, temperature, density and other parameters. These are friction, heat transfer coefficients, the relative phases velocity in two-component mixtures, etc. They are all integral characteristics of the flow, which with a certain approximation, reflects the exchange of momentum, heat, matter, existing in the real flow.

The use of empirical coefficients and the mentioned dependences allows to refuse from considering the real three-dimensional flow, simplify some equations, and others exclude entirely. Such simplification is admissible, since empirical relations to some extent reflect the real three-dimensionality of the flow. As a result of these assumptions for determining of the dynamic characteristics it is possible to use a one-dimensional model, and sometimes - a model with lumped parameters.

In the models with the *concentrated* parameters all the system parameters do not depend on spatial coordinates and are only functions of time. Derivatives of the spatial coordinates are replaced by the ratio of the difference in function values between the input and output to full length of the channel. For the description of processes in such objects known conservation laws are usually written in the form of *balance equations* (mass, energy, motion).

5 Lecture №5. Analytical methods of modeling the objects with lumped parameters

The content of the lecture: the analytical approach of Control Objects modeling.

The goal of the lecture: to learn the use of general conservation laws at the examples.

In the models with the *concentrated* parameters all the system parameters do not depend on spatial coordinates and are only functions of time. Derivatives from the spatial coordinates are replaced by finite difference. For describing the processes in such objects, the conservation laws are usually written in the form of *balance equation*s (mass, energy and motion).

According to the material balance the change of substance mass in time in a closed space is equal to the algebraic sum of the input and output streams:

$$\frac{dG}{dt} = \sum_{i=1}^{k} D_i - \sum_{j=1}^{r} D_i,$$

here D_i *(i=1,k)* is a mass rate of the *i-th* input stream, D_j *(j=1,r)* is a mass rate of the *j-th* output stream, G is the mass of matter in the considered volume, and t is time.

Similarly, in accordance to the heat balance equation, the change of enthalpy of a body in time equals to the algebraic sum of heat streams, that input (or output) the heat to the considered body:

$$\frac{dI}{dt} = \sum_{i=1}^{k} Q_i - \sum_{j=1}^{r} Q_i,$$

where Q_i *(i=1,k)* is the *i-th* input heat stream, Q_j *(j=1,r)* is the *j-th* output stream of heat, I is the body enthalpy.

There is no possibility to give a complete theory of modeling all kinds of processes in their various manifestations. To illustrate the application of the fundamental mass, motion, energy conservation laws to the most characteristic processes let us consider some examples. These examples are considered for the simplest cases with neglecting a number of irrelevant factors.

In industrial facilities there are often found tanks which, depending on the characteristics of the inflow, outflow and design, possess certain dynamic properties.

Example 1. The studied object is a tank from which liquid flows in the atmosphere. In this case we have the independent input flow $G_{in}(t)$ and the dependent output flow $G_{out}(t)$.

The control variable is the liquid consumption G_{in} flowing into the tank, a controlled variable is the liquid level in the tank H, the external disturbance is liquid flow from the tank.

The flow at the output is determined by the liquid level H over the output hole and by the square of section area of output hole f_c.

The liquid amount in the tank above the output hole is determined from the relation:

$$\mathbf{M} = \mathbf{F} \cdot \mathbf{H} \cdot \rho,$$

where F is the square of the tank cross sectional area;

ρ is the density.

In accordance with the material balance equation of the system

$$\frac{dM(t)}{dt} = G_{in}(t) - G_{out}(t).$$

Then we can write the following dependence

$$F \cdot \rho \cdot \frac{dH(t)}{dt} = Q_{in} - Q_{out}. \tag{5.1}$$

Liquid flows in the atmosphere, so to determine the output flow we use the equation, reflecting the motion conservation law (the law of hydrodynamics):

$$G_{out} = \mu \cdot f_c \cdot \sqrt{2g\rho \cdot H}, \tag{5.2}$$

where μ is discharge coefficient;

g is free fall acceleration;

ρ is density

Considering F and ρ as constants, that is, $F=F_0$, $\rho = \rho_0$ and by substituting expression (5.2) into equation (5.1), we obtain the differential equation describing the object dynamic, *the dynamic model*:

$$\frac{dH(t)}{dt} = \frac{1}{F_0 \rho_0}\left[G_n(t) - \mu\sqrt{2g\rho_0} \cdot \sqrt{H(t)} \cdot f_c(t)\right]. \tag{5.3}$$

Consider the object behavior at equilibrium state; that is the coordinates of state (here H and G_{out}) do not change in time. It is easy to see that at the equilibrium state the amount of substance in the tank will not change, that is:

$$\frac{dM}{dt} = 0.$$

Therefore, the equation of material balance is expressed by the relation:

$$G_{in}(t) = G_{out}(t).$$

On the other hand, from a mathematical point of view the concept of equilibrium is expressed by the equality to zero of all derivatives of state coordinates:

And this, in turn, means that:

$$\frac{dG_{in}}{dt} = \frac{d^2 G_{in}}{dt} = \ldots = 0, \quad or \quad G_{in}(t) = G_{in}.$$

$$\frac{dG_{out}}{dt} = \frac{d^2 G_{out}}{dt} = \ldots = 0, \quad \Rightarrow G_{out}(t) = G_{out}.$$

In the end, we can write

$$G_{in} = G_{out} = const. \tag{5.4}$$

Substituting (5.4) into (5.2) we get the possibility to calculate the liquid level in the tank:

$$H = \frac{1}{2g\rho\mu^2} \cdot \frac{G_{in}^2}{f_{out}^2}. \tag{5.5}$$

Model (5.5) is a *static* model; the name is associated with a static state of an object in equilibrium.

Let's get the linearized dynamic model, assuming that the initial equilibrium state of the system corresponds to the time t_0.

To evaluate state coordinates values in the initial equilibrium state, we can use a static model. Then

$$G_{in_0} = G_{out_0} = G_0; \qquad H_0 = \frac{1}{2g\rho \cdot \mu^2} \cdot \frac{G_0^2}{fc_0^2}.$$

The main nonlinearity in the system equations is given in the expression (5.2).

Decompose it in a Taylor series, and keep only the linear members:

$$G_{out} \approx G_{out_0} + \left(\frac{\partial G_{out}}{\partial H}\right)_0 \cdot (H - H_0) + \left(\frac{\partial G_{out}}{\partial f_{out}}\right)_0 \cdot (f_{out} - f_{out}). \tag{5.6}$$

We introduce the following notations for the coefficients and identify them, differentiating (5.2):

$$k_1 = \left(\frac{\partial G_{out}}{\partial H}\right)_0 = \mu \cdot f_{out} \cdot \sqrt{\frac{g\rho_0}{2H_0}}; \qquad k_2 = \left(\frac{\partial G_{out}}{\partial f_{out}}\right)_0 = \mu \cdot \sqrt{2g\rho_0 \cdot H_0}.$$

Taking into account the initial equilibrium state, we will examine only the deviations of variables from their initial values, that is:

$$\Delta G_{in} = G_{in} - G_{in_0}; \; \Delta G_{out} = G_{out} - G_{out_0};$$
$$\Delta H = H - H_0, \; \Delta f_{out} = f_{out} - f_{out_0}.$$

Then

$$\Delta G_{out}(t) = k_1 \cdot \Delta H(t) + k_2 \cdot \Delta f_{out}(t).$$

It is the first equation of the *linearized* model.

Keep in mind that:

$$\frac{dH_0}{dt} = 0 \text{ and } G_{in_0} = G_{out_0},$$

substitute the linearization result (5.6) in (5.3) and get:

$$F_0\rho_0 \frac{d[\Delta H(t)]}{dt} = \Delta G_{in}(t) - k_1 \Delta H(t) - k_2 \Delta f_{out}(t) \cdot$$

If we denote:

$$x_1 = \Delta G_{in,}\ x_2 = \Delta f_{out},\ y_1 = \Delta G_{out},\ y_2 = \Delta H,$$

the linearized dynamic model of the tank with the liquid can be expressed by the following system of equations (as it is usually written in math):

$$T\frac{dy_2}{dt} + y_2 = k_3 \cdot x_1 - k_4 \cdot x_2,$$
$$y_1 = k_1 \cdot y_2 + k_2 \cdot x_2,$$

here

$$k_3 = \frac{1}{k_1}, k_4 = \frac{k_2}{k_1}, T = \frac{F_0 \cdot \rho_0}{k_1} \quad .$$

Equation coefficients k_i and T are parameters of the linearized models, depend on the equilibrium state prior to the transition process.

Therefore, the model requires the reconfiguration of its parameters if initial conditions (H_0, G_{c_0}) of the simulated object change.

If the characteristics of the object is essentially nonlinear it is necessary to use a piecewise linear approximation. In this case a nonlinear object is described by a set of linear relations and the logical relations that define the validity area of each linear expression.

Example 2. If there are two communicating tanks (figure 5.1) the object is described by a system of equations

$$S_1 \frac{dH_1}{dt} = Q_{in1} - Q_{out1} - Q_{12},$$
$$S_2 \frac{dH_2}{dt} = Q_{in2} - Q_{out2} + Q_{12}, \qquad (5.7)$$

where $Q_{12}=Q_{12}(H_1-H_2)$ is some, generally non-linear monotonic function, S_1, S_2 are the the cross section's areas of the tanks.

Equation (5.7) represents the mathematical description of the object, where each of the vectors of controllable variables and influences has two coordinates $\bar{u} = \{ Q_{oit1,}\ Q_{out2}\}$, $\bar{y} = \{ H_{1,}\ H_2\}$.

Depending on the presence of devices which control the flows Q_{c1} and Q_{c2}, influences vector $\{Q_{c1},Q_{c2}\}$ there can be controlled or uncontrolled disturbances.

Controlled coordinates H_1, H_2 can be adopted as the state vector of the object $\bar{x} = \{x_1, x_2\} = \bar{y}$.

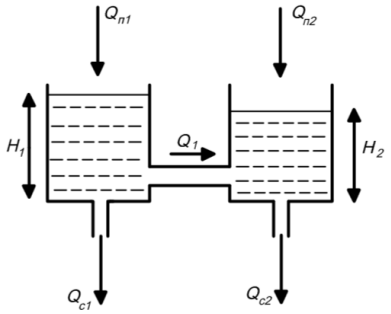

Figure 5.1 – System of two tanks

Example 3. Consider the system in the form of a cascade of two tanks (figure 5.2).

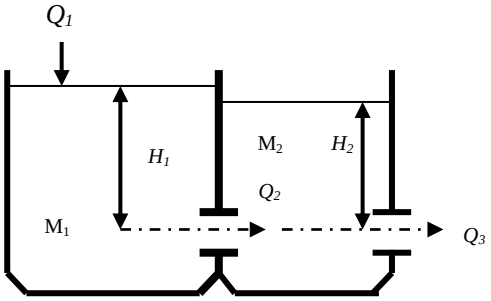

Figure 5.2 – Tanks cascade

The model is described by a system of equations:

$$\frac{dM_1}{dt} = Q_1 - Q_2, \qquad \frac{dM_2}{dt} = Q_2 - Q_3$$

In this case, together with the balance equations we have the following dependencies:

$$Q_3 = \mu_1 \cdot f_1 \cdot \sqrt{2 \cdot g \cdot \rho \cdot \mathrm{H}_2} = k_1 \cdot \sqrt{\mathrm{H}_2};$$

$$Q_2 = \mu_2 \cdot f_2 \cdot \sqrt{2 \cdot g \cdot \rho \cdot (\mathrm{H}_1 - \mathrm{H}_2)} = k_2 \cdot \sqrt{(\mathrm{H}_1 - \mathrm{H}_2)};$$

$$M_1 = \rho \cdot F_1 \cdot \mathrm{H}_1, \; M_2 = \rho \cdot F_2 \cdot \mathrm{H}_2.$$

These equations are the equations of the cascade of tanks.

However, it is often necessary to find the relationship between specific variables; in this case you must fulfill some transformation of this system. For example, to find the relation between the output Q_3 and the input of Q_1, after transformations we get a nonlinear second order equation. The equation order is determined by the number of tanks in the cascade.

6 Lecture №6. Modeling of a level control object

The content of the lecture: analytical approach to the modeling of control objects.

The goal of the lecture: to learn the examples of modeling of the technological object with lumped parameters.

To obtain the final product in the textile industry raw material is subjected to various processing on a variety of industries. One of the technological processes is the process of finishing fabrics, which includes the step of dyeing by various dyes.

One of the stages of technological process is the step of coloring the fabrics. The fabric moves along rollers of the coloring device and repeatedly passes through the coloring solution.

As control factors we can choose: temperature control in baths of dyeing machines and dryer units, control of solution level and concentration, humidity, tension, fabric width and shrinkage, speed modes, etc.

Disturbance factors include the large number of parameters, which characterize not only a specific technological operation but also the preceding stages of material processing. The factors are: physical and mechanical material properties (humidity, quality of the previous chemical treatment, the type of fiber, mixture and etc.), the solutions and colorants quality and stability, the material tension, thermal mode, etc.

Output coordinates of the multidimensional control object are indicators of technological process efficiency, which are determined by its productivity, products quality, materials and energy costs.

Let's consider a bath of the coloring machine, which is one of textile industry apparatuses, as another example of a level control object.

Fabric with initial dampness m_1 is supplied to the tub of the machine.

When leaving the machine, the fabric is squeezed to dampness of $m_2 > m_1$. Otherwise, the tub will overflow.

The *G kg/sec* dry cloth passes through a bath. The supporting colorant solution in the amount of Q_{in} *l/sec* flows into the bath through the control valve.

The fluid flow (consumption) is a function of the variables m_1, m_2 :

$$Q_{consumption} = f(G, m_1, m_2).$$

Here we use the following relation:

$$Q_{consumption} = \frac{G(m_2 - m_1)}{\rho}.$$

In steady state the inflow of the supporting solution is equal to the consumption flow $Q_{consumption}$ from the bath:

$$Q_{ino} - Q_{consumption0} = 0,$$

where

$$Q_{consumption0} = \frac{G_0(m_{20} - m_{10})}{\rho}, \qquad (6.1)$$

here ρ is the density of the solution, carried out by cloth;

the values which characterize the steady state are marked by the index "0".

In transient mode

- the change of inflow to the value of $Q_{n1} = Q_{n0} + \Delta Q_n$;

- or the change in consumption to a value of $Q_{p1} = Q_{p0} + \Delta Q_p$

in the bath will vary the amount of fluid and the volume of the liquid in the time dt will change to the amount:

$$d\mathrm{V} = S_0 \cdot dH,$$

where S_0 is the cross-sectional area of the bath corresponding to the predetermined liquid level H_0 (displacement of tissue in the bath are neglected).

For the time dt:

$$(Q_{n1} - Q_{p1})dt = S_0 \cdot dH \tag{6.2}$$

Divide equation (7.2) on *dt* and subtract from the equation (6.1), the result is:

$$S_0 \cdot \frac{dH}{dt} = \Delta Q_n - \Delta Q_p . \tag{6.3}$$

For transition to dimensionless variables we introduce the notations:

$\varphi = \dfrac{\Delta H}{H_0}$ is the relative change in the controlled variable, in this case level;

H_0 *is* nominal (or specified) value of controlled variable;

$\mu = \dfrac{\Delta Q_n}{Q_{n\max}}$ is the relative change in the inflow (regulated influence);

$f = \dfrac{\Delta Q_p}{Q_{n\max}}$ is the relative change in flow (disturbance).

When introducing these indications, the equation (6.3) will become as follow:

$$\frac{S_0 H_0}{Q_{n\max}} \cdot \frac{d\varphi}{dt} = \mu - f$$

or

$$T_a \frac{d\varphi}{dt} = \mu - f,$$

where $T_a = \dfrac{S_0 H_0}{Q_{n\max}}$ is time constant of object.

Thus, the bath as the object of solution level control has the properties of an integrating (astatic) unit to its input, in addition to the controlling influence μ the disturbing influence is applied:

$$f = \frac{\Delta Q_p}{Q_{n\max}} = \frac{1}{Q_{n\max}}[(\frac{\partial Q_p}{\partial G})_0 \Delta G + (\frac{\partial Q_p}{\partial m_1})_0 \Delta m_1 + (\frac{\partial Q_p}{\partial m_2})_0 m_2] =$$

$$= \frac{1}{Q_{n\max}}(\frac{m_{20} - m_{10}}{\rho}\Delta G - \frac{\Delta m_1}{\rho}G_0 + \frac{\Delta m_2}{\rho}G_0)$$

(6.4)

Maximum inflow

$$Q_{n\max} = \frac{G_{\max}(m_{2\max} - m_{1\max})}{\rho}$$

After substitution $Q_{n\max}$ into equation (6.4) and transformations we receive:

$$f = k_1 f_1 - k_2 f_2 + k_3 f_3,$$

where

$f_1 = \frac{\Delta G}{G_{\max}}$; $f_2 = \frac{\Delta m_1}{m_{2\max} - m_{1\max}}$; $f_3 = \frac{\Delta m_2}{m_{2\max} - m_{1\max}}$ are certain types of disturbances;

$k_1 = \frac{m_{20} - m_{10}}{m_{2\max} - m_{1\min}}$; $k_2 = k_3 = \frac{G_0}{G_{\max}}$ are the corresponding coefficients.

Thus, the disturbances are caused by a change in the speed of the fabric or its weigh, and by the changes of humidity values of incoming and outgoing fabric from the bath.

With these disturbances the equation of model has the form:

$$T_a \frac{d\varphi}{dt} = \mu - k_1 f, + k_2 f_2 - k_3 f_3.$$

The object transfer function in relation to the controlling influence:

$$W(p) = \frac{1}{T_0 p}.$$

The object transfer functions in relation to each of the disturbances:

$$W_1(p) = -\frac{k_1}{T_0 p};$$

$$W_2(p) = \frac{k_2}{T_0 p};$$

$$W_3(p) = -\frac{k_3}{T_0 p}.$$

7 Lecture №7. Modeling of heat exchange processes

The content of the lecture: the use of heat balance equations in analytical modeling of heat exchange processes.

The goal of the lecture: to learn the examples of modeling of the heat exchange process.

When we study the processes of heat transfer from one fluid to another through a wall, several basic stages can be distinguished: transfer of heat from the hot coolant to the cooler wall, the heat absorption by the wall material and its heating, heat distribution by volume of the wall, the heat transfer from the wall to the cold fluid.

If the process of heat exchange is stationary, the temperature at each point of the material (heat transfer and wall) does not change in time. The use of models with lumped parameters (i.e., when spatial coordinates are not included in the mathematical description) leads to the algebraic relations between the temperatures in the system. If the temperature varies over time, a mathematical description is obtained in the form of ordinary differential equations system (the argument is time).

The temperature dependence of the geometric coordinates leads to the mathematical description of the statics in the form of ordinary differential equations (if there is one spatial coordinate) or differential equations in partial derivatives. The independent variables are the spatial coordinates. Dynamic model in the presence of spatially distributed effects are described by partial differential equations, and one of the independent variables is time.

As noted earlier, in many practical cases, the laws of real flow motion are found on the basis of experimental data. The use of these dependencies allows to refuse from considering the real three-dimensional flow, simplify some equations, and others - exclude entirely.

Let's consider the heat exchanger with intensive mixing, in which a liquid Q_1 with a known temperature θ_1 receives. The average amount of substance in the heat exchanger V. From the heat exchanger the flow Q_2 with a temperature θ_2 flows.

The amount of substance in the heat exchanger is constant: the heat exchanger is hermetically closed, that is, the consumption of the substance is replenished by the input flow or, if the substance is ejected by some forces, the level in the heat exchanger is maintained by other apparatuses.

Since $V=const$, then $Q_1= Q_2= Q_{avg}$. The substance is intensively mixed, so the temperature in the volume θ and the temperature of the output stream is equal to θ_2, i.e. $\theta = \theta_2$.

We derive the equations of the heat exchanger model assuming that the input heat flow H_1 is the model input variable, the temperature of the output flow is the model output variable θ_2. The change of the input value H_1 may depend on the change in the flow Q_1, as well as on its temperature. In this regard, consider the following progressively more complex situations:

a) the heat exchanger is perfectly isolated, that is, there is no heat exchange with the environment.

When liquids are mixed intensively, we can consider the object as an object with lumped parameters and use the following balance equation:

$$\rho \cdot C \cdot V \cdot \frac{d\theta}{dt} = \sum H_i$$

where ρ is the material density, t/m^2;

C is the wall material heat capacity, Mcal/(т, grad);

H_i is the amount of heat supplied to the heat exchanger per unit time (positive value) or removed from it (negative value).

The input stream Q_1 corresponds to the supplied heat amount:

$$H_1 = \rho \cdot c \cdot Q_1.$$

With Q_2 flow, we lose the output heat amount:

$$H_2 = \rho \cdot c \cdot Q_2.$$

Substituting H_1 and H_2 in the heat balance equation, we obtain:

$$V \cdot \frac{d\theta}{dt} + Q_2 \cdot \theta_2 = Q_1 \cdot \theta_1;$$

$$V \cdot \frac{d\theta}{dt} + Q_2 \cdot \theta_2 = \frac{1}{\rho \cdot \mathrm{c}} \cdot H_1.$$

Because $\theta = \theta_2$, we get:

$$T \cdot \frac{d\theta_2}{dt} + \theta_2 = k_n \cdot H_1,$$

here $T = \frac{V}{Q_{cp}}$ is time constant, hour;

$k_n = \frac{1}{(\rho \cdot c \cdot Q_{cp})}$ is transfer coefficient, grad·hour/Mcal;

b) there is heat exchange with the environment by the law

$$H_3 = h \cdot S \cdot (\theta - \theta_c),$$

here h is heat transfer coefficient, Mcal/(hour, m^2, grad);

S is the heat exchanger surface, m^2;

θ_c is the environment temperature, ^{0}C.

In this case, the balance equation looks like:

$$\rho \cdot c \cdot V \cdot \frac{d\theta}{dt} = H_1 - H_2 - H_3 = H_1 - \rho \cdot c \cdot \theta_2 \cdot Q_2 - h \cdot S \cdot (\theta - \theta_c).$$

After conversion:

$$T \cdot \frac{d\theta}{dt} + \theta = k_n \cdot (H_1 + h \cdot S \cdot \theta_c).$$

here $T = \frac{V}{Q_{cp} \cdot (1 + \frac{h \cdot S}{\rho \cdot c \cdot Q_{cp}})}$, $k_n = \frac{1}{\rho \cdot c \cdot Q_{cp} + h \cdot S}$.

Compared with the case of a) the time constant *T*t decreased, since the value $\frac{h \cdot S}{\rho \cdot c \cdot Q_{cp}}$ is always positive.

If the ambient temperature is constant by moving the origin of the temperature coordinates to the $\underline{\theta_c}$ point, the term $h \cdot S \cdot (\theta - \theta_0)$ can be removed, otherwise θ_c can be considered as external action;

c) it is necessary to take into account the thickness of the walls of the heat exchanger, so it is necessary to take into account the heat capacity of the heat exchanger walls.

The heat flow from the volume *V* on the wall:

$$H_3^1 = h_M \cdot S \cdot (\theta - \theta_M),$$

here $h_м$ is the heat transfer coefficient from flow to the walls;

$\theta_м$ is wall temperature.

Heat flow from the walls to the environment:

$$H_4 = h_c \cdot S \cdot (\theta_M - \theta_{2,}),$$

here h_c is the heat transfer coefficient from the wall to the environment.

Since the walls of the heat exchanger have a heat capacity, we write down two equations (for the flow and for the walls):

$$\rho \cdot c \cdot V \cdot \frac{d\theta}{dt} = H_1 - H_3 - H_3^1 = H_1 - \rho \cdot c \cdot \theta_2 \cdot Q_2 - h_{м} \cdot S \cdot (\theta - \theta_M);$$

$$\rho_M \cdot c_M \cdot V_M \cdot \frac{d\theta_M}{dt} = H_3^1 - H_4 = h_M \cdot S \cdot (\theta - \theta_M) - h_c \cdot S \cdot (\theta_M - \theta_c).$$

Meaning that $\theta = \theta_2$, $Q_2 = Q_{cp}$, we express θ_i from the first equation and substitute this expression in the second equation.

After transformations we will receive:

$$\frac{\rho_M \cdot c_M \cdot V_M \cdot \rho \cdot c \cdot V}{h_M \cdot S} \cdot \frac{d^2\theta}{dt^2} + (\frac{\rho_M \cdot c_M \cdot V_M \cdot \rho \cdot c \cdot Q_{cp}}{h_M \cdot S} + \rho_M \cdot c_M \cdot V_M +$$

$$+\frac{(h_c+h_M)\cdot\rho\cdot c\cdot V}{h_M})\cdot\frac{d\theta}{dt}+\left(\frac{(h_C+h_M)\cdot\rho\cdot c\cdot Q_{cp}}{h_M}+h_C S\right)\cdot\theta=$$

$$=\frac{\rho_M\cdot c_M\cdot V_M}{h_M S}\cdot\frac{dH_1}{dt}+\left(1+\frac{h_C}{h_M}\right)\cdot H_1+h_C\cdot S\cdot\theta_C.$$

θ is output value, H and θ_c are the input variables.

We receive of the second order equation as there are two lumped objects: the heat exchanger and the walls.

8 Lecture №8 Modeling of an object with distributed parameters

The content of the lecture: analytical method of modeling an object with distributed parameters.

The goal of the lecture: to learn the example of modeling of an object with distributed parameters.

There are objects for which some state coordinates (process parameters) require orientation in geometric space. For example, the temperature of the flame at different points of the furnace is different. Models of such objects are the models with distributed parameters, and geometric coordinates $z = (z_1, z_2, z_3)$ must be used in the model operator.

The most important heat power engineering objects with distributed parameters include the heat exchangers with single-phase and two-phase coolant. In the analytical study of the dynamic properties of distributed heat exchangers, the flow of the working medium is usually considered one-dimensional, that is, the physical parameters of the medium on the cross section of the pipe are assumed to be constant. The changes of kinetic and potential energy of a moving environment are usually also neglected, because these values are small in comparison with the changes in heat energy that occur during the transient processes. Therefore, the basic equations for the working medium, which are accepted as the initial ones in the analytical study of distributed heat exchangers, are greatly simplified.

Consider an example. A section of the pipeline with a length L and a constant section f is given. Along this section an incompressible flow (liquid) moves, characterized by a flow rate G, a pressure p and a temperature θ relating to each of the pipeline sections. The pipeline is not isolated, i.e. the process is isothermal.

The studied object is distributed in length (z), the working liquid parameters are considered to be the same in cross section (one-dimensional object).

The basic equations:

- substance amount conservation equation:

$$\frac{\partial G}{\partial z}+f\cdot\frac{\partial\rho}{\partial t}=0; \qquad (8.1)$$

- motion amount conservation equation:

$$\rho \frac{\partial w}{\partial t} = -\frac{\partial w}{\partial z} - \frac{\partial p_{fr}}{\partial z}; \tag{8.2}$$

- the equation of the working substance state (for ρ) is presented in the form:

$$\rho = F(p, \theta). \tag{8.3}$$

This system of three equations is the original form of the pipeline model representation. However, its practical solution cannot be found even for the simplest boundary conditions. Analytical solution of the system is possible only under simplifying assumptions. Such an assumption may consist in the fact that the pipeline is divided into a number of elementary segments, each of which is a lumped object. The approximation to the real system is better if the size of the elements is less. The first approximation is obtained by replacing the system with one element and composing the equations of the lumped model. The best approximation is obtained by replacing the system by several elements.

We will consider the dynamics of pressures and consumption not in every section (that is, not for any z), but only the relationship of the input and output sections (this corresponds to the initial formulation of the problem). Then, instead of the original pipeline scheme, it is possible to consider its equivalent, characterized by the selecting of the concentrated volume $V = L \cdot f$, as well as the hydraulic resistance considered to the output section:

$$p_0 = p + \lambda_{fr} \frac{L}{f},$$

where p is the total coefficient of local resistance of the pipe;

λ_{fr} - the friction coefficient of the working medium on the pipe wall.

In addition, we neglect the acceleration of the flow caused by the dynamics of the pressure change along the length of the pipeline, i.e. we will take:

$$\frac{\partial w}{\partial t} = 0.$$

Then (8.2) will take the form:

$$\frac{\partial p}{\partial z} + \frac{\partial p_{fr}}{\partial z} = 0. \tag{8.4}$$

Since the entire mass of the working medium is concentrated in an equivalent volume V, under pressure p_1, instead (8.3) we accept:

$$\rho_1 = \rho(p_1, \theta). \tag{8.5}$$

The conservation equation (8.1) is converted with regard to the length of the pipeline:

$$f \cdot \frac{d\rho_1}{dt} = -\frac{(G_2 - G_1)}{L},$$

or

$$V \cdot \frac{d\rho_1}{dt} = G_1 - G_2. \tag{8.6}$$

Instead of (8.2) is similar to get

$$\frac{(p_2 - p_1)}{L} + \frac{p_{mp}}{L} = 0 \quad \text{or} \quad p_1 - p_2 = p_{mp}. \tag{8.7}$$

We use the known from hydraulics ratio, which determines the amount of friction losses:

$$p_{fr} = \frac{\lambda_{fr} \cdot G_2^2}{2 \cdot L \cdot \rho,}. \tag{8.8}$$

Hence, we express G_2 and using (8.6), we obtain the equation of the motion amount conservation:

$$G_2 = k \cdot \sqrt{\rho_1 \cdot (p_1 - p_2)}, \;\; k = L \cdot \sqrt{\frac{2}{\lambda_{fr}}}. \tag{8.9}$$

Thus, a system of three equations (8.5), (8.6), (8.9) is formed with respect to the six physical characteristics of the flow G_1, G_2, p_1, p_2, ρ_1, θ.

We will study the behavior of the object state coordinates G_2, p_1. Then the state equation (8.5) is used to exclude ρ_1:

$$V \cdot \frac{d(\rho_1(p_1, \theta))}{dt} = G_1 - G_2;$$

$$G_2 = k \cdot \sqrt{(\rho_1 \cdot (p_1 - p_2)}. \tag{8.10}$$

This system is a lumped nonlinear dynamic stationary model of the pipeline.

Let's assume that the compressed stream moves through the pipeline. The main assumptions adopted in the formation of the model in this case is the assumption that the properties of the compressible flow are close to the properties of the ideal gas. This gives grounds to use the well-known thermodynamics equations for determination of the properties of the compressible medium. In particular, for the isothermal process:

$$p = \rho \cdot R \cdot (273 + \theta).$$

Then in (9.6) instead of ρ_1, we can substitute a specific expression:

$$\rho_1 = \frac{p_1}{R \cdot (273 + \theta)}.$$

Now we consider a linearized model obtained by applying Taylor series to nonlinear relations. In particular, by writing down a system of equations in variable increments relative to the initial state marked by an additional index "0" for variables, we obtain:

$$V\frac{d(\Delta\rho_1)}{dt}=\Delta G_1-\Delta G_2;$$

$$\Delta G_2=\frac{k_0}{2}\cdot\sqrt{\frac{\rho_{1_0}}{p_{1_0}-p_{2_0}}}\cdot(\Delta p_1-\Delta p_2)+\frac{k_0}{2}\sqrt{\frac{p_{1_0}-p_{2_0}}{\rho_{1_0}}}\cdot\Delta\rho_1;$$

$$\Delta\rho_1=\left(\frac{\partial\rho}{\partial p}\right)_{\rho 10}\cdot\Delta \mathrm{P}_1+\left(\frac{\partial\rho}{\partial\theta}\right)_{\rho 10}\cdot\Delta\theta.$$

Substituting $\Delta\rho_1$ in the first two equations, we obtain:

$$\frac{d(\Delta\rho_1)}{dt}=a_1\cdot(\Delta G_1-\Delta G_2)-a_2\cdot\frac{d(\Delta\theta)}{dt};$$

$$\Delta G_2=a_3\cdot\Delta P_1-a_4\cdot\Delta P_2+a_5\cdot\Delta\theta,$$

where

$$a_1=\left[\left(\frac{\partial\rho}{\partial p}\right)_0\cdot V\right]^{-1};\quad a_2=\frac{\left(\frac{\partial\rho}{\partial\theta}\right)_0}{\left(\frac{\partial\rho}{\partial p}\right)_0};\quad a_3=\frac{k_0}{2}\cdot\left[\sqrt{\frac{\rho_{10}}{p_{10}-p_{20}}}+\sqrt{\frac{p_{10}-p_{20}}{\rho_{10}}}\left(\frac{\partial\rho}{\partial p}\right)_0\right];$$

$$a_4=\frac{k_0}{2}\cdot\sqrt{\frac{\rho_{10}}{p_{10}-p_{20}}};\quad a_5=\frac{k_0}{2}\cdot\sqrt{\frac{p_{10}-p_{20}}{\rho_{10}}}\cdot\left(\frac{\partial\rho}{\partial\theta}\right)_0.$$

If necessary, an equivalent model formed by a set of six "input-output" channels can be obtained. Determination of the characteristics of each channel can be performed in accordance with the rules of equivalence known in the theory of automatic control.

9 Lecture №9. Basic concepts of Identification theory

The content of the lecture: general approaches to the identification problem.
The goal of the lecture: learn basic definitions of Identification theory.

9.1 Basic information on identification of control objects

The previously discussed analytical methods use the laws of physics, mechanics, chemistry, etc. for description of the object. These models are the informative models. The essential feature of these models is a reflection of the

object or phenomenon mechanisms in the structure of the operator model, i.e., all causal relations to the object. When these relationships are not accounted, the cognitive aspect of the model is significantly affected, as for the cognition it is necessary to know not only how, but why.

This approach gives a positive result, if the object is rather simple in structure and is well studied.

Besides, functioning systems are subject to various external and internal disturbances, whereby their characteristics change. Therefore, it is almost impossible to create a sufficiently accurate mathematical model of a complex system only on the basis of theoretical studies of current physical processes. In this regard, if the object is insufficiently studied or is so complicated that the analytical description of mathematical model is impossible, the experimental methods are used.

Defining of the object characteristics using the results of input and output signals measuring is called *Identification*. This class of models is created with a single purpose – to use them to solve control problems. These models may not reflect the internal mechanisms of the phenomenon, which is necessary for cognitive models. They need only to ascertain the presence of certain formal relations between object inputs and outputs. The nature and characteristics of these relations are the basis of the model obtained in the identification process. So, identification studies the methods of creating the mathematical models of functioning systems using a priori and experimental information and is one of the main methods in the theory and practice of various physical natural complex control objects.

In general form identification problem is the problem of defining the object operator that transforms input signals to output signals. Mathematically, the correspondence between the input and output functions can be written in the form of an expression

$$y(t) = A\{u(t)\},$$

where A is the mathematical operator that is unknown beforehand and is to be determined;

$y(t)$ is the vector of object output coordinates;

$u(t)$ is the control vector (input);

u and y are respectively input and output vector variables of identifiable system which can be both deterministic and random. It is necessary keep in mind that the values of u and y are measured in the presence of random noise.

All experimental methods of studying the object dynamics are based on the processing of information contained in its input and output coordinates. There are two types of such information:

- *a priori* information is the information available before the observation of the object inputs and outputs. A priori information defines the structure of the identified object. For example, it is possible to identify four symptoms (although the structure is not limited to): dynamics, stochasticity, nonlinearity and discreteness.

Naturally, the opinion about the model can change after analyzing the posteriori information that is, after observing the behavior of the input and output of the object;

- *a posteriori* information has a quantitative nature, i.e. it is the result of observations object inputs and outputs. For continuous objects there are records of continuous functions (measurements of inputs and outputs during the observation period). In the discrete case, the minutes of observation are a measurement table.

In the identification problem it is required to estimate the unknown operator *A*. When solving this problem, it is assumed to conduct an appropriate *active* or *passive* experiment in order to accumulate the necessary data about the system, and then, using them and appropriate methods, to obtain explicitly the necessary information about the mathematical model of the system.

Active identification methods are characterized by the fact that the object inputs are served by predetermined test signals, and the output signal is examined.

In many cases, the disruption of the normal functioning of the object by artificial test signals is unacceptable. In these cases we use the *passive* identification methods, usually statistical, which are random, natural variations of the input signal. For effective use of these methods a large observation interval is required.

Depending on the amount of a priori information about the system the following types of identification are distinguished: *structural* identification, *parametric and non-parametric* identification.

The *structural identification* is the process of determination of the structure and type of the object operator, or, in other words, the type of mathematical model of the object. The determination of the model structure is one of the main problems of the identification theory. As a rule, the structure is postulated a priori with accuracy up to some set of unknown parameters. The decision is made on a given class of models-applicants. But any formal approaches and methods, allowing you to select the model structure based on the observable information of a plurality of object are absent. This is because some elements of the model are not amenable to mathematical interpretation.

In the case when the operator model is given by the accuracy of parameters vector, we have the problem of *parametric identification*. It is formulated like this: on the basis of experimental data to specify the values of the parameters vector for which the model is the best (or accurate enough) approximate object operator.

If the model operator contains unknown functions, identification in terms of such operators is called *nonparametric*.

The specific choice of a mathematical model depends on the type of the object. As mathematical models of technical systems the differential equations in ordinary and partial derivatives are applied. Moreover, when solving the control problems the preference is given to the models in state space and structural models described by differential equations in ordinary derivatives. As previously mentioned, any dynamic characteristics of the control object can be used as its model. Let us remember the control object dynamic characteristics: differential

equation (differential equation derived by analytical way, differential equation derived from transition functions), transfer function (transfer function obtained from the transition function, transfer functions derived from the differential equation, transfer function obtained from the experimental amplitude-phase characteristics), amplitude-phase characteristics (obtained from the experiments, obtained using Fourier transform, derived from transfer functions).

Now consider the classification of the identification problem. Classification of identification methods is carried out in different ways, depending on what are the main features of the basis, therefore any classification is relative. We consider the following classes:

a) classification of identification methods depending on the *class* to which belongs the studied system: continuous, discrete, linear non-linear, stationary, non-stationary.

We can reduce the model to make it easier. So, obviously, the behavior of a dynamic object can be described by the static model, if the dynamics of the object is not too pronounced; a nonlinear object can be approximated by linear, etc. Of course, the effectiveness of control based on this model will decrease. But if this reduction is small and the gain in identification is significant, then this choice should be considered optimal;

b) according to the *method of the object characteristics presentation* (in time or frequency domain): differential equations, difference equations, impulse response, transition function, frequency characteristics;

c) according to the method of information *processing:* statistical or non-statistical. When the object is stochastic, it is necessary to carry out series of experiments to obtain the average value of the indicators;

d) according to the *method of the experiment* on the object: active, passive and mixed identification methods.

The mathematical description should reflect regularities existing in the real object with the precision determined by the requirements of the control problem. The quality of control depends on it.

9.2 The identification object

Any real system is an object in which different variables interact at all kinds of system time and space scales and that produces observable signals. A graphical representation of a system, suitable for the system identification problem, is represented in Figure 9.1a. The system variables may be scalars or vectors, continuous or discrete functions of time. The sensor box, which will be considered as a static element, is added to emphasize the need of monitoring the systems to produce observable signals. In what follows, the sensor is considered to be a part of the dynamic system, so we can represent a system as in the Figure 9.1b. In Figure 9.1 the following system variables can be distinguished.

- *input u*: the input *u* is an exogenous, measurable signal. This signal can be manipulated directly by the user;

- *disturbance w*: the disturbance *w* is an exogenous, possibly measurable signal, which cannot be manipulated. It originates from the environment and directly affects the behavior of the system. If the disturbance is not measurable, it is considered as possibly structured uncertainty in the input *u* or in the relationship between *u* and *x*, and indicated as system noise;

- *state x*: the system state *x* summarizes all the effects of the past inputs *u* and disturbances *w* to the system. Generally the evolution of the states is described by differential or difference equations. Hence, the dynamic behavior of the system is affected by variations of the exogenous signals *u* and *w* and laws describing the internal mechanism of the system. In what follows, static systems, which do not show a dynamic behavior, are considered as special cases of dynamic systems and are simply described by algebraic relationships between *u, w*, and x;

- *disturbance v*: as *w*, the output disturbance *v* is an exogenous signal, which cannot be manipulated. It represents the uncertainty (noise) introduced by the sensor, and is generally indicated as sensor noise; at the figure 10b we unite the two types of disturbances;

- *output y*: the output *y* is the sensors output. It represents all the observable signals that are of interest to the user. In general, *y* is modeled as a function of the other signals. Since the sensor dynamics are ignored, the static relationship between *y* and *x, v* is expressed in terms of algebraic equations.

Indicated signals can be vectors of different sizes, as notice at Figure 10b.

Input and output signals of the object are sources of information when identifying dynamic objects. The object inputs are often stochastically functions of time, the statistical properties of which in general case are unknown.

Data about the disturbance *w*, as a rule, are absent. It is assumed that the structure of this random function, i.e. its nature is known. It usually refers to the case that *w* is a normal random process, direct observation of which is impossible.

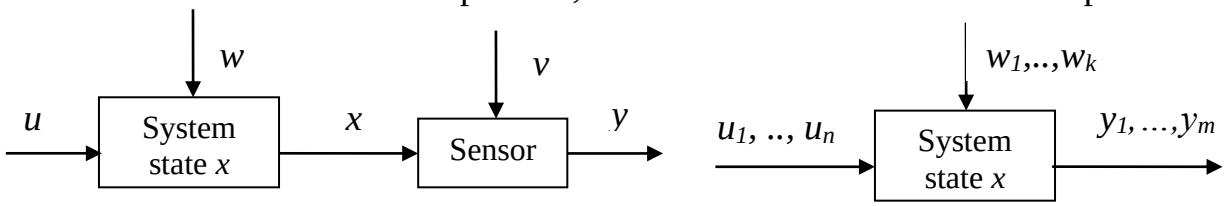

Figure 9.1 – Identification object

Without loss of generality we can put all these noises to the exit of the system (figure 9.2), where ỹ is the output signal "without noise".

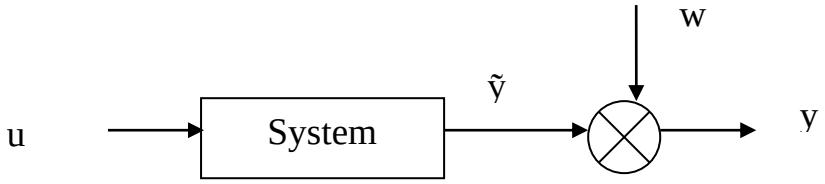

Figure 9.2 – Classical representation og dynamic system

Information about the object. We already said that a priori information defines the structure of the identified object. A posteriori information is the result of experiments. For continuous objects we have a record of continuous functions: the results of all measurements of object inputs $\overline{x(t)}$ and measurements of its outputs $\overline{y(t)}$ for the same observation period (the interval 0<=t<=T). The minute is written in the form: (<$\overline{x(t)}, \overline{y(t)}$>, 0<=t<=T). This means that the behavior of the object is recorded as $n+m$ different curves: $x_1(t),..., x_n(t); y_1(t),..., y_m(t)$ in this interval.

In the discrete case we write the minute $< \overline{x_1}, ..., \overline{x_N}); \overline{y_1}, ..., \overline{y_N}>$, or in the form (<$\overline{x_i}, \overline{y_i}$>, $i=1,..., N; j = 1,...,N$), where $\overline{x_i} = (x_{1i}, ..., \overline{x_{ni}}), y_i = (y_{1i}, ..., y_{mi})$. This minute is a table with $n+m$ columns and N rows:

x_{11}	x_{21}	...	x_{n1}	y_{11}	y_{21}	...	y_{m1}
x_{12}	x_{22}	...	x_{n2}	y_{12}	y_{22}	...	y_{m2}
x_{1N}	x_{2N}	...	x_{nN}	y_{1N}	y_{2N}	...	y_{mN}

9.2 The identification problem

We have the object. In the process of the normal functioning of this object we can simultaneously measure its input $x(t)$ and output $y(t)$ signals (these are in general a random function). Using the results of measurements of $x(t)$ and $y(t)$ it is required to build a model of the specified object, that is, to find the operator determining the relations between output $y(t)$ and input $x(t)$ functions. We define not the operator of the object, only its *approximate* value, and its *estimation*. That is, we are searching the model operator, which is in a sense close to the object operator.

Let the characteristic of the object is represented by the operator A_0, where output signal $y(t)$ corresponds to an arbitrary input signal $x(t)$:

$$y(t)=A^0\{x(t)\}.$$

The identification problem is to determine some estimation that is used as an approximation of the operator A^0, i.e.:

$$y_м(t) = A\{x(t)\}.$$

A^0 is object characteristics, A is model characteristics.

To speak about the correspondence between model and object is possible only if the estimation of the operator A is close in a way to its true value. "Proximity" is very relative, since the operators A and A^0 may have a different structure, can be formulated in different languages, have a different number of inputs. In this regard, it is natural to estimate the proximity of the operators by their response to the same input signals $x(t)$, that is, by the outputs $y(t)$ and $y_m(t)$.

The scheme of identification procedure is shown in the Figure 9.3.

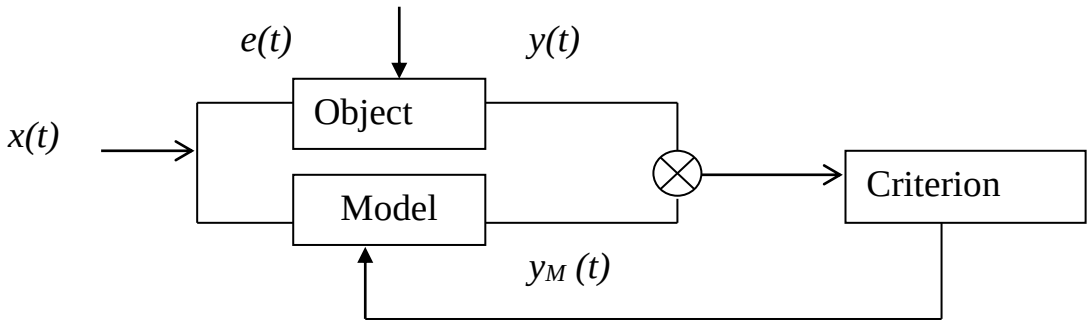

Figure 9.3 - The scheme of identification procedure

10 Lecture №10. Identification of linear dynamic objects. Direct methods

The content of the lecture: methods of identification of linear objects; identification by using special signals**.**

The goal of the lecture: study the methods of linear objects identification by using the transition function.

10.1 Main groups of methods of identification of linear objects

Determination of a dynamical model is very often impossible due to the complexity of the process, whose dynamics may be even (partially or completely) unknown. Even if we have a mathematical model, sometimes it is too complex to use for controller design (large state dimensions, nonlinearities, etc.). Model reduction is a way to simplify, and *linear models* are required.

Now we will consider the identification methods of objects that are linear or approximated with sufficient accuracy by linear models. The experimental methods for determining the dynamic characteristics of such an object can be classified in various ways. Most commonly there is accepted the division of all methods into the following three main groups:

- the first one includes the so-called *direct methods* that allow you to determine the sequence of discrete values of the operator. The following characteristics are defined by the methods of this group: amplitude $|W(j\omega)|$ and $\varphi(\omega)$ phase characteristics in the frequency domain; the impulse response *g(t)* and transient function *h(t)* in the time domain;

- *parametric identification* or *the methods of determining the parameters of a model with known structure* form a second group. In general, the estimation of the parameters of the model with given structure is carried out by minimizing the selected criterion. As a result of application of this group methods it is possible to define coefficients of the differential equation of linear one-dimensional object, and, therefore, and coefficients of a transfer function;

- the third group includes methods based on approximation of unknown dynamic characteristics of the object by analytical expressions, which are selected on the basis of available to the researcher a priori information about the object.

Statistical methods are used here, which allow using random natural signals of the identified object as a source of information.

In accordance with the considered above classification of experimental methods of Control Objects modeling let's consider the first group of methods - the *direct methods*. Specific test signals are used for identification by the direct methods. The first identification methods implemented in control systems were based on the use of step, impulse and frequency inputs. Most of these methods are limited to linear processes where the input / output relations received for one type of input signals are stored for all other types of input signals. They can also be used in linearized systems if signal levels are low.

10.2 Approximation of the object model by the typical dynamic elements

The idea of the method is the following. One of the three typical disturbance effects is supplied on the active object through the input: a step, an impulse, and harmonic oscillations of different frequencies. Step signal is most often used. The response of the object to such disturbances is a graph of the change in the time of the object output signal; it is called *an experimental acceleration curve*. Next, a special mathematical apparatus is used: a set of six typical dynamic elements. If we consider the object as a "black box", it turns out that control objects differencing by nature of the technological process, by the volume and configuration in dynamic mode are mathematically described in the form of the same model equation. In the automatic control theory were selected only 6 types of equations of the relationship of the object output signal with the input signal, which are called *typical dynamic elements.* Mostly typical equations of the relationship of typical dynamic elements are differential.

The method of using a set of typical dynamic elements is as follows: each typical dynamic element has its own *typical acceleration curve* and a number of other typical characteristics. The experimental acceleration curve obtained at the operating object is compared with a set of six typical acceleration curves of typical dynamic elements, and by coincidence of the nature of the change in time of the experimental and any typical acceleration curve, the approximation of the object under study is carried out by this typical dynamic element. Then the equation of this typical dynamic element becomes the equation of the object output and input signals relationship, or the desired mathematical model of the object. The values of the coefficients included in this typical dynamic element model equation are found by the object experimental acceleration curve.

10.3 Identification using transient function

The simplest input used for identification is a step signal. Such a signal at the input of the system can be formed, for example, by the sudden opening (or closing) of the input valve, switching on (or off) the control voltage or current, etc., as it is almost always possible without the use of special equipment. The ideal step signal has a rise time of zero, which is physically impossible, since the rise rate should be

infinitely high. Therefore, any real step input signal is only an approximation of the ideal step signal. However, if the signal rise time is much shorter than the higher harmonic period, the identification error becomes insignificant. In processes with disturbances or where measurements contain noise (which is usually in practice), appropriate noise filtering is required.

Identification with the transition function is carried out autonomously, outside the control process, and therefore is applicable only to stationary processes. However, since the step signals affect many systems during switching on or during normal operation, the transient functions can be recorded without disturbing the normal operation of the system. This is an additional advantage of this method. It is obvious that it is necessary to assume that the system is stationary, since the identification results are considered reliable after the application of the step signal. In addition, it is assumed that the system is linear in the amplitude range of the step signal.

In many cases, the transient function graphic is used to determine the transfer function of a system. This method is applicable to most types of linear systems (1 and 2 orders and to higher order periodic systems). The most correctly graphical identification method using transient functions is applied to the first-order processes. Consider this method.

The transition function graph is given. At time $t_0=0$ input value x is changed to the value a by jumping. It is necessary to write the equation of the object. The required equation for the first-order object has the form:

$$T \cdot \frac{dy}{dt} + y = k \cdot x \quad \text{or} \quad \frac{Y(p)}{X(p)} = \frac{k}{T \cdot p + 1}.$$

It is necessary to determine the parameters of the equation T and k. Consider several methods for defining these parameters:

a) analytical solution of the equation for initial conditions: $y=0$ for $t=0$ and $x=a$ for $t>0$ is written in the following form:

$$y(t) = k \cdot a \cdot \left[1 - e^{\frac{-t}{T}} \right].$$

Take two points on the graph (that's enough). Substituting the coordinates of these points in the solution expression, we obtain two equations for determining two unknown T and k:

$$y_1(t) = k \cdot a \cdot \left[1 - e^{\frac{-t_1}{T}} \right], \quad y_2(t) = k \cdot a \cdot \left[1 - e^{\frac{-t_2}{T}} \right].$$

However, these equations are transcendental, so it is very difficult to calculate T and k from them. In addition, the accuracy of the solution is small, since only two points of the graph were used;

b) to obtain a more accurate solution, divide the graph into y_1, y_2 , y_3,... ordinates with step Δt. Using a common solution for the resulting points it can be written:

$$y_0 = k \cdot a \cdot (1 - e^{-\frac{0}{T}}),\ y_1 = k \cdot a \cdot (1 - e^{\frac{-\Delta t}{T}}),\ y_2 = k \cdot a \cdot (1 - e^{\frac{-2\Delta t}{T}}),\ y_3 = k \cdot a \cdot (1 - e^{-\frac{3\Delta t}{T}})$$

and so on. We have

$$y_1 - y_0 = k \cdot a - k \cdot a \cdot e^{\frac{-\Delta t}{T}},\ y_2 - y_1 = k \cdot a \cdot e^{\frac{-\Delta t}{T}} - k \cdot a \cdot e^{\frac{-2 \cdot \Delta t}{T}},\quad y_3 - y_2 = k \cdot a \cdot e^{\frac{-2\Delta t}{T}} - k \cdot a \cdot e^{\frac{-3 \cdot \Delta t}{T}}$$

and so on

Denote $e^{\frac{-\Delta t}{T}} = q$, then

$$y_1 - y_0 = k \cdot a \cdot (1 - q),\ y_2 - y_1 = k \cdot a \cdot q \cdot (1 - q),\ y_3 - y_2 = k \cdot a \cdot q^2 \cdot (1 - q)$$

and so on.

We get

$$q_1 = \frac{y_2 - y_1}{y_1 - y_0};\quad q_2 = \frac{y_3 - y_2}{y_2 - y_1};\ q_3 = \frac{y_4 - y_3}{y_3 - y_2}$$ and so on.

The difference in q values is due to errors in experimental data and the determination of $y(t)$ values. After calculating the arithmetic mean $\bar{q}$ of the obtained q_i values, we can obtain a refined value of the time constant from the following expression:

$$\bar{T} = -\frac{\Delta t}{\ln \bar{q}}\ (\text{из } e^{\frac{-\Delta t}{T}} = \bar{q},\ -\frac{\Delta t}{T} = \ln \bar{q}).$$

Similarly, calculating

$$k_1 = \frac{y_1 - y_0}{a \cdot (1 - q)};\quad k_2 = \frac{y_2 - y_1}{a \cdot q_2 \cdot (1 - q_2)};\ k_3 = \frac{y_3 - y_1}{a \cdot q^2{}_3 \cdot (1 - q_3)}$$ so on,

we determine the updated mean value $\bar{k}$;

c) in practice, the following simpler method is used more often.

At $t \to \infty$ we have $y(t) = k \cdot a$, that is, through the ordinate of asymptotes (ordinate of asymptotes $b = k \cdot a$) we can determine k. Coefficient k is the ratio between the system output steady state and the amplitude of the input signal.

At $t = T$ the function

$$y\ (t) = b \cdot \left[1 - e^{\frac{-t}{T}}\right] = b \cdot\ (1\text{-}e^{-1}) = b \cdot\ (1\text{-}0.37) = 0.63 \cdot b.$$

That is, the time constant T of the first order system is equal to the period of time during which the transient function reaches 63% of its steady-state value. Noting in the graph 63% of the steady value of the transition process, determine the abscissa of this point (parameter T);

d) a constant T can also be defined as follows. Differentiate the common decision and $t \to 0$, then

$$y' = \frac{dy(t)}{dt} = k \cdot a \cdot \frac{1}{T} \cdot e^{-\frac{t}{T}}, \quad \frac{dy(0)}{dt} = k \cdot a \cdot \frac{1}{T} = \frac{b}{T} = tg\alpha$$

here α is the angle of the tangent to the graph of the function when $t = 0$. Then $T = \frac{b}{tg\alpha}$. Thus, the value T is the t-segment of the axis is equal to the distance from the origin to the point at which the tangent intersects with the asymptote (because y' is the tangent angular coefficient).

This solution is the simplest, however, not the most accurate, as it is very difficult to make an exact tangent and determine the exact ordinate *b* of asymptotes.

If the transition function is time-delayed, that is, 0 for a period of time after the application of the step signal, then the system has a purely time delay, for which the Laplace transform is $e^{-\tau s}$. Therefore, if the transient function of the system is equal to:

$$y(t) = \begin{cases} 0, t <= \tau \\ k \cdot (1 - e^{\frac{-(t-\tau)}{T}}), \ t > \tau \end{cases},$$

the transfer function of the system is obtained in the form of

$$W(s) = \frac{k \cdot e^{-\tau \cdot s}}{T \cdot s + 1};$$

e) graphic identification using the transition functions of the second order processes.

The second-order object is described by the equation

$$T_1 \cdot T_2 \cdot \frac{d^2 y}{dt} + (T_1 + T_2) \cdot \frac{dy}{dt} + y = k \cdot x.$$

The input signal is a step function. We compute the *T1, T2, k* constants provided that at $t>0$ the input signal is step signal $x = a=1$.

As in the previous case, first write down the General solution of the equation:

$$y(t) = C_1 \cdot e^{-\frac{t}{T_1}} + C_2 \cdot e^{-\frac{t}{T_2}} + k \cdot a$$

the sought particular solution has the form:

$$y(t) = k \cdot a \cdot (1 + \frac{T_1}{T_2 - T_1} \cdot e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2} \cdot e^{-\frac{t}{T_2}}).$$

Now, substituting the coordinates of the three points of the graph into this expression, we can get three equations for finding the desired values. These equations are transcendental, so it is very difficult to find a numerical solution. You can apply the method that was used for the first-order object again.

11 Lecture №11. Identification of the control objects using the direct methods

The content of the lecture: identification of the control objects using the specific test signals.

The goal of the lecture: learn methods of the identification of linear objects using the impulse transition function and frequency characteristics.

11.1 Graphic identification using the impulse transition function

Sometimes due to technological conditions it is not permitted to keep a single spike at the input of the object for a long time. Then there is given the ~~indignation~~ disturbance of the type of a single pulse, the duration of which is sufficient for a noticeable change of the output signal. Almost a single impulse is considered as two consecutive single jumps, only the first has an amplitude (+1) and the second (-1). At the output of the object we obtain the experimental impulse response – a graph of the object output signal changes in time.

As it is known,

$$W(p) = \frac{Y(p)}{X(p)},$$

where *Y(p) и X(p)* are the transformations of the input and output signals to frequency domain.

If the input signal is the single impulse, then Laplace transformation for him equals *1*: *X(p) = 1*. Then the Laplace transformation of output *Y(p) = W(p)* and impulse transient function *g(t)* of the linear system is identical to the reverse Laplace transformation of its transfer function:

$$y(t) = L^{-1}\{Y(p)\} = L^{-1}\{W(p)\} = g(t).$$

The first order systems can be described by the following transfer function:

$$W(p) = \frac{k}{T \cdot p + 1}.$$

Then the impulse transfer function is written as

$$g(t) = L^{-1}[W(s)] = \frac{k}{T} \cdot e^{-\frac{t}{T}}. \qquad (11.1)$$

At the object output we have a graph of the impulse transition function. When $t \to 0$, function (11.1) asymptotically approaching the *y* axis, and when $t \to \infty$ this function tends to zero, remaining positive. Taking this into account, from (11.1) at the starting point we can record:

$$y_0 = \frac{k}{T} \cdot e^{-\frac{0}{T}} = \frac{k}{T},$$

and at $t=T$

$$y = \frac{k}{T} \cdot e^{-\frac{T}{T}} = \frac{k}{T} \cdot e^{-1} = 0.37 \cdot \frac{k}{T},$$

that is, at time T, $g(t)$ reaches 0.37% of its steady-state value. From these expressions we define the required parameters of the transfer function.

The following method can be also applied. By the geometric meaning, the derivative of the function at some point is the slope of the tangent at that point. For the function

$$g(t) = L^{-1}[W(s)] = \frac{k}{T} \cdot e^{-\frac{t}{T}}$$

we have

$$\frac{dg}{dt} = -\frac{k}{T^2} \cdot e^{-\frac{t}{T}}. \qquad (11.2)$$

The equation of tangent is the equation of the line passing through the initial point $t=0$ with angular coefficient (11.2). The equation of this tangent line $l(t)$ (equation of a line with known angular coefficient, passing through a given point) has the form:

$$l(t) = \frac{k}{T} - \frac{k}{T^2} \cdot t.$$

And at $t = T$, $l(t) = \frac{k}{T} - \frac{k}{T^2} \cdot t = 0,$ that is, the constant T is the intersection point of the tangent at the starting point of the graph $g(t)$ with the time axis.

11.2 Identification using the frequency characteristic

Identification by using frequency characteristic is based on the application of sinusoidal signals and signals approximating a sine wave, the frequency of which varies in the considered interval.

If at the object input a sinusoidal exposure $A_0 \cdot \sin(\omega t)$ at different frequencies is applied, then the measured steady-state value of the output signal will be:

$$y(t) = A_1 \cdot \sin[\omega t + \varphi(\omega)] + n(t),$$

where $n(t)$ is the measurement error.

The frequency characteristics $W(j\omega)$ is determined by applying sinusoidal input signals $A_0 \cdot \sin(\omega t)$ at different frequencies ω and recording the corresponding output signals $A_1 \cdot \sin[\omega t + \varphi]$. For the purpose of obtaining the required frequency characteristic, the magnitude $\frac{A_1}{A_0}$ and φ are determined for each of the considered frequency ω. That is, the records of input and output signals determine the

amplitude ratio at the frequency ω_i and receive a $|W(j\omega_i)|$. Phase shift $\varphi(\omega_i)$ is derived from the comparison of the positions of maxima of the curves *x(t)* and *y(t)*.

As a result of the conducted experiments, by measuring the input and output signals, and then determining, as described above, the amplitude $A(\omega)$ and phase $\varphi(\omega)$ characteristics of the object, we can record:

$$P(\omega_i) = A(\omega_i) \cdot \cos\varphi(\omega_i),$$
$$Q(\omega_i) = A(\omega_i) \cdot \sin(\omega_i)$$

for each considered frequency, where $P(\omega i)$, $Q(\omega i)$ are the coefficients, respectively, of real and imaginary parts of complex transfer functions.

Let's remember that the structural parameters of the model (in this case, the order of the equation) are determined at the stage of structural identification. We are set by some (alleged) order of the equation. For definiteness, let's suppose that it is the object of the third order. Then

$$W(p) = \frac{Y(p)}{X(p)} = \frac{b_1 \cdot p^2 + b_2 \cdot p + b_3}{p^3 + a_1 \cdot p^2 + a_2 \cdot p + a_3}.$$

We need to determine the coefficients of the transfer functions a_i, b_j. Let's replace p on $j\omega$ and write the transfer function as a sum of real and imaginary parts:

$$W(j \cdot \omega) = \frac{-b_1 \cdot \omega^2 + j \cdot b_2 \cdot \omega + b_3}{-j \cdot \omega^3 - a_1 \cdot \omega^2 + j \cdot a_2 \cdot \omega + a_3} = P(\omega) + j \cdot Q(\omega).$$

From this

$$-b_1 \cdot \omega^2 + j \cdot b_2 \cdot \omega + b_3 = (-j \cdot \omega^3 - a_1 \cdot \omega^2 + j \cdot a_2 \cdot \omega + a_3) \cdot [P(\omega) + j \cdot Q(\omega)].$$

Equating the coefficients of the imaginary and real parts of these complex expressions, we get the system of equations which is valid for all values of ω. Substituting in these equations the different values of frequencies ω_i and the corresponding $P(\omega i)$, $Q(\omega i)$, we obtain the system of algebraic equations to determine the unknown coefficients of the transfer function. To clarify the values of the coefficients, the calculations are repeated several times on different frequencies, and an average of these calculations is taken.

If the order of the object is higher than the supposed one, then in repeated calculations the values of the coefficients will significantly differ from the first ones. That is that strong difference of the coefficients indicates that the order of the object is lowered (but does not indicate the error of the experiment).

12 Lecture №12. Parametric identification

The content of the lecture: set of the task and the methods of parameter identification of linear dynamic objects.

The goal of the lecture: to learn the procedure of minimization of the residual functional for the linear dynamic model.

12.1 Problem formulation

Parametric identification is the methods of finding of the model with *known* structure parameters using *measured data*. We assume that the structure and the order of the model of the object are already known.

We define not the operator of the object, only its approximate value, its assessment. That is, we build the model operator, which in a sense close to the object operator. The proximity of the operators is assessed on their reaction to the same input signals $x(t)$, that is, at the outputs of the object $y(t)$ and the model $y_м(t)$ according to some accepted criterion. In this case it is required to work out such a plan of observation over x and y and find such method of computing estimates, which would let receive in some sense the best estimation of the unknown parameters of the model.

In the general case we use the function $\rho(y,y_м)$, which depends on y, $y_м$, and does not depend on A and is called the *loss function* or the *residual function* (the discrepancy function). This function has the following properties:

1) $\rho(y,y_м)>=0$ for any $y,y_м$.

2) $\rho(y,y_м) = 0$ then, and only then, when $y=y_м$.

3) $\rho(y,y_м)$ is continuous and convex that is, this function always lies above a line segment joining any two points y,y_m.

A common measure of proximity in the whole observation interval may be the following functional:

$$Q = \int_0^T \rho(y, y_м)dt\,.$$

The functional Q is called the *residual* one, this functional depends on the operator A.

If by the physical sense of the problem the importance of information at different points in time vary, it is advisable to introduce a weighting function $h(t)>0$ with natural regulation:

$$\int_0^T h(t)dt = const,$$

then

$$Q = \int_0^T \rho(y, y_M)h(t)dt.$$

The choice of the function $h(t)$ is determined by the value of the information.

For the discrete case, the functional Q is written by this way:

$$Q = \sum_{i=1}^{N} \rho(y_i, A)h_i,$$

where $h_i > 0$ $(i=1,\ldots,N, \sum h_i = N)$ is weight of information at the time i.

Thus, the degree of *residual* (degree of discrepancy) of the model and the object operators can be expressed in the form of the functional that depends explicitly on the model operator A.

It is natural to build the identification process in such a form so it minimizes the residual, i.e. to solve the problem of the functional Q minimization on operator A:

$$Q(A) \to \min_{A \in \Omega}.$$

We minimize this functionality by varying operator A not arbitrarily, and in a certain class of operators' Ω. The result is the operator A^* (not necessarily unique) with the property:

$$Q^* = Q(A^*) = \min_{A \in \Omega} Q(A),$$

that is, the discrepancy in this operator is minimal.

The using of a minimization procedure for solving the problem of parametric identification is important.

The choice of the parameters estimation criterion depends on the interference nature, i.e. their statistical properties. In a normal distribution of noise the highest accuracy gives the mean-square criterion. The normal distribution is used most often to describe the properties of various random variables. A theoretical justification of the role of the normal distribution is the *central limit theorem*. According to this theorem, in most situations, when independent random variables are added, their properly normalized sum tends toward a normal distribution even if the original variables themselves are not normally distributed. This does not mean that any random variable, unless otherwise proven, is the subject to this distribution. The theorem is a key concept in probability theory because it implies that probabilistic and statistical methods that work for normal distributions can be applicable to many problems involving other types of distributions. Normal distribution has the advantage that it has convenient mathematical properties. Therefore, most statistical methods are built on the assumption that the investigated value submits to normal distribution, although in practice this assumption always requires special inspections.

12.2 Dynamic deterministic model identification

In the one-dimensional case, the relationship between the input $x = x(t)$ and output $y = y(t)$ variables is an ordinary differential equation:

$$\sum_{i=0}^{p} a_i \cdot y^{(i)} = \sum_{j=0}^{l} b_j \cdot x^{(j)}, \qquad (12.1)$$

where $y^{(i)} = \frac{d^i y}{dt^i}$, $x^{(i)} = \frac{d^j x}{dt^j}$, $a_p = 1$, $l \le p$

together with the initial conditions for $\frac{d^i y}{dt^i}$, $i = 0,1,...,n-1$.

The model is determined by $(p+l+1)$ parameters $c = (a_0,...,a_{p-1},b_0,...,b_l)$.

To describe the dynamics of the objects that are characterized by discrete values of input and output signals instead of differential equations, one can use difference equations. Denoting the discrete values of the input and output signals respectively $x_{k-j} = x[(k-j)]$ and $y_{k-j} = y[(k-j)]$, the difference equation (analogue of the differential) can be written in the form:

$$y_\kappa + a_1 y_{k-1} + ... + a_p y_{k-p} = b_1 x_k + b_2 x_{k-1} + b_3 x_{k-2} + ... + b_l x_{kl+1}. \qquad (12.2)$$

It is also necessary to specify initial conditions.

Structural parameters of the model are p and l, which must be selected in the process of structural identification.

The initial information for constructing the identification procedure is structure of identifiable models and observations $\{x_t, y_t\}$ in the interval $[0,T]$. We need to determine the coefficients of the equations a_i, b_j.

In general, at the substitution of observations in the model equation (12.1) (or (12.2)) the equality in this equation is not fulfilled. It is necessary to choose a_i, b_j in such a way that in equation (12.1) (or (12.2)) the right and the left sides differ in the smallest way.

Let's construct the residual function in the form of the mean squared difference of the right and left side of equation (12.1) after the substitution there of the functions x_t, y_t – the observations of the object:

$$Q(c) = \int_o^T [\sum_{i=0}^{p} a_i \cdot y_t^{(i)} - \sum_{j=0}^{l} b_j \cdot x_t^j]^2 dt .$$

(12.3)

We minimize this functional by a_i *u* b_j:

$$Q(c) \to \min.$$

The minimization result is a vector c^*; *it* gives the values of the identifiable parameters.

The problem of the functional (12.3) minimization are solved by the usual methods of finding the minimum of a smooth function. Equating to zero the derivative of (12.3) for all unknown parameters (since the function is smooth), we get the system of linear equations whose solution is the solution of the minimization problem:

$$\begin{cases} \frac{\partial Q(c)}{\partial a_i} = 0, i = 0,..., p-1, \\ \frac{\partial Q(c)}{\partial b_j} = 0, j = 0,..., l. \end{cases}$$

After obvious transformations we get the system of linear algebraic equations:

$$\begin{cases} \sum_{i=0}^{p} a_i \mu_{ik} - \sum_{j=0}^{l} b_j \lambda_{jk} = 0, \ k = 0,..., p-1, \\ \sum_{i=0}^{p} a_i \lambda_{qi} - \sum_{j=0}^{l} b_j \xi_{iq} = 0, \ q = 0,..., l, \end{cases} \tag{12.4}$$

where

$$a_p = 1, \ \mu_{ik} = \int_0^T y_t^{(i)} \cdot y_t^k dt; \ \lambda_{jk} = \int_0^T x_t^{(j)} \cdot y_t^k dt; \ \xi_{iq} = \int_0^T x_t^{(i)} \cdot x_t^{qk} dt.$$

The resulting system can be solved by standard computational methods. However, to calculate the coefficients of this system we need to know the derivatives of the input x_t and output y_t signals of the object. It is simply, if these signals are given in an analytical form. Otherwise, it is necessary to choose a method of obtaining derivatives; it depends on specific conditions and must be determined each time specially:

1. We can use numerical differentiation, that is, to use the estimation of the derivatives:

$$\dot{z}(t) \approx \frac{z(t) - z(t - \Delta t)}{\Delta t}; \ \ddot{z}(t) = \frac{\dot{z}(t) - \dot{z}(t - \Delta t)}{\Delta t},..., \ z^{(i)} = \frac{z_t^{(i-1)} - z_{t-\Delta t}^{(i-1)}}{\Delta t},$$

where Δt is the interval characterizing a accuracy of estimations.

This method has two disadvantages. Firstly, the accuracy of the estimation of the derivatives decreases rapidly with increasing the order of derivatives. Practically a good rate derivative above second order is not possible. Secondly, to estimate the *i-th* derivative at $t=0$ we must have the value of the function at $t<0$, that is $z(-\Delta t), z(-2\Delta t), ..., z(-i\Delta t)$. Since these values are not in measurements, we have to take in (12.3) the limits of integration not $[0, T]$, and $[i \bullet \Delta t, t]$, where i is the number of the function maximum derivative.

2. We can use the decomposition in series by a given system of functions (due to the presence of the operation of the smoothing error in the definition of derivative will be much less).

In order for the identification problem has a solution, it is necessary that the determinant of the system (12.4) was not equal to 0. If this condition is not met, it is necessary to take a different implementation of the object inputs and outputs, or to reduce the number of identifiable parameters (i.e. p and l), that is to reduce the model order.

13 Lecture №13. Nonparametric identification of linear dynamic objects

The content of the lecture: the problem of nonparametric identification of linear dynamic objects.

The goal of the lecture: to study the methods of solution of nonparametric identification problem.

13.1 The general approach to the definition of the nonparametric model

Earlier the focus was on methods that directly use the specific object responses, in particular the step, impulse and sine-wave responses. The methods of determining the dynamic characteristics of the investigated object using artificial test signals and measuring the response is often inapplicable for the following reasons:

- undesirability or impossibility to submit to the object's input special test signals, as this leads to the disruption of the normal way of the processes in the object;

- very often the uncontrolled disturbances are superimposed on these signals, so it is impossible to determine the dynamic characteristics by the typical input signals.

In many applications noise is clearly present. Now methods that are less sensitive to noise, and thus very useful under practical circumstances, are presented. It is the method based on statistical representations. The statistical method allows using the random natural signals of identifiable object as the source of information. This approach is used for nonparametric identification.

If the model operator contains unknown functions that are to be defined in the process of identification procedure, the identification is called *nonparametric*. We will characterize the nonparametric model by the impulse response. This is due to the fact that the specificity of a linear dynamic object is uniquely identified by its response to a single impulse input.

In the stationary case (considered by us) the impulse transition function depends only on one variable – time:

$$g = g(t), \quad 0 <= t < \infty.$$

The input–output relationships with single input and single output (SISO) can be represented in the following general form:

$$y(t) = \int_0^{\infty} g(\theta) \cdot x(t-\theta) d\theta, \qquad (13.1)$$

which is also indicated as the *impulse response* model representation.

The output *y(t)* is presented in terms of the convolution integral of *g(t)* and *x(t)*. Therefore these models are also called *convolution models.*

This model is formed the basis of classical data-based or nonparametric identification methods. The adjectives "data-based" and "nonparametric" express the very limited prior knowledge used in the identification procedure; the prior knowledge is limited to assumptions with respect to linearity and time-invariance of the system under consideration.

The weight function of stable systems has the following obvious property:

$limg(t) = 0$ at $t \to \infty$.

Physically this means that a stable system after pulsed effect always returns to its original state. Therefore, there is no need to integrate in (13.1) to infinity, enough to some T for which

$$|g(t)| \le \alpha \cdot g_{\max} \text{ at } t>T,$$

that is, the weight function, starting from the moment *T*, does not exit the 100α-percent corridor (usually $\alpha = 0.05$).

Now the convolution integral can be written in the following form:

$$y(t) = \int_0^T g(\theta) \cdot x(t-\theta) d\theta \qquad (13.2)$$

Let's consider the equation (13.2) as the supposed model, which gives the possibility to determine the desired dynamic characteristic of the object: pulse transfer function.

As we mentioned above, the results are sensitive to noise as raw input–output data sets are used. Therefore, the correlation methods have been developed to overcome this noise sensitivity.

In order to arrive at these correlation methods, let us first introduce the *autocorrelation function* $R_{xx}(\tau, t)$ of a signal *x(t):*

$$R_{xx}(\tau, t) = M\{x(t)u(t+\tau)\},$$

where τ is the lag time.

The notation *M{.}* stands for the expectation operator, or in other words, it signifies the mean value of the particular function.

According to e*rgodic hypothesis*: for a stationary random process *an average by the set* equal to *the average by time.* In what follows, this expectation will always be interpreted as the time average:

$$R_{xx}(\tau) = \lim \frac{1}{2T} \int_{-T}^{T} x(t)\, x(t+\tau) dt.$$

Notice that this function is now only a function of lag τ and not of *t*. Hence, it includes some time-invariance or stationarity property. The integral is taken over the interval $[-T,T]$ with $T \to \infty$, because at this stage transient responses will be excluded. This function is even function: $R_{xx}(\tau) = R_{xx}(-\tau)$.

In addition to the autocorrelation function, the *cross-correlation function* $R_{xy}(\tau, t)$ between two different signals *[(t)* and *y(t)* is introduced and is defined as:

$$R_{xx}(\tau, t) = M\{x(t)y(t+\tau)\}.$$

Similarly,

$$R_{xy}(\tau) = \lim \frac{1}{2T} \int_{-T}^{T} x(t)\, y(t+\tau) dt.$$

Although the cross-correlation function also exists for negative lags, it is not an even function. Notice that for negative lags, the correlation between inputs at time instant *i* and outputs at *i + l*, with *l* < 0, is calculated. These correlations are seldom of interest, because in causal systems the output does not depend on future inputs. Hence, for practical interpretation, only the function values for positive lags are of interest.

In future we will use the following simplified formulas:

$$\hat{R}_{xx} \approx \frac{1}{T}\int_0^T x(t)\cdot x(t+\tau)dt,$$
$$\hat{R}_{xy} \approx \frac{1}{T}\int_0^T y(t)\cdot x(t+\tau)dt. \qquad (13.3)$$

It is also important to note here that both the auto- and cross-correlation functions are important in the data-based identification of LTI systems, because they are closely related to the unit-pulse response of the system.

13.2 Wiener-Hopf equation

Determination of the impulse transient function directly from the convolution equation is undesirable due to the following reasons: integral equations of the form (13.1) are *poorly conditioned* equations. In addition, as a result of measurements, the values of random processes at the input and output of the object are obtained with large errors that need to be smoothed. To increase the quality of restoration of impulse transfer function there is necessary the pre-processing working out of the signals. To minimize the effect of noise the correlation functions of the signals are used.

We have an expression (13.2), but *y(t)* is related to *x(t)* by equations (13.3):

$$R_{yx}(\tau) \approx \frac{1}{T}\cdot\int_0^T y(t)\cdot x(t+\tau)dt = \frac{1}{T}\cdot\int x(t+\tau)dt\cdot\int_0^T g(\theta)x(t-\theta)d\theta =$$

$$= \int_0^T g(\theta)d\theta\cdot\frac{1}{T}\cdot\int_0^T x(t+\tau)\cdot x(t-\theta)dt = \int_0^T R_{xx}(\tau-\theta)\cdot g(\theta)d\theta.$$

Hence

$$R_{yx}(\tau) = \int_0^\infty g(\theta)\cdot R_{xx}(\tau-\theta)d\theta. \qquad (13.4)$$

This equation is called the *Wiener-Hopf equation*. It can be interpreted as equation (13.2) if $R_{xx}(t)$ is considered as input action and $Ryx(t)$ as response.

Identification is reduced to the solution of the equation (13.4) in the interval *[0,T]*. But since *g(t)* is a fading function, that is, *lim g (t) = 0 at* $t\to\infty$, , then, starting from a certain point in time *Tg*, its values are uninformative. Usually *Tg* is determined prior to identification. For example, you can determine the time T_R at

which $|R(\tau)| <= 0.05 \cdot R_{max}$. Here T_R is different for $R_{xx}(t)$ and $R_{yx}(t)$. But since we are interested in the dynamic properties of the object, and they are reflected in $R_{yx}(t)$, this value is determined by $R_{yx}(t)$.

Taking into account the above, the basic equation of statistical identification takes the form of:

$$R_{xy}(\theta) = \int_0^{T_g} g(\tau) R_{xx}(\tau - \theta) d\tau. \qquad (13.5)$$

Thus, the problem of dynamic characteristics determination is divided into the following stages:

1) Recording of random processes on the input and output of the object.

2) Calculation of the correlation functions of the input signal and the cross-correlation function of the input and output signals.

3) The definition of the parameter T_R.

4) Solution of the integral equation (8.2.7).

Thus, the problem of determining the impulse transient function is presented as the problem the Wiener-Hopf equation solution. Next, we consider methods for solving the Wiener-Hopf equation.

14 Lecture №14. Impulse Response Identification using the Wiener–Hopf equation

The content of the lecture: the methods of determining the smoothed impulse transition function

The goal of the lecture: learn the methods of solving the problem of nonparametric identification.

To determine the dynamic characteristics of the object - impulse transition function $g(t)$ as the intended model we consider the convolution equation:

$$y(t) = \int_0^T g(\theta) \cdot x(t - \theta) d\theta, \qquad (14.1)$$

where $x(t)$ and $y(t)$ are the input and output signals of the object.

14.1 Numerical method of Wiener-Hopf equation solution

One of the methods for solving the equation (14.1 is algebraic. We present (14.1) as a system of linear algebraic equations. To do this, move on to discrete time, we write the integral in the form of a sum. The interval is divided into m equal intervals $t, 2t, \ldots, mt$ and the integral is written approximately as a sum:

$$\sum_{t=0}^{m} x(kt) \cdot R_{xx}(\tau - kt) \cdot t \approx R_{yx}(\tau),\ t, \tau = 0,1,\ldots, m. \qquad (14.2)$$

Denoting $g_i = g(it)$ and assuming $\tau = t, 2t, \ldots, mt$ we will get the system:

$$R_x \cdot G = Q, \qquad (14.3)$$

where G = $[g_1, g_2, \ldots, g_m]^T$, Q= $[q_1, q_2 \ldots, q_m]^T$, $q_i = \frac{Ryx(\tau y)}{\Delta}$,

R_x *is* a square symmetric matrix *mxm:*

$$R_x = \begin{vmatrix} R_x(0) & R(t) & R_x(2t) & \ldots & R[(N-1)\cdot t] \\ R(t) & R_x(0) & R(t) & \ldots & R[(N-2)\cdot t] \\ R_x(2t) & R(t) & R_x(0) & \ldots & R[(N-3)\cdot t] \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ R[(N-1)\cdot t] & R[(N-2)\cdot t] & R[(N-3)\cdot t] & \ldots & R_x(0) \end{vmatrix}.$$

Thus, we obtain a system of *m* linear equations for determining *m* values of the ordinates of the weight function *g(t)* at points *t, 2t,..., mt*.

In real objects usually *g (0) = 0*, so in the right parts of the equations (14.3) the first components can be removed. In addition, to preserve the diagonal symmetry of the system determinant, it is often $R_{yx}(\tau)$ shifted by one step, that is, the system (14.3) is considered not from the first row, but from the second. If the interval Δ is small enough, the error will be insignificant.

14.2 Using approximation procedure in the problem of nonparametric identification

The Wiener-Hopf equation is poorly-conditioned: even small variations in the initial values of correlation functions lead to large variations of solutions. In addition, instead of the true values of correlation functions, their estimates were used. Therefore, the solution of the Wiener-Hopf equation is obtained with large errors. Although even obtained impulse transient functions have a small root mean square error close to a minimum, these functions do not correspond to the physical sense of the processes in the object. The physical meaning have a smooth solution.

We consider several methods for the use of approximating polynomials in solving the problem of nonparametric identification.

1. The original model is the Wiener-Hopf equation (14.3). Solving this equation by the algebraic method, we determine the discrete values $g_0, g_1, \ldots, g_m$ of impulse transition function.

The resulting sequence of discrete values can be presented using any approximating polynomial:

$$g(\tau) = \sum_{k=0}^{N} a_k \cdot \varphi_k(\tau), \qquad (14.4)$$

.

where $\{\varphi_k(\tau)\}$ is any system of approximating orthogonal functions.

In the case of orthogonal approximating functions, the approximation coefficients are determined according to the formula:

$$a_k = \sum_{\tau=0}^{m} g(\tau) \cdot \varphi_k(\tau), \qquad (14.5)$$

The question arises about the choice of the degree *N* of the approximating polynomial. This issue is very complicated and is currently not solved until the end. In this problem there are the following approaches:

a) in some cases the nature of the impulse response function is known. This allows to determine the degree *N* of the approximating polynomial (14.3) taking into account the specific form of the approximating functions;

b) one approach to the determination of the approximating polynomial degree of *N* is based on the following principle. The value of *N* is found using minimization of functional:

$$\Omega_N = \frac{\sum_{i=0}^{m} (\hat{R}_{yx}(it) - R_{yx}(it))^2}{N},$$

where $\hat{R}_{yx}$ is the result of the substitution of the polynomial (14.4) in the system of equations (14.3) instead of impulse function *g(t).* That is, the variance R_{yx} is minimized.

2. We apply a general approach to the construction of an object's linear model: the method of minimizing the mean square criterion. The identification criterion can be written in the form:

$$Q = \int_0^T [y(t) - y^*(t)]^2 dt \to \min.$$

Here *y(t)* is the object output signal, *y*(t)* is the model output signal, which is determined from the convolution equation:

$$y^*(t) = \int_0^T g^*(\theta) \cdot x(t-\theta) d\theta. \qquad (14.6)$$

We use the procedure of functions approximation. The approximating functions are usually chosen in the class of orthogonal functions.

We will approximate the impulse transition function with a polynomial:

$$g(\tau) = \sum_{k=0}^{N} a_k \cdot \varphi_k(\tau), \qquad (14.7)$$

where $\{\varphi_k(\tau)\}$ is the system of the approximating orthogonal functions;

N is the polynomial order.

The coefficients a_k of the decomposition (14.3) is still unknown.

Substituting in equation (14.6) the expression for the impulse transition function (14.6), we have:

$$y^*(t) = \int_0^T g^*(\theta) \cdot x(t-\theta) d\theta = \int_0^T \sum_{k=0}^{N} a_k \cdot \varphi_k(\theta) \cdot x(t-\theta) d\theta = \sum_{k=0}^{N} a_k \int_0^T \varphi_k(\theta) \cdot x(t-\theta) d\theta.$$

Then the criteria of identification will get the form:

$$Q = \int_0^T [y(t) - y^*(t)]^2 dt = \int_0^T [y(t) - \sum_{k=0}^{N} a_k \int_0^T \varphi_k(\theta) x(t-\theta) d\theta]^2 dt \to \min .$$

To solve the problem of minimization it is necessary to determine the derivatives $\frac{\partial Q}{\partial a_i}$ with respect to all unknown parameters. The best choice is for $\frac{\partial Q}{\partial a_i} = 0$. In the end, we get the system of linear algebraic equations for determining unknown coefficients of the decomposition (14.7). As a rule, in practice the value of $N<<m$, i.e. the received system has a low order and well conditioned due to the smoothness of the system of functions $\{\varphi_k(\tau)\}$.

However, the problem of choosing the degree N of the approximating polynomial remains.

3. The original model is the Wiener-Hopf equation:

$$R_{yx}(\tau) = \int_0^T g(t) \cdot R_{xx}(\tau - \theta) d\theta. \qquad (14.8)$$

Impulse transition function is approximated by the expression (14.7), where the unknown coefficients of the approximation are defined as:

$$a_k = \sum_{\tau=0}^{m} g(\tau) \cdot \varphi_k(\tau),$$

where m is the number of points for which the interval of modeling is divided.

We present the cross-correlation functions of the input and output signals of the object using approximating polynomials on the same system of functions:

$$R_{yx}(\tau) = \sum_{j=0}^{N} b_j \cdot \varphi_j(\tau),\ R_{xx}(\tau) = \sum_{j=0}^{N} c_j \cdot \varphi_j(\tau),$$

in these expressions the coefficients

$$b_j = \sum_{t=0}^{m} R_{yx}(t) \cdot \varphi_j(t),\ c_j = \sum_{t=0}^{m} R_{xx}(t) \cdot \varphi_j(t)$$

can be considered as given, since the values of correlation functions at the nodes are given and orthogonal functions $\{\varphi(\tau)\}$ are known.

We substitute approximating polynomials in the Wiener-Hopf equation. Due to the orthogonality of the functions $\{\varphi_j(\tau)\}$ we obtain a system of linear algebraic equations for determining the unknown decomposition coefficients a_k, which also has a low order and is well conditioned by the smoothness $\{\varphi_j(\tau)\}$. This makes it possible to obtain sufficiently accurate estimates of pulse transient functions by simple calculations.

In the process of solving, there are difficulties associated with the choice of the number of approximating functions, which were mentioned earlier.

4. Input and output signals of the object are approximated on the interval observation of a linear combination of some functions. The desired pulse transition function also approximated the same system function. For example, suppose that signals are distributed into series of orthogonal polynomials *{pi(t)}:*

$$\begin{cases} x(t) \approx \sum_{i=0}^{n} a_i p_i(t), \; y(t) \approx \sum_{i=0}^{m} b_i p_i(t), \end{cases}$$

where

$$p_i(t) = \sum_{j=0}^{i} p_j^i \cdot t^j.$$

So, input and output signals of the object are presented in the form of power series:

$$x(t) = \sum_{i=0}^{n} x_i \cdot \frac{t^i}{i!}, \quad y(t) = \sum_{i=0}^{n} y_i \cdot \frac{t^i}{i!},$$

here x_i y_i are known coefficients (since we know the dimension of the input and output signals):

$$x_i = i! \cdot \sum_{j=1}^{n} a_j \cdot p_j^i, \quad y_i = i! \sum_{j=1}^{n} b_j \cdot p_j^i$$

Impulse transition function also to be found in the form of a power series:

$$g(t) = \sum_{i=0}^{n} g_i \cdot \frac{t^i}{i!},$$

where the coefficients of the series g_i are required to be determined.

Substituting these expressions in the convolution integral, we get the system of equations:

$$\sum_{i=0}^{n} y_i \cdot \frac{t^i}{i!} = \int_0^T \sum_{i=0}^{n} x_i \cdot \frac{\tau^i}{i!} \sum_{j=0}^{n} g_i \cdot \frac{(t-\tau)^j}{j!} d\tau,$$

and equating the coefficients at equal degrees of variables we get the system of algebraic equations for determining unknown coefficients of series g_i:

$$g_0 = \frac{y_1}{x_0}, g_1 = \frac{y_2 - x_1 \cdot g_0}{x_0}, \; g_2 = \frac{y_3 - (x_1 \cdot g_1 - x_2 \cdot y_0)}{x_0}, \ldots,$$

$$g_i = -\frac{y_{i+1} - \sum_{j=1}^{i} \sum_{e=j}^{0} x_i \cdot g_e}{x_0}$$

15 Lecture №15. Identification of nonlinear objects

The content of the lecture: the characteristics and methods of identification of nonlinear objects.

The goal of the lecture: to learn the methods of identification of nonlinear objects.

The identification of nonlinear dynamic objects is a very complex problem, since there are an infinite variety of types of nonlinear operators describing the objects. In addition, the transition process of the non-linear object depends not only on the shape but also on the amplitude of the incoming signal and presents complex and conflicting requirements to the selection of a test signal with active identification.

As in the linear case, the identification problem consists in approximating the behavior of the object with such operator *F(x)*, which would minimize the residual functional. Usually *F(x)* can be represented as $F(x, \bar{c})$, i.e. in the form of a known operator with unknown parameters $\bar{c}$, which must be estimated in the identification process.

Thus, the following problem is solved:

$$Q(\bar{c}) = \int_0^T [F(x(t), \bar{c}) - y(t)]^2 dt \to min,$$

for continuous case, and

$$Q(\bar{c}) = \sum_{i=1}^{N} [F(x_i, \bar{c} - y_i]^2 \to min,$$

for the discrete case.

The solution is the values of the unknown parameters $\bar{c}$ vector.

There are different ways to represent the function $F(x, \bar{c})$: in the form of piecewise-linear or linear functions, in the form of an expansion in a given system of functions, etc.

The model in its simplest one-dimensional case is expressed by the nonlinear differential equation:

$$y^{(p)} = f(y^{(p-1)}, ..., y, x^{(l)}, ..., x),$$

where f is a nonlinear scalar function of $p+l+1$argument, which must be identified by observations $< x_t, y_t >, 0 \le t \le T$.

In the vector form this equation has the form:

$$\dot{y} = F(y, x'),$$

where $y = (y_1, ..., y_p)$, $x' = (x, x^{(l)}, ..., x^{(l)})$ are the vector functions of vector arguments.

The observations $< x_t, y_t >$ we reduce to

$$x_t^{'} = (x_t, x_t^{'}, ..., x_t^{(l)}), y_t = (y_t, y_t^{'}, ..., y_t^{(P-1)})$$

(by differentiating the original functions and using the smoothing procedure).

For practical application we recommend the following methods.

1. The *functional models*. Let *F* is a known function with unknown parameters. In this case, the system of equations:

$$\dot{y} = F(y, x', \bar{c})$$

is integrated numerically (for example by Runge-Kutta methods) at the given initial conditions and fixed values of the identifiable parameters $\bar{c}(c_1,\ldots, c_k)$.

The obtained solution $y = y(t,\bar{c})$ is compared with observations y_t and the the residuals function is obtained:

$$Q(c) = \int_0^T [y(t,\bar{c}) - y_t]^2 \, dt,$$

the minimization of which solves the problem of identification.

If the structure of the model is selected in the class of differentiable functions, then this problem is solved by the system of transcendental equations (which is also not an easy problem):

$$\frac{\partial Q}{\partial C_j} = 2 \cdot [F - y_t, \frac{\partial F}{\partial C_j}] = 0, \; j = 1,\ldots, k,$$

where [,] is the scalar product.

Otherwise you can use the search methods of minimization, which are implemented with recursive expressions that define the transition from (*i-1*) approximation to the *i*-th:

$$c_i = c_{i-1} + \Delta c_i,$$

where step Δc_i depends on c_{i-1} and the searching algorithm.

The most commonly used algorithm is the *gradient method*:

$$c_i = c_{i-1} - \alpha \cdot grad q_i^2(c_{i-1}),$$

where q_i is the discrepancy on the *i*-th step when the values of the parameters in (*i-1*)-th step.

To realize the search we need only the value of the function *F* at different c_j, so it is possible to create a model not only in the class of analytic descriptions (in this regard, this approach is called *functional*).

2. *Models, linear with respect to the estimated parameters*. They are the special case of functional models and are formed in the result of the decomposition of the desired function for a given system of functions:

$$F(x,\bar{c}) = \sum_{j=1}^{k} c_j \varphi_j(x),$$

where $\varphi_j(x) = \{\varphi_1(x),..., \varphi_k(x)\}$ is the given system of functions, which is determined at the structural identification stage. Approximation can be done, for example, with the help of polynomials.

The task of finding the expansion coefficients is solved by known methods.

3. The *methods of linearization*, that is, the approximation of nonlinear relations in a given range of arguments by linear expression. This is the most developed group of identification methods of nonlinear systems. The basis of linearization of the nonlinear equations is the assumption that in the investigated dynamic process the variables are changed so that their deviations from the steady-state values are sufficiently small. The condition of sufficient smallness of the dynamic variables of deviations from some steady-state values for the automatic control system is usually performed. The linearization is performed by decomposing nonlinearities in a Taylor series in the neighborhood of the initial stationary mode saving only the linear parts of the decomposition and subsequent subtraction of the statics equations. With the help of this procedure, there are obtained the equations of the model not in regard to its variables, but in regard to deviations of variables from the initial steady state.

4. *Piecewise-linear models*. In this case, there determined the *supporting* sequence $(x_i, y_i), j = 1, \dots, d$ to define the values of functions. Then the task (1) is solved, where *F* is the piecewise linear function given by its base sequence. This task is solved by one of the search methods.

5. *Identification of nonlinear functions of a priori known types*.

If there is a priori information about the type of nonlinearity, the parameters of nonlinear functions can be identified with the use of change of variables in the initial analytical expression or by transforming the initial expressions to more simple relations.

For example, we should to identify the parameters of a following nonlinear model:

$$y = a \cdot e^{b \cdot x}.$$

Let's take a logarithm of this expression:

$$\lg y = \lg a + bx.$$

Denote: $lgy=Y$, $x=X$, $lga=A$,

then get the linear model:

$$Y=A+b \cdot X.$$

The parameters of this linear model are determined by one of the considered methods. Now, applying the potentiating procedure, we will get the parameters of the initial nonlinear model.

In all cases, identification can be performed *only* in the assumption of some specific type of nonlinear approximating functions, the parameters of which are subject to identification.

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Content

Composite plan 2018, pos.220

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Abstracts of lectures for the specialty 5B070200 - Automation and Control

Editor L.Ya. Korobeinikova, Senior teacher Dept. LS

Specialist of standardization N.K. Moldabekova

Signed for printing __.__.__.
Circulation 50 copies
Volume 4.1 e.-p. sheets

Format 60x84 1/16
Printing paper №1
Order . Price 2100 tg.

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