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ALMATY UNIVERSITY OF
POWER ENGINEERING
& TELECOMMUNICATIONS**

Department of
Space Engineering

PHYSICS

Lecture notes for students of specialty 5B071800 – Electric power engineering

Almaty 2019

COMPOSERS: M. Karsybayev, L. Mazhitova, A. Salamatina. PHYSICS. Lecture notes for students of specialty 5B071800 - Electric Power Engineering. - Almaty: AUPET, 2019 - 99 p.

A brief summary of the lectures on the discipline "Physics" for the electric power specialties of the bachelor degree is given.

The lecture summary "Physics" is another element of the system of methodological support of the educational process in discipline and can be used as a handout in lecture classes, as well as in independent work on theoretical material in preparation for practical, laboratory studies and examinations.

Figures - 38, tables – 1, bibliography 4.

Reviewer: senior teacher, Master of Science E.S. Kim

Printed according to the Publishing plan of the Non-profit Joint-stock Company "Gumarbek Daukeev Almaty University of Power Engineering and Telecommunications" for 2019 y.

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1 Lecture №1. Introduction. Kinematics

Objective of the lecture: familiarize with the main objectives of the physics and the subject of physical science; introduce basic kinematic concepts and quantities.

1.1 Introduction. Physics and technology. The subject of physics

Physics originated from the needs of material production and is very closely related to technical activity. This connection is two-way, all branches of technical knowledge have grown from physical discoveries and are based on physical theories. Based on the achievements of physics, new technologies that are more advanced. In turn, the perfect technique equips scientists, allows them to push the boundaries of knowledge and reveal the deeper secrets of the universe.

The role of physics at a technical university is extremely large. The study of physics forms the students' scientific worldview and general culture; the general theoretical basis for the study of engineering and special technical disciplines, allows you to see how the various fields of science and technology are related; develops thinking and intellectual culture. The study of physics, especially problem solving, develops the ability to create ideal models of complex processes of phenomena; extract private conclusions from the general; see the general of the private; apply mathematical methods and use analogies; flexibly reorganize your thinking to the perception of fundamentally new approaches.

The object of physics is the whole diversity of inanimate nature: fields, quarks, elementary particles, atoms, molecules, macrobodies, planets and stars, their systems, galaxies, space environments and physical vacuum. The size of the observable Universe is 10^{26} m! Its age is 13.77 billion years or $4 \cdot 10^{17}$ s! Atoms have sizes from 10^{-10} m, and their nuclei - from $2 \cdot 10^{-15}$ m.

Physics is the science of the simplest forms of motion of matter; it studies the most general laws of nature and their particular manifestations at the micro-, macro- and mega-levels of organization of matter.

It is fundamentally important that the transition from one spatial-temporal scale at which physical phenomena take place to another is accompanied by qualitative changes in the nature of the laws that describe them. So, the natural velocity scale in nature is the speed of light propagation in vacuum with $= 3 \cdot 10^8$ m/s. A qualitative difference between nonrelativistic movements ($v \ll c$) and relativistic movements ($v \sim c$) is associated with it. With the Planck constant $\hbar = 1,054 \cdot 10^{-34}$ J·s, the distinction between physics into classical and quantum is associated. Classical mechanics includes Newtonian mechanics and A. Einstein's SRT. At the beginning of the twentieth century, after M. Planck put forward the hypothesis of quanta, a modern - quantum - physics - arose.

Physics is an experimental science. A scientific experiment is the observation of the phenomena studied and the measurement of physical quantities under precisely controlled conditions. The experimental method is closely related to theoretical meth-

ods, such as analysis, synthesis, systematization, abstraction, generalization, formalization, modeling, deduction, induction, development of the theory. Theoretical and empirical methods are mutually complementary.

Physical theory consists of: a) reliably established basic experimental facts and data; b) the main provisions, assumptions, axioms, laws and principles; c) logical and mathematical apparatus; e) consequences arising from the main provisions of the theory and subject to experimental verification. Physical theory provides an explanation of a whole range of natural phenomena from a single point of view, and predicts new phenomena.

1.2 Mechanical motion as the simplest form of motion of matter. Space and time

Mechanics (classical) is a branch of physics in which the mechanical motion of bodies and the interaction between bodies are studied. By mechanical movement is understood the change in time of the relative position of bodies or their parts in space.

All processes, phenomena and events in our world unfold in space and time. Space and time are forms of the existence of matter. Space expresses the order of coexistence of material objects; time reflects the order of change of phenomena. I. Newton considered space and time to be absolute, independent of each other and the presence of bodies. It has now been established that Space and Time are closely connected, forming four-dimensional space-time, and do not exist separately from matter: as follows from the general theory of relativity, large gravitating masses distort space and affect the course of time.

In physics, simplified models of real physical systems are used. In mechanics, there are a material point, a rigid body, an elastic collision, a continuous medium. Abstract mathematical models of space and time, for example, Euclidean space to a greater or lesser extent reflect the properties of real space and time.

1.3 Models in mechanics. Kinematics of a material point. Frame of reference. The trajectory of the motion of a material point. Displacement

Kinematics studies the spatial-temporal characteristics of motion independent of the interaction between bodies. A material point is a body whose shape and size can be neglected under the conditions of this problem. In other words, *a material point* is understood to mean a *physical object, in the geometrical sense, equivalent to a mathematical point, but possessing mass. Rigid body is a solid body in which deformation during its movement is zero or so small it can be neglected.*

Any movement is relative. To describe the mechanical motion, it is necessary to choose a *frame of reference* (or *reference frame*) — the totality of associated with the reference body coordinate system and a clock. As a reference body, you can choose the center of the Sun (heliocentric RF), the center of the Earth (geocentric RF)

or any point on the Earth's surface (laboratory RF), a car, a ship, a rocket or an air-plane.

A rectangular (Cartesian) coordinate system consists of an ordered triplet of mutually perpendicular lines - axes OX , OY and OZ , which intersect at one point O , called an origin. The position of an arbitrary point A at some point of time is given by three numbers: the coordinates x , y , z . The position of the point can also be set using the position vector (radius vector) \vec{r} - the directed segment of a straight line connecting an origin O and this point A (figure 1.1). Using unit vectors \vec{i} , \vec{j} and \vec{k} we can relate them:

$$\vec{r} = \vec{i} \cdot x + \vec{j} \cdot y + \vec{k} \cdot z. \quad (1.1)$$

The motion of a particle is completely determined if its position vector or coordinates are given as functions of time t :

$$\vec{r} = \vec{r}(t); \quad x=x(t); \quad y=y(t); \quad z=z(t). \quad (1.2)$$

Equations (1.2) are called the kinematic equations of motion.

The line that the particle describes during its movement is called a trajectory. Depending on the shape of the trajectory, rectilinear motion, circular motion, curvilinear motion are distinguished. On figure 1.2 shown trajectory of a body thrown at an angle to the horizon. The trajectory equation may be obtained by excluding the parameter t from the kinematic equations.

Let a particle move along a certain trajectory from point 1 to point 2. The path S traveled is the distance between points 1 and 2 counted along its trajectory. The straight line segment drawn from the initial position 1 to the final 2 is called the displacement vector \vec{s} :

$$\vec{s} = \Delta\vec{r} = \vec{r}_2 - \vec{r}_1. \quad (1.3)$$

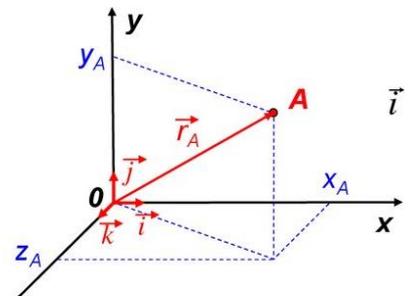


Figure 1.1

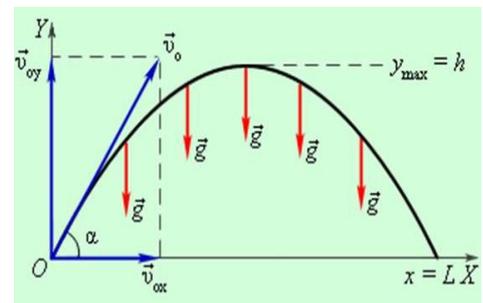


Figure 1.2

1.4 Velocity and acceleration as the derivatives of the radius vector (position vector) with respect to time

The average velocity $\langle \vec{v} \rangle$ for the time interval Δt is the ratio of the displacement to this time interval:

$$\langle \vec{v} \rangle = \frac{\Delta\vec{r}}{\Delta t}. \quad (1.4)$$

Instantaneous velocity - a vector quantity equal to the first derivative of the radius vector with respect to time; its components v_x , v_y , v_z are equal to the derivatives of the corresponding coordinates with respect to time:

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \vec{i} \cdot \frac{\partial x}{\partial t} + \vec{j} \cdot \frac{\partial y}{\partial t} + \vec{k} \cdot \frac{\partial z}{\partial t}. \quad (1.5)$$

The instantaneous velocity vector is directed tangentially to the trajectory in the direction of motion (figure 1.2). Its modulus is equal to the first time derivative of the path S : $v = \left| \frac{d\vec{r}}{dt} \right| = \frac{|d\vec{r}|}{dt} = \frac{dS}{dt}$, hence: $dS = v \cdot dt$ and

$$S = \int_{t_1}^{t_2} v(t) dt. \quad (1.6)$$

The acceleration characterizes the speed of change of the velocity vector, it is equal to the first time derivative of the velocity vector of the material point, or, the second derivative of the radius vector of this point:

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{i} \cdot \frac{\partial v_x}{\partial t} + \vec{j} \cdot \frac{\partial v_y}{\partial t} + \vec{k} \cdot \frac{\partial v_z}{\partial t} = \frac{d^2 \vec{r}}{dt^2}. \quad (1.7)$$

In the case of plane curvilinear motion, it is convenient to represent the acceleration as the sum of the tangential \vec{a}_τ and normal \vec{a}_n accelerations (figure 1.3):

$$\vec{a} = \vec{a}_\tau + \vec{a}_n. \quad (1.8)$$

The tangential acceleration directed along the tangent to a trajectory and is equal to the speed of the speed change in magnitude:

$$a_\tau = \frac{dv}{dt}. \quad (1.9)$$

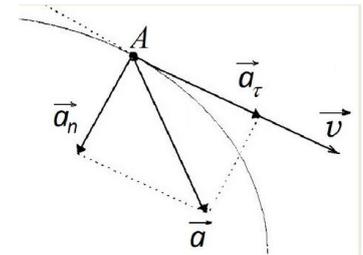


Figure 1.3

The normal (centripetal) acceleration is directed normal to the trajectory to the center of its curvature O and characterizes the speed of the change in the direction of the velocity. Normal component of acceleration is:

$$a_n = \frac{v^2}{R}. \quad (1.10)$$

The modulus of any vector quantity is equal to the square root of the sum of squares of its mutually perpendicular components:

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{a_\tau^2 + a_n^2}. \quad (1.11)$$

1.5 Kinematics of rotational motion. Angular values

During the rotational motion of a rigid body, its entire points move along circles which centers lie on one straight line, called the axis of rotation. Since the radii

of different points of the body are different, they travel different paths during the period of revolution T . In this case, the movement of the body can be specified by the rotation angle $\Delta\varphi$ about the axis. The elementary angular displacement $d\vec{\varphi}$ is a vector quantity whose modulus is equal to the infinitesimal angle of rotation $d\varphi$, and the direction coincides with the direction of translational motion of the right screw. Angular velocity:

$$\vec{\omega} = \frac{d\vec{\varphi}}{dt} = \dot{\vec{\varphi}}, \quad (1.12)$$

angular acceleration:

$$\vec{\varepsilon} = \frac{d\vec{\omega}}{dt} = \dot{\vec{\omega}} = \frac{d^2\vec{\varphi}}{dt^2} = \ddot{\vec{\varphi}}. \quad (1.13)$$

Units of angular velocity is 1 rad/s , of angular acceleration is 1 rad/s^2 .

All relations between linear and angular quantities follows from the equality of the length of the arc of a circle to the product of its radius by the value of the central angle: $l = \Delta\varphi \cdot r$.

2 Lecture №2. Dynamics of a material point, of a system of particles and of a rigid body

Objective of the lecture: to introduce the basic concepts of dynamics; introduce the basic methods of dynamics.

2.1 The concept of state in classical mechanics

Dynamics - the doctrine of the causes and laws of motion. It studies the mutual relationship between mechanical motion and the interaction of bodies.

The main task of mechanics is to determine the mechanical state of the considered system of bodies at any given time. For this, it is necessary, firstly, to establish those quantities whose assignment completely determines the state of the system; secondly, to establish laws - equations of motion, in accordance with which this state changes. Both of these problems were resolved by I. Newton. He established that the mechanical state of a material point (particle) at some instant of time t is completely determined by setting its spatial position - three coordinates x, y, z , and three projections of velocity v_x, v_y, v_z .

2.2 The first law of Newton. Inertial frame of reference

Classical mechanics is based on Newton's three laws. Newton's first law (the law of inertia): the body maintains a state of rest or uniform and rectilinear motion until the action of other bodies takes it out of this state. A body that can be neglected by other bodies is called free. A free body either rests or moves uniformly and rectilinearly.

The reference frame in which Newton's first law holds are called inertial. Reference frames in which this law is not implemented are non-inertial. Inertial are heliocentric, geocentric and, with some reservations, laboratory reference frames. It can be proved that any reference frame moving uniformly and rectilinearly relative to an inertial reference frame is also inertial. Accordingly, if some body moves with acceleration relative to an inertial reference frame, then the reference frame associated with this body is non-inertial.

2.3 Mass and momentum. Force. Newton's second law. The equation of the dynamics of a material point

The desire of the body to maintain a state of rest or uniform rectilinear movement is called inertia. Inertia is one of the main properties of matter. Inertness is a property due to which bodies differently change the state of their movement under the influence of the same force. The measure of a body inertia is called mass m .

The dynamic effect of a moving body on other bodies depends on both its velocity and mass. Therefore, as a dynamic characteristic, a momentum \vec{p} is introduced - the product of body mass by its velocity:

$$\vec{p}=m\vec{v}. \quad (2.1)$$

Force is a general measure of various types of mechanical action on a body by other bodies or fields, as a result of which the body acquires acceleration or changes shape and size - it is deformed. Force is a function of the state of the system, that is, it depends on the relative position of the bodies (coordinates) and their velocities. Strength is a vector quantity, this means that it is characterized not only by a numerical value (modulus of force), but also by direction. For a certain class of problems, the point of application of force matters.

At speeds much lower than the speed of light in vacuum $c = 3 \cdot 10^8$ m/s, the principle of superposition is valid, which reflects the law of independence of the action of forces: the force with which one particle acts on another depends only on the relative position and speeds of these two particles; the presence of other particles does not affect it. The principle of superposition: the simultaneous action of several forces on a material point is equivalent to the action of one - the resultant force, equal to their vector sum:

$$\vec{F}_R=\vec{F}_1+\vec{F}_2+\vec{F}_3+ \dots+\vec{F}_N. \quad (2.2)$$

Newton's second law: acceleration \vec{a} , acquired by a material point proportional to the force \vec{F} causing it, coincides with it in direction and inversely proportional to body mass m :

$$\vec{a}=\vec{F}/m \quad \text{or} \quad m\vec{a}=\vec{F}. \quad (2.3)$$

In this formulation, Newton's 2-nd law is valid only for bodies whose mass does not change with motion ($m = \text{const}$).

A more general formulation of Newton's 2-nd law: the rate of change of momentum of a material point is equal to the force acting on it:

$$\frac{d\vec{p}}{dt} = \vec{F}. \quad (2.4)$$

Impulse is the change of momentum of an object when the object is acted upon by a force for an interval of time:

$$d\vec{p} = \vec{F} \cdot dt. \quad (2.5)$$

Equation (2.4) is the basic equation of the dynamics of a material point. With the translational motion of a rigid body, all its points have one speed and one acceleration, therefore, equation (2.4) is the main equation for the dynamics of the translational motion of a rigid body.

2.4 Mechanical system. External and internal forces. Newton's Third Law. The center of mass of a mechanical system and the law of its motion

A mechanical system is a set of material points (bodies), considered as a single whole. The forces acting between the bodies that are included in the system under consideration are called internal, and those forces that act on the bodies of the system from bodies that are not included in the system are called external.

Newton's third law: *every action of material points on each other has the character of interaction; the forces with which the material points act on each other are always equal in magnitude and directed in opposite directions along the straight line connecting these points:*

$$\vec{F}_{12} = -\vec{F}_{21}. \quad (2.6)$$

These forces are applied to different bodies, act in pairs and have one nature. Newton's third law allows you to move from the consideration of a single material point to their system. The state of a mechanical system consisting of N material points is determined by setting $6N$ quantities — the coordinates x_i, y_i, z_i , and the projections of the momenta p_{ix}, p_{iy}, p_{iz} of each of the particles.

Consider a system of two material points (particles) 1 and 2, with masses m_1 and m_2 , respectively. They interact with forces \vec{F}_{12} and \vec{F}_{21} , and external forces \vec{F}_1 and \vec{F}_2 act on them. We write equation (2.4) for each of the particles:

$$\frac{d\vec{p}_1}{dt} = \vec{F}_{12} + \vec{F}_1; \quad (2.7)$$

$$\frac{d\vec{p}_2}{dt} = \vec{F}_{21} + \vec{F}_2. \quad (2.8)$$

Let us add equations (2.7) and (2.8) term-by-term and take into account (2.6), which means that the sum of all internal forces in the system is zero. Hence:

$$\frac{d(\vec{p}_1 + \vec{p}_2)}{dt} = \vec{F}_1 + \vec{F}_2. \quad (2.9)$$

Under the sign of the derivative in (2.8) is the sum of the momenta of all the material points of a given system — this is the momentum \vec{p} of the system. In general:

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots + \vec{p}_N = \sum_{i=1}^N \vec{p}_i. \quad (2.10)$$

On the right-hand side of (2.9) is the sum of all external forces acting on the bodies of the system, or the total external force $\vec{F}_{\text{external}} = \vec{F}_1 + \vec{F}_2$.

So, the basic equation of dynamics for a mechanical system has the form:

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{external}}. \quad (2.11)$$

The meaning of equation (2.11) is that only external forces change the momentum of the system.

The center of mass (or center of inertia) of a system of material points is an imaginary point C , the radius vector of which \vec{r}_c is equal to:

$$\vec{r}_c = \frac{\sum_{i=1}^N m_i \vec{r}_i}{m}, \quad (2.12)$$

where $m = m_1 + m_2 + \dots + m_N = \sum_{i=1}^N m_i$ is the mass of the system.

Differentiation (2.12) gives an expression for the velocity of the center of mass:

$$\vec{v}_c = \frac{d\vec{r}_c}{dt} = \frac{\vec{p}}{m}. \quad (2.13)$$

Thus, the momentum of the system is equal to the mass of the system multiply by the velocity of its center of mass:

$$\vec{p} = m\vec{v}_c, \quad (2.14)$$

then (2.11) takes the form:

$$m \frac{d\vec{v}_c}{dt} = \vec{F}_{\text{BHEIII}}. \quad (2.15)$$

Equation (2.15) expresses the law of motion of the center of mass: the center of mass of the system moves as a particle would move with a mass equal to the total

mass of the system, under the action of a force equal to the sum of the external forces applied to the bodies of the system. The motion of the center of mass can be identified with the translational motion of the system as a whole. The latter is also true for a solid, which is considered as a system of rigidly interconnected material points.

2.5 The angular momentum. Moment of force. The equation of moments for a material point and a system of material points

An angular momentum (rarely, moment of momentum, or rotational momentum) \vec{L} of a material point relative to a fixed center O is the vector product of the radius vector \vec{r} of this particle drawn from the center O and its momentum \vec{p} :

$$\vec{L} = [\vec{r}, \vec{p}] = [\vec{r}, m\vec{v}]. \quad (2.16)$$

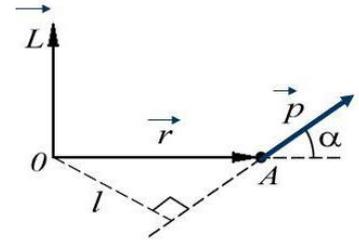


Figure 2.1

An angular momentum modulus is equal to the product of the modulus of the factors by the sine of the angle between their directions:

$$L = r \cdot p \cdot \sin\alpha,$$

where $l=r \cdot \sin\alpha$ - is the *length of the moment arm* - a line dropped perpendicularly from the origin O onto the path of the particle (figure 2.1). The direction of such a vector is perpendicular to the directions of each of the factors and is found by the rule of the right screw.

We differentiate (2.16) with respect to time:

$$\frac{d\vec{L}}{dt} = \left[\frac{d\vec{r}}{dt}, \vec{p} \right] + \left[\vec{r}, \frac{d\vec{p}}{dt} \right] = [\vec{v}, \vec{p}] + [\vec{r}, \vec{F}] = [\vec{r}, \vec{F}], \quad (2.17)$$

where it is taken into account that the vector product of the vectors of the same direction \vec{v} and \vec{p} is equal to zero, and the time derivative of the particle momentum, taking into (2.4), is replaced by the resulting force vector \vec{F} . The vector product of the radius vector the point of a force application and the force vector is called the moment \vec{M} of the force relative to the center O :

$$\vec{M} = [\vec{r}, \vec{F}]. \quad (2.18)$$

The resulting equation is called the *equation of the moments* for a particle:

$$\frac{d\vec{L}}{dt} = \vec{M}. \quad (2.19)$$

It means that the rate of change in the angular momentum of a particle, determined with respect to a certain center O , is equal to the total moment relative to the same point O of the forces applied to the particle.

The moment of force characterizes the ability of the force to cause and change the rotation; therefore, it is also called the torque.

The modulus of the moment of force is:

$$M = rF\sin\beta, \quad (2.20)$$

where $l = r \cdot \sin\beta$ - represents the shortest distance from the center O to the line of action of a force and is called the *arm of the force*.

The direction of the vector \vec{M} coincides with the direction of translational motion of the right screw when it is rotated from the first factor \vec{r} in the shortest direction to the second factor \vec{F} .

2.6 Equation of the moments for a mechanical system

The angular momentum of a system of particles relative to the center O is equal to the sum of the angular momenta of all particles of the system:

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \dots + \vec{L}_N = \sum_{i=1}^N \vec{L}_i. \quad (2.21)$$

Differentiating by time, we obtain:

$$\frac{d\vec{L}}{dt} = \sum_i \frac{d\vec{L}_i}{dt} = \sum_{i=1}^N \vec{M}_i. \quad (2.22)$$

Consider two interacting particles 1 and 2. According to Newton's 3rd law $\vec{F}_{12} = -\vec{F}_{21}$. Then the total moment of internal forces:

$$\vec{M}_{internal} = [\vec{r}_1, \vec{F}_{12}] + [\vec{r}_2, \vec{F}_{21}] = [\vec{r}_1, \vec{F}_{12}] - [\vec{r}_2, \vec{F}_{12}] = [(\vec{r}_1 - \vec{r}_2), \vec{F}_{12}] = 0.$$

Since the force \vec{F}_{12} is directed along the straight line $(\vec{r}_2 - \vec{r}_1)$ connecting both particles, their vector product is zero. It follows that the moments of internal forces compensate each other in pairs and the sum of the torques of internal forces is zero.

Thus, the rate of change of the angular momentum of a system of particles is equal to the sum of the moments of all external forces applied to the particles of the system in question:

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^N \vec{M}_i^{external} \quad (2.23)$$

Equation (2.23) is called the equation of moments for a system of particles; it is true for a rigid body too.

2.6 The basic equation of the dynamics of the rotational motion of a rigid body relative to a fixed axis. Moment of inertia of a body

In practice, the rotation of a rigid body around a fixed axis is important, at which each point of the body moves in a circle. The centers of these circles lie on one straight line, called the axis of rotation. Consider the rotational motion of some solid body relative to the fixed axis OZ . Projecting equation (2.23) on this axis, we obtain:

$$\frac{dL_z}{dt} = M_z. \quad (2.24)$$

To determine the angular momentum L_z of a body relative to the axis, it is necessary to project the total angular momentum relative to point O of all particles of this body to this axis:

$$L_z = \sum_{i=1}^N L_{iz}. \quad (2.25)$$

This expression can be converted to:

$$L_z = \omega \sum_i m_i R_i^2 = J\omega. \quad (2.26)$$

The sum of the products of elementary masses per square of their distances to the axis

$$J = \sum_{i=1}^N m_i R_i^2 \quad (2.27)$$

called the moment of inertia of the body about the axis. The moment of inertia depends on the distribution of body mass around the axis, characterizes the inert properties of the body during rotational motion, and is an analog of mass.

More precisely in terms of the integral:

$$J = \int R^2 \cdot dm, \quad (2.28)$$

where integration is performed over the entire volume of the body. If the body is homogeneous, then its density $\rho = \frac{dm}{dV} = \text{const}$, then:

$$J = \int R^2 \cdot dm = \rho \iiint R^2 \cdot dV. \quad (2.29)$$

Steiner's theorem: the moment of inertia J of a body about an arbitrary axis is equal to the sum of the moment of inertia J_c relative to an axis parallel to that and

passing through the center of mass and the product of body mass m by the square of the distance b between the axes:

$$J = J_c + mb^2. \quad (2.30)$$

So, the basic equation for the dynamics of the rotational motion of a rigid body relative to a fixed axis OZ is:

$$J \cdot \varepsilon_z = M_z, \quad (2.31)$$

according to which the angular acceleration of the body is directly proportional to the moment of the forces applied to the body, and inversely proportional to its moment of inertia. All equations of dynamics are valid only in inertial frame reference.

3 Lecture №3. Energy and work. Conservation laws in mechanics

Objective of the lecture: to introduce the basic concepts of work and energy; establish the relationship between work and energy; to familiarize with the fundamental laws of conservation in mechanics.

3.1 Energy as a universal measure of all forms of movement and interaction. Work done by a force. Power

Each natural phenomenon represents a particular form of motion and interaction of matter. In nature, there are many forms of motion of matter. Mechanical movement is the simplest of them. For its quantitative description, we used the concept of momentum \vec{p} or angular momentum \vec{L} . To describe thermal phenomena, other characteristics of the state are introduced, for example, temperature T and entropy S . All these quantities reflect the qualitative features of various forms of matter motion. Consequently, it is necessary to introduce a physical quantity that relates to all forms of motion of matter and reflects their mutual transformability. Such a physical quantity is energy - one of the basic concepts in physics. *Energy is a universal measure of various forms of motion and interaction of matter.*

The concept of energy links together all natural phenomena. Various forms of energy: mechanical, thermal (internal), electromagnetic, atomic, nuclear, and so on are associated with various forms of motion of matter.

In nature, processes are continuously going on in which mechanical motion and energy are transferred from one body to another. A change in the mechanical motion of a body is caused by forces acting on it by other bodies. To quantitatively characterize the process of energy exchange between interacting bodies, work of a force applied to a given body is considered in mechanics. Work is a measure of the change in energy in the processes of force interaction.

Work done by a constant force is a physical quantity equal to the scalar product of a force vector \vec{F} and a displacement vector \vec{s} :

$$A = \vec{F} \cdot \vec{s} = F \cdot s \cdot \cos\alpha = F_s \cdot s, \quad (3.1)$$

where α is the angle between the force vector and the direction of movement; F_s is the projection of the force on the direction of movement.

Formula (3.1) is applicable when a force is constant in magnitude and direction, while the body moves rectilinearly. In the case of a variable force and a curved path, this definition is applicable only for elementary work δA on an infinitesimal displacement $d\vec{s}$:

$$\delta A = \vec{F} \cdot d\vec{s} = F \cdot ds \cdot \cos\alpha = F_s \cdot ds. \quad (3.2)$$

Work done by a force on the displacement from initial point 1 to final point 2 is the line integral of its scalar tangential component along the path of its application point:

$$A_{12} = \int_1^2 \vec{F} \cdot d\vec{s} = \int_1^2 F_s \cdot ds. \quad (3.3)$$

If the dependence of F_s on s is set on the graph, then the area of the curvilinear trapezoid formed by the graph and the abscissa axis s is numerically equal to A_{12} .

The power N of a force \vec{F} is the rate at which work is done:

$$N = \frac{\delta A}{dt} = \frac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F} \cdot \vec{v} = F \cdot v \cdot \cos\alpha, \quad (3.4)$$

where \vec{v} is the instantaneous velocity at which the point of application of the force \vec{F} moves,

α is the angle between the directions of the force and velocity vectors.

Thus, instantaneous power developed by a force is equal to the scalar product of the force and velocity vectors; its unit is $1 \text{ W} = 1 \text{ J/s}$.

3.2 Kinetic energy of a particle and a system of particles. The relation of the kinetic energy with the work. Kinetic energy and work of a rigid body rotation

The force acting on the particle and causing a change in this movement does the work. We show, using Newton's second law and the definition of the instantaneous velocity, that the particle energy increases by the amount of work expended:

$$\delta A = \vec{F} \cdot d\vec{s} = m \cdot \frac{d\vec{v}}{dt} \cdot d\vec{s} = m \cdot \vec{v} \cdot d\vec{v} = d\left(\frac{mv^2}{2}\right). \quad (3.5)$$

The scalar value under the differential sign is called the kinetic energy:

$$W_k = \frac{mv^2}{2}. \quad (3.6)$$

Kinetic energy of an object is the energy that it possesses due to its motion, it depends on the mass of the body and its speed; since speed is a relative quantity, kinetic energy depends on the choice of a reference frame.

We integrate both sides of equality (3.5) along the particle trajectory from point 1 to point 2:

$$\int_1^2 \delta A = \int_1^2 dW_k. \quad (3.7)$$

The left side of the resulting equality is the work of the resultant of all the forces applied to the particle, and the right is an increment of kinetic energy:

$$A_{12} = W_{k2} - W_{k1}, \quad (3.8)$$

thus, the work of the resultant force acting on a particle goes to increase the kinetic energy of the particle. This statement is called the principle of work and kinetic energy.

Relation (3.8) can be generalized to a mechanical system (system of material points); in this case, the kinetic energy of the system is defined as the sum of the kinetic energies of all particles of a given system:

$$W_k = \sum_{i=1}^N \frac{m_i v_i^2}{2}. \quad (3.9)$$

When a rigid body rotates around a fixed axis, its kinetic energy is the sum of the kinetic energies of all particles of this body. Since the particle velocities in this case are different, then using the angular velocity, we obtain:

$$W_k = \sum_{i=1}^N \frac{m_i v_i^2}{2} = \sum_{i=1}^N \frac{m_i \omega^2 R_i^2}{2} = \frac{\omega^2}{2} \sum_{i=1}^N m_i R_i^2 = \frac{J \omega^2}{2}, \quad (3.10)$$

here J is the moment of inertia of the body relative to the axis of rotation;
 ω is the angular velocity of the body rotation around the axis.

The work of a force during the rotation of a rigid body is determined by the integral:

$$A_{12} = \int_1^2 M_z \cdot d\varphi, \quad (3.11)$$

where M_z is the projection of the moment of external forces on the axis OZ ;
 $d\varphi$ is an infinitesimal rotation angle around the same axis.

3.3 Conservative and non-conservative forces. Potential energy of a particle and its connection with the field force

In the general case, the work done by a force when moving a particle depends on the shape of its trajectory. Conservative is a force whose work is determined only

by the initial and final positions of the body and does not depend on the shape of the path. All central forces, gravitational and elastic forces are conservative. Non-conservative forces include friction and resistance. Since the work of a conservative force does not depend on the path, then on any closed path this work is equal to zero:

$$\oint \vec{F} \cdot d\vec{s} = 0. \quad (3.12)$$

A force field is a region of space at each point of which a force \vec{F} acts regularly on a particle placed there, which varies regularly from point to point. The force field (in the theory of short range) plays the role of a carrier of interaction between bodies distant from each other.

Fields of conservative forces form a class of potential fields and have a number of characteristic properties. In particular, such a field at each point can be characterized by a certain scalar function $W_p(\mathbf{r})$, which depends on the position of the point in space and the nature of the force in such a way that the work of the conservative force when moving the particle between points 1 and 2 equal to decrease this function:

$$A_{\text{конс.12}} = W_{p1} - W_{p2}. \quad (3.13)$$

This function $W_p(x,y,z)$ is called the potential energy of the particle. Potential energy is the interaction energy, it depends on the relative position of the bodies and the nature of the interaction forces between them, as well as on the choice of the zero level of potential energy.

The relationship of the field strength at some point with the potential energy of a particle located at the same point is given by the relation:

$$\vec{F} = -\left(\vec{i} \frac{dW_p}{dx} + \vec{j} \frac{dW_p}{dy} + \vec{k} \frac{dW_p}{dz}\right) = -\text{grad}W_p(x,y,z) = -\nabla W_p(x,y,z), \quad (3.14)$$

where $\vec{i}, \vec{j}, \vec{k}$ are unit vectors, and the components of the force are the corresponding projections:

$$F_x = -\frac{\partial W_p}{\partial x}; \quad F_y = -\frac{\partial W_p}{\partial y}; \quad F_z = -\frac{\partial W_p}{\partial z}. \quad (3.15)$$

The potential energy of a system of interacting particles located in an external potential field is the sum of the potential energies of each of the particles in this field and the potential energy of interaction of each particle with all other particles of this system.

3.4 Mechanical energy of a particle and of the particles system. The law of change in mechanical energy

The mechanical energy of a body is the energy of its movement and interaction, is equal to the sum of its kinetic and potential energies:

$$W = W_k + W_p. \quad (3.16)$$

Consider a particle under the action of conservative \vec{F} and no conservative \vec{F}^* forces. According to (3.8) and (3.13)

$$A_{12} = A_{\text{KOH.C.12}} + A^*_{12} = W_{k2} - W_{k1}; \quad A_{\text{KOH.C.12}} = W_{p1} - W_{p2}.$$

After substituting the second equation in the first and elementary algebraic transformations, we come to the formulation of the law of change in mechanical energy:

$$W_{k2} - W_{k1} + W_{p2} - W_{p1} = (W_{k2} + W_{p2}) - (W_{k1} + W_{p1}) = W_2 - W_1 = A^*_{12}.$$

Thus, a change in the mechanical energy of a particle when moving from state 1 to state 2 is equal to the work of non-conservative forces on this movement:

$$W_2 - W_1 = A^*_{12}. \quad (3.17)$$

It can be proved that this is true for a mechanical system.

3.5 Closed mechanical system. The law of conservation of momentum

Consider a system of material points (particles) that interact with each other and at the same time are under the influence of external forces. According to the basic law of dynamics, the total momentum of such a system changes under the influence of only external forces. However, if the mechanical system is closed (isolated), that is, its particles interact only with each other and no external bodies act on them, then the total momentum of such a system will not change over time.

So, according to the law of conservation of momentum, the total momentum of a closed (isolated) system of bodies does not change over time, that is, it remains:

$$\vec{p} = \sum_{i=1}^N m_i \vec{v}_i = \text{const}. \quad (3.18)$$

In this case, the momenta of the individual particles of the system changes under the action of the forces of interaction between them, but their vector sum remains constant.

The momentum of a system of particles can be expressed in terms of the mass of the system and the velocity of its center of mass $\vec{p} = m\vec{v}_c$, therefore, the center of mass of a closed system of bodies always moves uniformly and rectilinearly: $\vec{v} = \text{const}$.

In solving specific problems, the law of conservation of momentum is also applied in the case of open systems. Firstly, if the momentum of external forces is much less than the momentum of internal forces. For example, when a projectile breaks, when the momentum of external forces — gravity and air resistance forces — is much less than the internal forces that break the projectile and tell its flying fragments quite high speeds during Δt of this gap.

Secondly, if in an open system there is such a direction, for example, the OX axis, the projection of external forces onto which is equal to zero $F_x = 0$, then, projecting the vector equation (2.10) onto this axis, we obtain:

$$\frac{dp_x}{dt} = F_x = 0,$$

whence follows:

$$p_x = \sum_{i=1}^N p_{ix} = \text{const.} \quad (3.19)$$

In a collision (impact) of two bodies, for example, two billiard balls, with masses m_1 and m_2 , the law of conservation of momentum is in the form of the equation:

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{u}_1 + m_2 \vec{u}_2, \quad (3.20)$$

here \vec{v}_1 и \vec{v}_2 are the velocities of the bodies before, and \vec{u}_1 и \vec{u}_2 are velocities after the collision.

If two balls collide inelastically, then after such an impact the bodies move as one whole body with a mass equal to the sum of the masses ($m_1 + m_2$). During inelastic collisions, bodies stick together due to internal forces, and the action of external forces during the period of impact, for example, gravity, can be neglected. The law of conservation of momentum for these balls has the form:

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{u}. \quad (3.21)$$

This equation (3.21) corresponds to momentum vector diagram in figure 3.1. To solve the problem to the end, you need to know the angle at which the balls fly upon impact, and then project the vector equation (3.21) on the OX and OY axes; then solve the resulting system of equations.

The most striking example of the application of the law of conservation of momentum is jet propulsion (figure 3.2).

Reactive is the movement of the body that occurs when a part of it is separated from it at a certain speed. In this case, the body itself receives an oppositely directed impulse. The simplest example of jet propulsion is the flight of a balloon from which

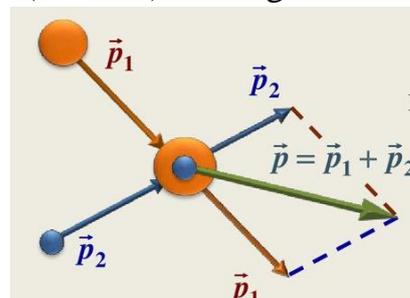


Figure 3.1

air escapes. If we inflate the ball and release it, it will begin to fly in the direction opposite to the movement of the air coming out of it.

An example of jet movement in nature: jellyfish, squid, and other inhabitants of the deep sea move by absorbing water and then throwing it away. When a rocket with a jet engine moves as a result of fuel combustion, a jet of liquid or gas (jet stream) is ejected from the nozzle. As a result of the interaction of the engine with the leaking substance, reactive force (thrust) appears. Since the rocket along with the ejected substance is a closed system, the momentum of such a system does not change with time. Another example of jet propulsion is a water-jet propulsion device (water-jet) - in which the force propelling the vessel (boat) is created by the jet of water pushed out of it.

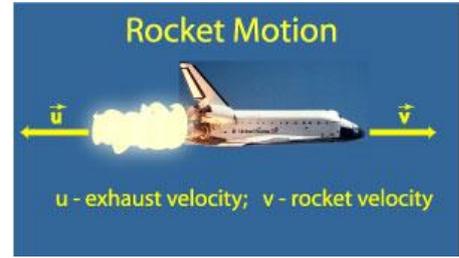


Figure 3.2

3.6 The law of conservation of angular momentum

According to the basic dynamics equation (2.8), the rate of change in the total angular momentum of a mechanical system is equal to the total moment of external forces applied to the particles of the system. External forces do not act on particles of a closed system; therefore, their moment is equal to zero: $M_{\text{external}} = 0$. It follows that the rate of change in the angular momentum of the system is zero, which means that the total angular momentum of a closed system of the particles does not change over time and remains constant:

$$\vec{L} = \sum_{i=1}^N \vec{L}_i = \text{const.} \quad (3.22)$$

This statement is the law of conservation of angular momentum. As well as momentum, the angular momentum of the system can be stored in open systems. For example, if for a certain period of time the total moment of external forces is negligible compared to the moments of internal forces, then during this time the angular momentum of the system remains almost unchanged.

If a rigid body, or a system of this bodies, is fixed on the axis of rotation OZ and the projection of the moment of external forces on the same axis is zero $M_z = 0$, then in this case $\frac{dL_z}{dt} = 0$. Therefore, the angular momentum relative to some axis, the projection of the moment of external forces on which is equal to zero, does not change over time:

$$L_z = \sum_{i=1}^N J_i \cdot \omega_i = \text{const.} \quad (3.23)$$

An excellent illustrative example of the action of the law of conservation of angular momentum - when the skater rotates on ice, and changes his moment of inertia, arms spread out to the sides, and then, pressing them to the body - the angular

velocity of its rotation noticeably changes. Also, Kepler's 2nd law is a manifestation of the conservation law of an angular momentum as applied to the revolution of the Earth around the Sun.

3.7 Law of conservation of energy

Mechanical systems on which bodies only conservative forces act are called conservative. In this case, the work of non-conservative forces on the particles of the system is not performed. It follows from equation (3.20) that the total mechanical energy of the conservative system does not change over time, that is, it is preserved:

$$W = W_k + W_p = \text{const.} \quad (3.24)$$

This statement is called the law of conservation of mechanical energy. Thus, in conservative systems, the total mechanical energy remains constant. Kinetic energy can only be converted into potential energy and vice versa in equivalent quantities so that the total energy remains unchanged.

In dissipative systems in which dissipative forces act on bodies (friction and resistance of the medium), mechanical energy gradually decreases. However, with the "disappearance" of mechanical energy, an equivalent amount of energy of another type always arises, for example, thermal. Therefore, the law of conservation and transformation of mechanical energy considered above is a special case of the general physical law of conservation and transformation of energy, which states that in a closed (isolated) system, the total energy does not change over time, that is, it is conserved. This means that energy can be neither created nor be destroyed, it can only be transformed from one species to another.

3.8 Conservation laws are fundamental principles of physics. Conservation laws as a consequence of the symmetry of space and time

The laws of conservation of momentum, angular momentum and energy are among the most fundamental principles of physics. As it turned out, their action goes far beyond the scope of mechanics. They are true both in the field of elementary particles and in the field of space systems, and therefore are one of the few general laws that underlie modern physics. These laws are universal laws of nature.

The laws of conservation of momentum, angular momentum and energy have become a very powerful and effective research tool, since:

- conservation laws do not depend on the trajectories of particles, nor on the nature of the acting forces. Therefore, they allow one to obtain a number of very general and significant conclusions about the properties of various mechanical processes without going into their detailed consideration with the help of equations of motion;

- this fact allows the use of conservation laws even when the forces are generally unknown (collisions of bodies, molecules, nuclear reactions and other transformations of elementary particles).

The laws of conservation of momentum, angular momentum and energy are of deep origin, associated with the fundamental properties of space and time. So, the law of energy conservation is based on the homogeneity of time, i.e. the equivalence of all moments of time. Different points in time are equivalent to each other in their physical properties in the sense that any physical process proceeds in the same way regardless of when it started. The law of conservation of momentum is based on the homogeneity of space, i.e. identical properties of space in all its points.

The law of conservation of angular momentum is based on the isotropy of space. This means that the properties of space at each point are the same in all directions.

4 Lecture №4. Elements of the special theory of relativity and relativistic dynamics

The objective of the lecture: to familiarize with the basic provisions and relations of the special theory of relativity.

4.1 The principle of relativity and Galileo's transformation

Galileo's principle of relativity (the mechanical principle of relativity) is a generalization of all the experimental facts known at that time (which related to mechanics): any mechanical phenomenon proceeds identically in all inertial reference systems. This means that no mechanical experiments carried out in an inertial reference frame (IRF) can not establish whether this reference frame moves rectilinearly and uniformly or is at rest.

Consider two IRFs, denoting them K and K' . Suppose that at the initial instant of time the origin of coordinates O and O' coincide, and the corresponding axes are pairwise parallel. If the relative motion (reference frame K relative to K') occurs along one of them (for example, along the OX axis) with constant speed V , then Galileo transformations (direct and inverse) have the form:

$$\left. \begin{aligned} x &= x' + Vt' \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned} \right\}; \quad (4.1)$$

$$\left. \begin{aligned} x' &= x - Vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned} \right\}. \quad (4.2)$$

Or, in the vector form:

$$\vec{r} = \vec{r}' + \vec{V}t'; \quad (4.3)$$

$$\vec{r}' = \vec{r} - \vec{V}t. \quad (4.4)$$

From the transformations directly (by time differentiation), the classical law of velocity addition follows:

$$\vec{v} = \vec{v}' + \vec{V}, \quad (4.5)$$

where \vec{v} is the particle velocity in the K system, and \vec{v}' is the velocity of the same particle in the reference frame K' .

Differentiation in time of equality (4.3) we get:

$$\vec{a} = \vec{a}'. \quad (4.6)$$

Physical quantities that remain unchanged during the Galilean transformations are called invariants of the Galilean transformations. Since the forces depend on the relative position and relative velocities of the interacting particles, they are invariant. From experience follows the invariance (invariance) of body mass, $m = m'$. Therefore, the dynamics equation (Newton's second law) is also an invariant of Galileo's transformations. In the general case, the form of the equations of mechanics does not change during the transition from one IRF to another.

Thus, in mechanics, all IRFs are completely equal (equivalent to each other) - it is impossible to single out any particular or main one.

4.2 The postulates of Einstein

A deep analysis of the contradictory situation in physics by the end of the 19th century led A. Einstein to revise the initial principles of classical physics, primarily ideas about the properties of space and time. As a result, in 1905 he developed a special theory of relativity (STR). The STR is the physical theory of space and time, in which it is assumed that space is homogeneous and isotropic, time is homogeneous. The basis of the special theory of relativity of Einstein are *two postulates*: the generalized *principle of relativity* and the *principle of the constancy of the speed of light in vacuum*:

- every physical phenomenon proceeds in the same way in all inertial reference frames;

- the speed of light in vacuum is the same in all inertial reference frames and does not depend on the motion of sources and receivers of light, i.e. is universal constant. It is equal to $c = 2,99793 \cdot 10^8$ m/s $\approx 3,0 \cdot 10^8$ m/s.

4.3 Lorentz transformations and their consequences

Relativistic transformations of coordinates and time, reflecting the properties of space and time in the special theory of relativity, are called Lorentz transformations. According to these transformations, the transition from the system K , moving relative to the system K' along the X axis at a speed V , to the system K' is carried

out according to the formulas (4.1), and from K to the system K' - according to the formulas (4.2).

On the basis of the obtained transformations of coordinates and time, one can give one more formulation of the principle of relativity: *physical laws are invariant with respect to Lorentz transformations.*

$$\left. \begin{aligned} x &= \frac{x' + Vt'}{\sqrt{1 - \frac{V^2}{c^2}}} \\ y &= y' \\ z &= z' \\ t &= \frac{t' + \frac{x'V}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \end{aligned} \right\}; \quad (4.7)$$

$$\left. \begin{aligned} x' &= \frac{x - Vt}{\sqrt{1 - \frac{V^2}{c^2}}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - \frac{xV}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \end{aligned} \right\}. \quad (4.8)$$

Properties of Lorentz transformations:

a) Lorentz transformations clearly demonstrate the inextricable interrelation of the spatial and temporal properties of our world. Thus, it is impossible to speak separately about space and separately about time, it will be correct to talk about space-time, in which our world exists. In other words, our world is *four-dimensional*;

b) at speeds of motion much lower than the speed of light in vacuum $V \ll c$, which corresponds to the limiting transition $c \rightarrow \infty$, the Lorentz transformations go over to the Galileo transforms;

c) for $V \geq c$ formulas (4.7) and (4.8) lose their physical meaning. This means that it is impossible to move at a speed equal to or greater than c .

Let's consider some consequences of the Lorentz transformations. On the basis of Lorentz transformations, the relative nature of simultaneity can be described; - time flows in different ways in different inertial frames of reference. The statement that a certain period of time has passed between the two events acquires meaning only when it is indicated to which reference frame this statement applies; events that are simultaneous in one reference system may not be simultaneous in another reference system.

Next consequence is relativity of time intervals of the same event in the IRFs K and K' . Suppose that at some point with the coordinate x' in the IRF K' some event occurs with the duration $\Delta t' = t'_2 - t'_1$, then in the system K the duration of the same event $\Delta t = t_2 - t_1$ is equal to:

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \beta^2}}, \quad (4.9)$$

where $\beta = \frac{V}{c}$.

Thus, the same clock in different inertial frames of reference goes differently: in that frame of reference with respect to which the clock moves, they go slower than in the frame of reference where they rest. In other words, a moving clock is slower than a resting one. This phenomenon is called *time dilation*.

Let a rod located parallel to the OX axis move relative to the K -frame of reference with a constant speed along the same axis. The length of the rod l_0 measured in the K' -reference frame, in which the rod is fixed, is called proper. The length l of the rod, measured in the K -system, is

$$l = l_0 \sqrt{1 - \beta^2}. \quad (4.10)$$

Thus, the length of the rod moving relative to the K -frame is reduced in the direction of movement. This phenomenon is called *Lorentz contraction of length*. The transverse dimensions of the body do not depend on the speed of its movement and are the same in all IRFs.

Relativistic law of velocity addition. Let a particle with a speed v move in the K -system in the x,y -plane, whose projections v_x and v_y . Using the Lorentz transformations, one can find the projection of the velocity of this particle in the K' -system, which, recall, itself moves relative to the first with velocity V in the direction of the OX axis:

$$v'_x = \frac{v_x - V}{1 - \frac{v_x V}{c^2}}, \quad (4.11)$$

$$v'_y = \frac{v_y \sqrt{1 - \beta^2}}{1 - \frac{v_x V}{c^2}}. \quad (4.12)$$

At low speeds ($V \ll c$ and $v \ll c$), the relativistic formulas (4.7) - (4.10) transform into classical ones, and the relativistic law of addition of velocities into the classical ones.

Thus, all relativistic effects are observed (experimentally confirmed) only at speeds comparable with the speed of light in vacuum.

4.4 Space-time interval. Invariants of Lorentz transformations

From the Lorentz transformations it follows that the speed of light is the same in different inertial frames of reference. This conclusion corresponds to the principle of the constancy of the speed of light. From the Lorentz transformation formulas it also follows that the speed of light is maximum.

In relativistic mechanics, the invariance of spatial and time intervals relative to the Lorentz transformations does not hold. The question arises whether it is impossible to indicate in the special theory of relativity the value associated with the spatial and temporal intervals between two events that would be invariant to the Lorentz transformations? This question is solved positively. Such an invariant in the special theory of relativity is the quantity determined by the relation:

$$\Delta s = \sqrt{c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2} = \sqrt{c^2 \Delta t^2 - \Delta l^2}. \quad (4.12)$$

It can be shown, applying the Lorentz transformations, that the interval is an invariant quantity, that is, in any other IRF:

$$\Delta s' = \sqrt{c^2 \Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2} = \sqrt{c^2 \Delta t'^2 - \Delta l'^2} = \Delta s = inv.$$

The invariants of the Lorentz transformations are also the speed of light in vacuum, proper time and proper length.

4.5 Relativistic momentum transformation. The basic equation of the dynamics of a relativistic particle

The relativistic expression for the particle momentum, ensuring the invariance of the momentum conservation law, has the form:

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (4.13)$$

where m is mass of a particle; it is invariant of Lorentz transformations. The basic equation of relativistic dynamics has the form:

$$\frac{d\vec{p}}{dt} = \frac{d}{dt} \left(\frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \right) = \vec{F}. \quad (4.14)$$

As follows from the equation, an acceleration of a relativistic particle in the general case is not proportional to the magnitude of the force and does not coincide with it in direction.

The relativistic expression for kinetic energy has the form:

$$T = \frac{m \cdot c^2}{\sqrt{1-\beta^2}} - m \cdot c^2. \quad (4.15)$$

A body at rest has energy called rest energy:

$$E_0 = m \cdot c^2. \quad (4.16)$$

Total energy of relativistic particle is:

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}. \quad (4.17)$$

In the theory of relativity, the total energy is equal to the sum of the kinetic energy of a particle and its rest energy:

$$E = E_0 + T. \quad (4.18)$$

Relations between total energy and momentum of a relativistic particle are:

$$E^2 = m^2c^4 + p^2c^2; \quad (4.19)$$

$$p = \frac{1}{c} \sqrt{E^2 - E_0^2} = \frac{1}{c} \sqrt{T(2E_0 + T)}. \quad (4.20)$$

From the last relations it follows that the existence of massless particles with non-zero energy and momentum is possible only under the condition that they move at the speed of light in vacuum. Then:

$$p = \frac{E}{c}. \quad (4.21)$$

5 Lecture №5. Statistical distributions

The objective of the lecture: to familiarize with the methods of studying the properties of macroscopic systems and statistical regularities in an ideal gas.

5.1 Statistical and thermodynamic research methods

Molecular physics is a branch of physics in which the structure and properties of matter are studied, as well as the processes occurring in it, on the basis of the molecular-kinetic theory. According to this theory, all substances are composed of molecules that are in continuous and erratic thermal motion. The proof of this is Brownian motion, which consists in observing that small particles suspended in a liquid under a microscope continuously and randomly move because of random impacts of the molecules.

To study what happens in macroscopic bodies, consisting of a huge number of molecules, processes, two methods are used: statistical and thermodynamic, which complement each other.

The statistical method is a microscopic approach, it is based on model atomic-molecular ideas about the structure of macro bodies and the application of laws that determine the motion and interaction of particles forming these bodies. Based on the laws of mechanics and using methods of probability theory and mathematical statistics, averaged values of the dynamic characteristics of these particles (momentum,

energy) are found. Moreover, the properties of bodies directly observed in the experiment (pressure p) are interpreted as the total averaged result of the action of individual particles. In this way, the basic equation of the molecular-kinetic theory of an ideal gas was obtained, which relates the gas pressure p to the average kinetic energy of the translational motion of the molecules $\langle \varepsilon_{transl} \rangle$ and their concentration n :

$$p = \frac{1}{3}nm_0\langle v^2 \rangle = \frac{2}{3}n \langle \varepsilon_{transl} \rangle. \quad (5.1)$$

In the thermodynamic method the internal structure of the systems under study and the laws of motion of individual particles are not considered. Thermodynamics studies the properties of macroscopic bodies and processes in them, establishing the connections between the macroscopic properties of matter. It is known that *the gas state* can be characterized by only three thermodynamic parameters - temperature, volume and pressure. Thermodynamics is based on two empirical fundamental laws. The third law (theorem of Nernst) is used in solving a small number of problems. The thermodynamic method makes it possible to study many physical properties of macroscopic systems, but it does not allow us to study phenomena in which the structure of bodies matter plays an important role.

Both methods despite the different approaches allow to study the properties of the substance and the processes occurring in them, complementing each other.

5.2 Maxwell distribution. Velocities of thermal motion of molecules

Consider an ideal gas, which consists of a huge number N of identical molecules and is in a state of thermodynamic equilibrium. Gas molecules move randomly in various directions and at different speeds, which from time to time, due to collisions, change randomly.

Let's single out a infinitesimal interval of thermal motion speeds $[v, v + dv]$; the number dN_v of molecules whose velocities fall in this interval is proportional to the width of this interval $dN_v \sim dv$. We introduce the distribution function:

$$F(v) = \frac{1}{N} \cdot \frac{dN_v}{dv}, \quad (5.2)$$

Then,

$$dN_v = N \cdot F(v)dv, \quad (5.3)$$

The velocity distribution function of molecules $F(v)$ determines the fraction of total number of molecules, whose speeds are in a unit interval around a given value of v . J.K. Maxwell theoretically derived the form of this function (1860):

$$F(v) = 4\pi \left(\frac{m_0}{2\pi kT} \right)^{3/2} \cdot v^2 \cdot \exp\left(-\frac{m_0 v^2}{2kT}\right), \quad (5.4)$$

where m_0 is the mass of a molecule;
 $k = 1.38 \cdot 10^{-23}$ J/K is the Boltzmann constant.

The function $F(v)$ satisfies the normalization condition:

$$\int_0^{\infty} F(v)dv = 1. \quad (5.5)$$

With increasing temperature, the maximum of the velocity distribution function of the molecules will shift to the right (figure 5.1). However, the area bounded by the curve remains unchanged.

The speed corresponding to the maximum of the function $F(v)$ is called the *most probable*, it is found from the condition that the derivative of the velocity distribution function is equal to zero.

The most probable, average and root mean square gas molecules speeds respectively are:

$$v_b = \sqrt{\frac{2kT}{m_0}} = \sqrt{\frac{2RT}{\mu}}, \quad (5.6)$$

$$\langle v \rangle = \int_0^{\infty} vF(v)dv = \sqrt{\frac{8kT}{m_0}} = \sqrt{\frac{8RT}{\mu}}, \quad (5.7)$$

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m_0}} = \sqrt{\frac{3RT}{\mu}}. \quad (5.8)$$

Maxwell's distribution of relative speeds has the form:

$$f(u) = \frac{1}{N} \frac{dN_u}{du} = \frac{4}{\sqrt{\pi}} e^{-u^2} \cdot u^2, \quad (5.9)$$

where $u = v/v_b$ is relative speed.

5.3 Barometric formula. Boltzmann distribution for particles in a potential field

All previously presented equations and the results of the molecular-kinetic theory were obtained without taking into account external force fields. Molecules of any gas are in the Earth's gravitational field. Gravity, on the one hand, and the thermal motion of molecules, on the other hand, lead to a certain stationary state of the gas, in which the gas pressure decreases with height:

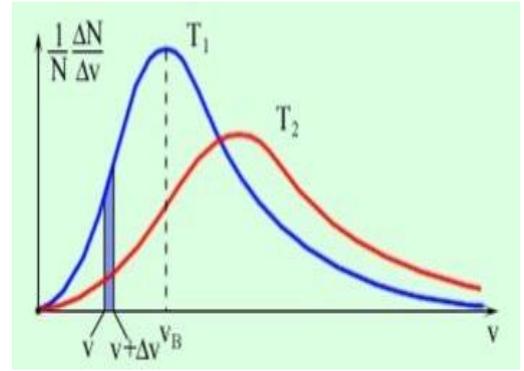


Figure 5.1

$$p = p_0 e^{-\frac{\mu gh}{RT}}, \quad (5.10)$$

where p is the gas pressure at a height h ;

p_0 is the gas pressure at the level $h = 0$.

This equation, which establishes the law of decreasing pressure with height, is called the barometric formula and is used in altimeters — instruments for determining altitude above the Earth by measuring pressure at a given height.

Given that $p = nkT$, it can be written in the form:

$$n = n_0 \cdot \exp\left(-\frac{\mu gh}{RT}\right), \quad (5.11)$$

where n is the concentration of molecules at height h ;

n_0 is the concentration of particles at the zero level of potential energy.

Since $\mu = m_0 N_A$ (N_A is the Avogadro constant, m_0 is the mass of one molecule) and $R = k \cdot N_A$ then

$$n = n_0 \cdot \exp(-\varepsilon_p/RT), \quad (5.12)$$

where $m_0 gh = \varepsilon_p$ is the potential energy of a molecule in the gravitational field.

The relation (5.12) is called *the Boltzmann distribution* for the particles in the external potential field. It follows from (5.12) that at a constant temperature the density of the gas is greater where the potential energy of its molecules is less. If particles with the same mass are in a state of chaotic thermal motion the Boltzmann distribution (5.12) is valid in any external potential field and not just in the field of gravity.

The relation (5.11) shows that as the temperature is lowered the number particles at all heights, except zero, decreases vanishing at $T = 0$. At $T = 0$ K all the molecules would be located on the surface of the Earth.

At high temperatures the concentration decreases slightly with height so the molecules are distributed in height almost uniformly. This can be understood if we take into account two factors: a) the attraction of molecules to the Earth, b) thermal motion with a tendency to equalize the concentration of molecules in height.

Boltzmann proved that his law is valid for systems of any identical particles that are in a state of chaotic thermal motion in any potential field of forces.

5.4 Average energy of molecules

Let's compare two expressions for pressure, namely:

$$p = (2/3)n\langle\varepsilon_{transl}\rangle \quad \text{and} \quad p = nkT.$$

Equating the right-hand sides of these equations we obtain

$$\langle \varepsilon_T \rangle_{transl} = \frac{2}{3} kT. \quad (5.13)$$

Equation (5.13) has a *deep physical meaning*: the thermodynamic temperature is directly proportional to the average energy of the translational motion of the molecules. This dependence is valid only in classical physics for gases and is violated in quantum physics. From this equation it follows that the average energy of the molecules depends only on the temperature and does not depend on the mass of the molecule. This circumstance can be used to establish a temperature scale that does not depend on the properties of the thermometric body. The readings of mercury or alcohol thermometers, coinciding with each other at 0°C and 100°C due to the corresponding calibration method, differ at other temperatures. We point out that the thermodynamic temperature scale established on the basis of the second law of thermodynamics has been spared from such shortcomings and, in addition, 1 *Kelvin* of this scale is equal 1°C .

In expression (5.13) only translational motion is represented, which is characteristic of the simplest monatomic molecules. In diatomic and more complex molecules, not only translational motion of molecules is observed, but also rotational and oscillational modes of motion. With these types of motion, additional energy of molecules is connected and it is determined by means of the statistical law on equipartition of energy in degrees of freedom.

First we introduce the concept of *degrees of freedom*. *The number of degrees of freedom* of a mechanical system is the number of independent coordinates by which the position of the system in space is specified. For example, to specify the location of a *material point* it is three coordinates x, y, z are given, i.e. it *has three degrees of freedom*. *Rigid body* has *six degrees of freedom* due to the absence (in its definition) of the oscillational degrees of freedom: three translational and three rotational degrees.

In molecules with a rigid bond, the number of degrees of freedom is equal to: for monatomic (He, Ne, etc.) molecules, only 3 translational degrees of freedom, for diatomic (H_2 , O_2 , etc.) - 5 degrees of freedom (3 translational and 2 rotational) and for polyatomic (CO_2 , H_2O , etc.) - 6 degrees of freedom (3 translational and 3 rotational).

The *law of equipartition of energy* states that molecules in thermal equilibrium, on the average, have the same energy associated with each degree of freedom and that the energy is:

$$\langle \varepsilon_1 \rangle = \frac{1}{2} kT. \quad (5.14)$$

Then the average energy of the molecule is determined by its number of degrees of freedom i as:

$$\langle \varepsilon \rangle = \frac{i}{2} kT, \quad (5.15)$$

where $i = n_T + n_R + 2n_O$ is the sum of the number of translational n_T , rotational n_R and oscillational n_O degrees of freedom. The presence of a coefficient 2 in front of the number of oscillational degrees of freedom is explained by the fact that the oscillational motion possesses not only kinetic energy but also potential energy. The oscillational degrees of freedom of gas molecules manifest at elevated temperatures. At room temperatures, they can be ignored.

6.5 Internal energy of an ideal gas

The internal energy of a body is the energy contained within the body, but does not include the kinetic energy of motion of the body as a whole, nor the potential energy of a body as a whole due to external force fields. The internal energy of the body is the sum of the kinetic energies of the thermal motion of the body's particles and the potential energy of interaction of the particles with each other.

An ideal gas is a gas of molecules that do not interact at a distance. Therefore, its internal energy added only from the kinetic energies of all its molecules:

$$U = \sum_{i=1}^N \varepsilon_i = N \cdot \langle \varepsilon \rangle = N \frac{i}{2} kT = \frac{i}{2} \nu RT, \quad (5.16)$$

where $\nu = N/N_A = m/\mu$ amount of substance in moles,

$R = 8,31 \text{ J}/(\text{mol} \cdot \text{K})$ – universal gas constant.

So, the internal energy is a function of the state; internal energy of an ideal gas depends only on amount of its substance, its temperature and the internal structure of its molecules:

$$U = \frac{i}{2} \nu RT = \frac{i}{2} \cdot \frac{m}{\mu} RT = \frac{i}{2} \cdot pV. \quad (5.17)$$

6 Lecture №6. The fundamentals of thermodynamics

The objective of the lecture: introduce the basic concepts and laws of thermodynamics.

6.1 Heat and work. First law of thermodynamics

When the thermodynamic system interacts with the environment, energy is exchanged. In this case, two qualitatively different methods of energy transfer are possible - heat and work.

The energy transferred to the system with a change in its external parameters (for example, volume) is called work A . A gas enclosed in a cylindrical vessel with an easily sliding piston, when expanded under constant pressure, does the work:

$$A = p \cdot \Delta V = p(V_2 - V_1). \quad (6.1)$$

If during any thermal process the gas pressure changes, then work done by the gas in this process is:

$$A = \int_{V_1}^{V_2} p dV. \quad (6.2)$$

On the p, V -diagram of the process, work done by the gas when its volume changes from V_1 to V_2 is numerically equal to the area of the curved trapezoid bounded by the V axis, the dependence curve $p = p(V)$, and the straight lines V_1 and V_2 . In isothermal expansion of an ideal gas work is:

$$A = \int_{V_1}^{V_2} p dV = \nu RT \int_{V_1}^{V_2} \frac{dV}{V} = \nu RT \ln \frac{V_2}{V_1} = p_1 V_1 \ln \frac{V_2}{V_1}. \quad (6.3)$$

For a system undergoing only thermodynamics processes, the change in the internal energy during the transition of the system from initial equilibrium state 1 to final equilibrium state 2 is equal to the sum of the work A' of external forces over the system and transferred to the system of the amount of heat Q :

$$\Delta U = Q + A', \quad (6.4)$$

which constitutes the first law of thermodynamics. The first law of the thermodynamics is a version of the law of conservation of energy, adapted for thermodynamic systems.

When equilibrium thermodynamic processes occur, the system's work A against external forces is equal to the work A' of external forces over the system with a minus sign: $A = -A'$. Thus, the first law of thermodynamics may be written in form:

$$Q = \Delta U + A, \quad (6.5)$$

i.e. the amount of heat transferred to the system is expended on increasing the internal energy of the system and the work done by the system over external bodies.

In differential form first law of thermodynamics:

$$\delta Q = dU + \delta A. \quad (6.6)$$

For an ideal gas, the first law has the form:

$$\delta Q = \frac{m}{\mu} \cdot C_v dT + p dV, \quad (6.7)$$

where $C_v = \frac{i}{2} R$ is the molar heat capacity at constant volume $V = \text{const.}$

The differential dU is complete and the internal energy is a function of the state. This means that when a system performs a circular process (cycle), i.e. the system returns to its initial (original) state, the change in internal energy is zero:

$$\oint dU = 0.$$

In contrast to the internal energy of the system the amount of heat and work are functions of process, so their integrals in the transition are expressed differently:

$$\int_1^2 \delta Q = Q_{12} \quad \text{and} \quad \int_1^2 \delta A = A_{12}.$$

6.2 Reversible and irreversible thermal processes

Thermodynamic processes can be carried out in one of two ways: reversibly or irreversibly. A thermodynamic process is reversible if the process can return back in such a that both the system and the surroundings return to their original states, with no other change anywhere else. It means both system and surroundings are returned to their initial states at the end of the reverse process. In these processes, a system undergoes through a series of equilibrium states, and when the process reverses, the system passes through exactly the same equilibrium states while returning to its initial state.

In simple words, the process which can reverse back completely is a reversible process. This means that the final properties of the system can perfectly reverse back to the original properties. The process can be perfectly reversible only if the changes in the process are infinitesimally small. In practical situations it is not possible to trace these extremely small changes in extremely small time, hence the reversible process is also an ideal process. The changes that occur during the reversible process are in equilibrium with each other. Reversibility means the reaction operates continuously at quasiequilibrium.

6.3 Heat engine and their efficiency. Carnot cycle. Carnot's theorems

Every single thermodynamic system exists in a particular state. When a system is taken through a series of different states and finally returned to its initial state, a *thermodynamic cycle* is said to have occurred. In the process of going through this cycle, the system *may perform work on its surroundings*, for example by moving a piston, thereby acting as a heat engine.

A heat engine is a periodically acting engine and involves a thermodynamic process that converts the heat supply in it into mechanical work. The types of heat engines are Steam engine, Internal combustion engine and Gas turbine.

There are three main parts in an engine: a hot body called *source*, a *working substance*, and a body called *sink* - it may be *environment*. There must be the *source of heat* of infinite thermal capacity, and that should be at a constant high temperature so that if any amount of heat is withdrawn from it, that does not affect its temperature. It must be some substance through which the heat is absorbed or rejected into the sink. This is the working substance, steam or any gas. There must be a *sink* of infinite thermal capacity, and that should be at a constant low temperature so that if any amount of heat is given to it, that does not affect its temperature.

The efficiency η of the heat engine is the ratio between its output of work A to the heat Q_1 supply of the heat engine (absorbed from the source):

$$\eta = \frac{A}{Q_1}. \quad (6.8)$$

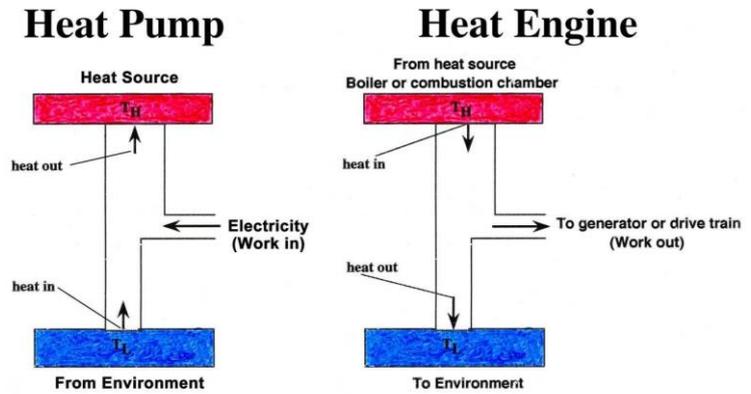


Figure 6.1

After each cycle, the engine returns to its original state so that it does not affect its internal energy $\Delta U = 0$. Then

$$A = Q_1 - Q_2. \quad (6.9)$$

The engine efficiency is:

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}. \quad (6.10)$$

The heat pump is a device used to pump heat into the system. The purpose of a heat pump is to transfer energy to a warm environment, such as interiors of a house in the winters (figure 6.1). The great advantage of using a heat pump to keep your house warm rather than just burning fuel in a fireplace or furnace is that a heat pump supplies.

The reversible cycle performed by the working substance during heat exchange with a heater and an infinitely large capacity condenser can consist of only two isothermal (at heater T_1 and cooler T_2 temperatures) and two adiabatic processes. This circular process (figure 6.2) was called the *Carnot cycle*.

A system undergoing a Carnot cycle is called a Carnot heat engine, although such a "perfect" engine is only a theoretical construct and cannot be built in practice.

Carnot's theorem, developed in 1824 by Sadi Carnot, also called Carnot's rule, is a principle that specifies limits on the maximum efficiency any heat engine can obtain. This theorem states that:

a) no engine working between two given temperatures can be more efficient than a reversible engine working between the same two;

b) all the reversible engines working between the same two temperatures have the same efficiency, whatever the working substance may be. The efficiency of a Carnot heat engine is:

$$\eta = \frac{T_1 - T_2}{T_1} \quad (6.11)$$

To the Carnot theorem, the reversible engine will always have a greater efficiency than the irreversible one.

The reversible heat engine operates on a reverse cycle and functions as a heat pump (or refrigerator), that transmit heat energy to a hot body, in particular a heating system. This energy is taken from an environment with a lower temperature and is generated as a result of mechanical work, for example, a compressor.

Heat pumps are more efficient for heating than resistance heaters because most of the energy they release comes from the external environment, only a fraction of which comes from energy required to run the device. In electrically-powered heat pumps, the heat transferred can be three or four times larger than the electrical power consumed, giving the system a coefficient of performance (COP) of 3 or 4, as opposed to a COP of 1 for a conventional electrical resistance heater, in which all heat is produced from input electrical energy.

6.4 The second law of thermodynamics. Entropy of the thermodynamic system and its properties

The second law is concerned with the direction of natural processes. The second law has been expressed in many ways. According to Clausius: *heat always flows spontaneously from hotter to colder bodies*, and never the reverse, unless external work is performed on the system. According to Kelvin: *It is impossible to convert heat into work completely in a cyclic process*.

The Kelvin–Planck statement (or the *heat engine statement*) of the second law states that *it is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work*.

All this empirical formulations are in agree with each other. The second law of thermodynamics agrees with the statement about *the impossibility of creating a perpetual motion machine*: perpetual motion of the second kind or a periodically acting

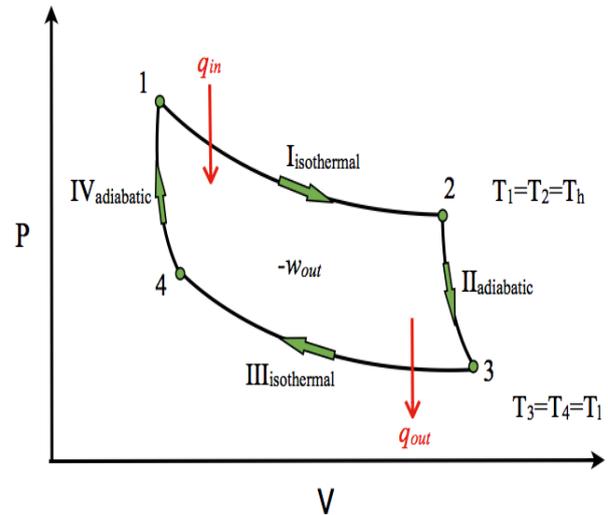


Figure 6.2

engine that receives heat from any reservoir and turns it completely into work *is impossible*.

The Clausius theorem (1854) states that in a cyclic process:

$$\oint \frac{\delta Q}{T} \leq 0. \quad (6.12)$$

The equality holds in the reversible case and the strict inequality holds in the irreversible case. The reversible case is used to introduce the state function entropy. This is because in cyclic processes the variation of a state function is zero from state functionality. According to the Clausius equality (6.12) for reversible processes the line integral

$$\int_1^2 \frac{\delta Q}{T} \quad (6.13)$$

is path-independent. So we can define a state function S called entropy, which satisfies: in reversible processes

$$dS = \frac{\delta Q_{rev}}{T}. \quad (6.14)$$

To find the entropy difference between any two states of a system, the integral

$$\Delta S = \int_1^2 \frac{\delta Q}{T} \quad (6.15)$$

must be evaluated for some reversible path between the initial and final states. Since entropy is a state function, the entropy change of the system for an irreversible path is the same as for a reversible path between the same two states.

The change in entropy of an ideal gas when it transit from initial state 1 to final state 2 is:

$$\Delta S = \int_1^2 \frac{(\nu C_V dT + p dV)}{T} = \nu C_V \ln \frac{T_2}{T_1} + \nu R \ln \frac{V_2}{V_1}. \quad (6.16)$$

In statistical physics, *entropy* is an extensive property of a thermodynamic system and relates with the *statistical weight* Ω . It is necessary at first to introduce the concepts of *microstates* and *macrostates* of a thermodynamic system. *The state of the system* determined with the help of macroscopic parameters – temperature T , pressure p and volume V , is called *macroscopic*. *In the equilibrium state* of the system (for example, gas), the macro parameters and, consequently, the macro state are preserved and the microstate constantly changes as a result of collisions of molecules and changes in their impulses. Thus the macro state is realized in various ways and different microstates correspond to it. The number of different microstates corresponding to a given macrostate is called its *statistical weight* Ω .

Entropy expresses the number Ω of different configurations that a system defined by macroscopic variables could assume. Boltzmann obtained that under the assumption that each microstate is equally probable, the entropy S is:

$$S = k \cdot \ln \Omega, \quad (6.17)$$

where k is the Boltzmann constant.

The relation (6.17) is called Boltzmann's formula.

The main properties of entropy are:

- a) entropy is a function of the state of a thermodynamic system;
- b) entropy is an additive quantity, i.e. if two systems with entropy S_1 and S_2 , respectively, are brought into contact with each other, their combined entropy equals $S_1 + S_2$;
- c) at reversible adiabatic process (without heat transfer between thermodynamic system and its surroundings) the entropy of the system remains constant. So, the reversible adiabatic process is isentropic.

The *second law of thermodynamics* states that the entropy of an isolated system can never decrease over time, and is constant if all processes are reversible. Isolated systems spontaneously evolve towards thermodynamic equilibrium, the state with maximum entropy.

Thus, the entropy of an isolated thermodynamic system can only increase and, after reaching the maximum value at equilibrium, will remain constant:

$$\Delta S \geq 0. \quad (6.18)$$

The macro state of thermodynamic system that is realized by a small number of microstates is called *ordered* or *nonrandom*. If very large number of configurations implements the macro state of system, such state is called *chaotic* or *random*. Thus, the *entropy is a quantitative measure of the disorder* in a system.

The Nernst theorem - the third law of thermodynamics – states that *the entropy is zero at absolute zero temperature*:

$$\lim_{T \rightarrow 0} S = 0. \quad (6.19)$$

7 Lecture №7. Electrostatic field in vacuum and substance

The objective of the lecture: to introduce the basic concepts and laws of electrostatics; familiarize with the basic methods of electrostatics.

7.1 Electrostatic field in vacuum. Intensity and potential of electric field. The main task of electrostatics and method of solving

Electromagnetic interaction is one of the four fundamental interactions between elementary particles. Electromagnetic interaction determines the majority of

physical properties of macroscopic bodies and the change of these properties in the transition from one aggregate state to another. The overwhelming majority of forces - elasticity, friction, surface tension, etc. - are of an electromagnetic nature. At the microscopic level, the electromagnetic interaction determines the structure of the electron shells of atoms, the structure of molecules and crystals.

The electromagnetic field is a fundamental physical field acting on electrically charged particles (bodies), currents, as well as particles and bodies that have their own electrical or magnetic moments. It is a combination of electric and magnetic fields that can, under certain conditions, generate each other. Maxwell first introduced the concept of an electromagnetic field into physics.

Electric charge q is a scalar physical quantity that determines the ability of bodies to participate in electromagnetic interaction. For the first time the electric charge was introduced in Coulomb's law in 1785.

a) There are two kinds of electricity, one of them is positive, and the other is negative. A positive charge appears on the glass, rubbed with the skin, and the negative one - on ebonite or amber, rubbed with wool. Charged bodies of the same name repel, unlike charged - are attracted to each other. Electric charge carriers are electrically charged elementary particles.

b) The quantization of electric charges - the charge q of any body is discrete: it is a multiple $q = N \cdot e$, where $e = 1.6 \cdot 10^{-19}$ C is an elementary charge, N is an integer. Elementary charge e is the smallest portion of electric charge that exists in nature. The smallest by mass of a stable particle, having one negative elementary electric charge, is an electron; the positive one - is positron.

c) The charge is additive - the charge of a system of bodies is equal to the sum of the charges of the individual bodies making up the system $q = \sum q_i$. *The law of charge conservation: the algebraic sum of the electric charges of bodies or particles forming an electrically isolated system remains unchanged for any processes occurring in the system:*

$$\sum_{i=1}^N q_i = \text{const.} \quad (7.1)$$

d) The electric charge is an invariant quantity, which means that its magnitude does not change when passing from one inertial reference frame to another.

Charge is a basic property of matter. Every constituent of matter has an electric charge with a value that can be positive, negative, or zero. Most bulk matter has an equal amount of positive and negative charge and thus has zero net charge.

According to Coulomb's law, the interaction force between two point charges q_1 and q_2 in a vacuum is proportional to the product of their charges and inversely proportional to the square of a distance r between them:

$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} \cdot \frac{\vec{r}_{12}}{r}, \quad (7.2)$$

where \vec{F}_{12} is the force acting on the charge q_2 on the charge side q_1 ;

\vec{r}_{12} is the radius vector connecting the charge q_1 with the charge q_2 ;
 $\epsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the electric constant.

Electric field, an electric property associated with each point in space when charge is present in any form. An *electrostatic field* is an electric field produced by static charges. It manifests itself by a force action on any charged particles, which is proportional to a charge q_0 of this particle, but also depends on the field characteristic at the point where this charge is placed:

$$\vec{F} = q_0 \cdot \vec{E}. \quad (7.3)$$

The magnitude and direction of an electric field are expressed by the quantity \vec{E} , called *the electric field strength* or *electric field intensity*, or simply *the electric field*. The magnitude of the electric field \vec{E} at a given point in space in direction is coinciding and numerically is equal to the force that would be exerted on a positive unit point charge at that position in space:

$$\vec{E} = \frac{\vec{F}}{q_0}. \quad (7.4)$$

The direction of the electric field depends on the sign of the charge q . The electric field is directed radially outward if point charge q is positive, and radially inward if q is negative. The field of a point charge q in vacuum at a distance r is:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \frac{\vec{r}}{r}. \quad (7.5)$$

In order to visualize the fields, lines of its intensity are introduced - these are imaginary lines whose tangents coincide in direction with the electric field \vec{E} at a given point. The lines are drawn so that the number of lines passing through a unit of surface perpendicular to them is equal to the magnitude of field at a given location. An electric field is called *uniform* if the electric field \vec{E} is the same at all points of the field. The lines of the electrostatic field, starting at the positive charge go to infinity, or, coming from infinity, terminate on a negative charge. Figure 7.1 illustrates a uniform field between the plates of a flat capacitor.

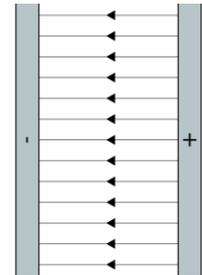


Figure 7.1

If in the electrostatic field of a point charge q , moves another point charge q_0 then the field force applied to the charge q_0 do the work. The work of the field force on an elementary displacement $d\vec{r}$ is equal to:

$$dA = \vec{F} \cdot d\vec{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q \cdot q_0}{r^2} \cdot \frac{\vec{r}}{r} \cdot d\vec{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q \cdot q_0}{r^2} \cdot dr. \quad (7.6)$$

When the charge q_0 moving from initial point 1 to point 2 the work is:

$$A_{12} = \int_{r_1}^{r_2} dA = \frac{q \cdot q_0}{4\pi\epsilon_0} \int_{r_1}^{r_2} \frac{dr}{r^2} = \frac{1}{4\pi\epsilon_0} \left(\frac{qq_0}{r_1} - \frac{qq_0}{r_2} \right). \quad (7.7)$$

This work does not depend on the trajectory of movement, but is determined only by the positions of start and finish points. Conclusion: *the electrostatic field is potential.*

Work done by a potential field force equal to the decrease of the potential energy:

$$A_{12} = W_{p1} - W_{p2} \quad (7.8)$$

A point charge q_0 in the field produced by another point charge q has a potential energy:

$$W_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r}. \quad (7.9)$$

The quantity, equal to the ratio:

$$\varphi = \frac{W_p}{q_0} \quad (7.10)$$

does not depend on the magnitude of this charge, it is called an electric potential.

The electric potential arising from a point charge q , at a distance r from the charge is:

$$\varphi(r) = \frac{q}{4\pi\epsilon_0 r} \quad (7.11)$$

The work of electric forces is equal to change of potential energy of a charge, which is moving:

$$A_{12} = W_{p1} - W_{p2} = q_0(\varphi_1 - \varphi_2). \quad (7.12)$$

The potential difference between two points of the electrostatic field:

$$\varphi_1 - \varphi_2 = \frac{A_{12}}{q_0}. \quad (7.13)$$

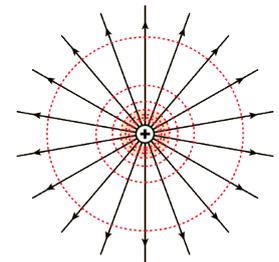


Figure 7.2

An imaginary surface all of whose points have the same potential is called the equipotential surface. Its equations has the form: $\varphi(x,y,z) = \text{const}$. The figure 7.2 shows the fields lines and equipotential surfaces (spheres) of a positive point charge.

7.2 The circulation theorem of an electrostatic field. Relationship between the electric field strength and potential

The theorem wording: the circulation of an electrostatic field along arbitrary loop is zero:

$$\oint_L \vec{E} \cdot d\vec{l} = 0, \quad (7.14)$$

that is an mathematical expression of the electrostatic fields potentiality.

The relationship between an electrostatic field strength and potential is:

$$\vec{E} = -grad\varphi = -\left(\vec{i}\frac{\partial\varphi}{\partial x} + \vec{j}\frac{\partial\varphi}{\partial y} + \vec{k}\frac{\partial\varphi}{\partial z}\right), \quad (7.15)$$

$$E_x = -\frac{\partial\varphi}{\partial x}, \quad E_y = -\frac{\partial\varphi}{\partial y}, \quad E_z = -\frac{\partial\varphi}{\partial z}. \quad (7.16)$$

for an uniform electric field $E = \frac{\Delta\varphi}{d}$.

Lines of \vec{E} directed towards steepest decreasing of potential along the normal to the equipotential surfaces, and its module is equal to the change in potential per unit length. Lines of the field and the equipotential surfaces are orthogonal. The field \vec{E} at every point is directed along a normal to the equipotential surface.

The principle of superposition states that every charge in space creates an electric field at point independent of the presence of other charges in that medium. The resultant electric field is a vector sum of the electric field due to individual charges:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_i + \dots + \vec{E}_N = \sum_{i=1}^N \vec{E}_i. \quad (7.17)$$

The superposition of the potentials follows from the superposition of the field \vec{E} and states that to find the electric potential at a point due to a collection of charges, simply add up the electric potential at that point due to each individual charge:

$$\varphi = \sum_i \varphi_i. \quad (7.18)$$

When considering macroscopic charges, they are distracted from their discrete (intermittent) structure and consider them distributed throughout the space in a continuous manner with finite density. Then, the superposition principle has the form:

$$\vec{E} = \int d\vec{E}, \quad \text{and} \quad \varphi = \int d\varphi. \quad (7.19)$$

The main objective of electrostatics is to find the main characteristics of the field at a given charge distribution. General solution method is based on the application of the superposition principle and formulas which determine the strength and potential of a point charge field, i.e. (7.5) and (7.11) respectively.

7.3 Electric flux. Gauss's law

The electric flux passing through a plane surface of area S in uniform field is:

$$\Phi = E \cdot S \cdot \cos \alpha = E_n \cdot S,$$

where α is the angle between the electric field \vec{E} and the unit normal \vec{n} to surface S . In general:

$$\Phi_E = \int_S \vec{E} \cdot d\vec{S} = \int_S E_n dS. \quad (7.20)$$

Electric flux through any surface is numerically equal to the number of the electric field lines passing through this surface.

Gauss's law states that "*the total electric flux through any closed surface in free space of any shape drawn in an electric field is equal to the total electric charge enclosed by the surface, divided by ϵ_0* ":

$$\oint_S E_n dS = \frac{1}{\epsilon_0} \sum_{i=1}^N q_i^{enclosed S}. \quad (7.21)$$

In the case of a closed surface, it is conventionally that the positive normal vector at each of its points is directed outwards. Gauss's theorem implies that the lines of the electrostatic fields are beginning at the positive charges and are ending at the negative charges.

7.4 Electric field in dielectrics. Polarization of dielectrics. The polarization density. Bound charges

Substance placed in the electric field can significantly change it. This is due to the fact that the substance consists of charged particles. In general, an atom and a molecule, as well as an ionic crystal in a normal state, are electrically neutral: the number of positive charges per unit volume of a substance is equal to the number of negative charges. Nevertheless, on the example of the simplest electrically neutral system - the electric dipole - we will see that the electrically neutral system also creates an electric field.

Two point charges, one with charge $+q$ and the other one with charge $-q$ separated by a distance l , constitute an *electric dipole*. The product of a charge q and displacement vector \vec{l} , pointing from $-q$ to $+q$, is the *electric moment* of a dipole:

$$\vec{p}_e = q\vec{l}. \quad (7.22)$$

The dipole field (Figure 7.3) at a point remote from the center of the dipole at a distance r decreases inversely with the distance in the cube.

In an external electric field, an electric dipole is affected by the moment of forces (torque), which tends to align the dipole with the field:

$$\vec{M} = [\vec{p}_e, \vec{E}] = \vec{p}_e \times \vec{E}. \quad (7.23)$$

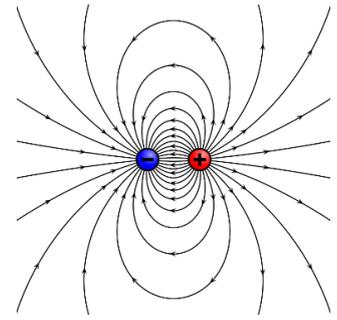


Figure 7.3

Many molecules have such dipole moments due to the inhomogeneous distribution of positive and negative charges in atoms. Consequently, a neutral molecule with an electric moment creates an electric field.

A *dielectric* is an electrical insulator that can be polarized by an applied electric field. When a dielectric is placed in an electric field, electric charges do not flow through the material as they do in an electrical conductor but only slightly shift from their average equilibrium positions causing dielectric polarization. Because of dielectric polarization, positive charges are displaced in the direction of the field and negative charges shift in the direction opposite to the field. This creates an internal electric field that reduces the overall field within the dielectric itself.

Dielectrics are divided into the following types:

a) non-polar dielectrics are substances whose molecules in the absence of an external electric field do not have an electric moment. Molecules acquire such a moment in an external electric field, due to the fact that its positive and negative charges are displaced by the opposite forces of the field. These displacements are much smaller in size than the molecules, but as a result, each molecule acquires an electric moment. Nonpolar dielectrics include H_2 , O_2 , N_2 , etc.

b) polar dielectrics are materials which consists of molecules which are permanent dipoles. In the absence of external electric field, the polar molecules are randomly oriented. As the result, these materials have no net dipole moment. In an electric field, the dipoles rotates along the field under the action of a torque (7.22), and the dielectric acquires a nonzero resultant moment. Polar dielectrics: H_2O , NH_3 , CO , etc;

c) crystalline dielectrics ($NaCl$, KCl) with an ionic lattice - substances whose internal structure is a spatial lattice with the correct alternation of ions of different signs. This crystal can be considered as a system of two ionic sublattices pushed into one another. Under the action of an external electric field, a relative displacement of the sublattices occurs, leading to the appearance of dipole moment.

There are other types of dielectrics, for example, ferroelectrics, electrets, piezoelectrics, pyroelectrics.

The polarization of dielectrics is a phenomenon associated with a limited displacement of bound charges in the dielectric or rotation of electric dipoles, under the influence of an external electric field, as a result of which the dielectric acquires an

electric moment. Accordingly, three polarization mechanisms are distinguished into three groups of dielectrics: a) electron or distortion polarization; b) orientation, or dipole, polarization; c) ionic polarization.

As a result of polarization, charges called bonded appears on the surfaces of the dielectric. Bound charges are opposite charges that are part of atoms (or molecules) and cannot move independently of each other under the influence of an electric field.

To quantify the polarization of a dielectric, polarization density is introduced, defined as the electric moment of a unit volume of a dielectric:

$$\vec{P} = \frac{1}{V} \sum_i \vec{p}_{ei}. \quad (7.24)$$

The normal to the surface component of the polarization is related to the surface density of bound charges σ' on this surface:

$$P_n = \sigma'. \quad (7.25)$$

The field in the dielectric, according to the principle of superposition, is equal to the geometric sum of the external field and the field created by the bound charges:

$$\vec{E} = \vec{E}_0 + \vec{E}'. \quad (7.26)$$

Consider the field in the dielectric as an example of a flat plate placed in a uniform electrostatic field \vec{E}_0 (Figure 7.4). The plate is located so that its surfaces are perpendicular to the direction of the field \vec{E}_0 . The appearance of bound charges on the plate surfaces leads to the appearance of an additional electric field \vec{E}' , which inside the plate is directed against the external field \vec{E}_0 and weakens it:

$$E = E_0 - E' = E_0 - \frac{\sigma'}{\varepsilon_0} = E_0 - \frac{P}{\varepsilon_0}.$$

For a large class of dielectrics, the polarization linearly depends on the field strength, and if the dielectric is isotropic, then it coincides with it in the direction:

$$\vec{P} = \alpha \varepsilon_0 \vec{E}, \quad (7.27)$$

where α is the dielectric susceptibility of the substance. Then:

$$E = \frac{E_0}{1+\alpha} = \frac{E_0}{\varepsilon}, \quad (7.28)$$

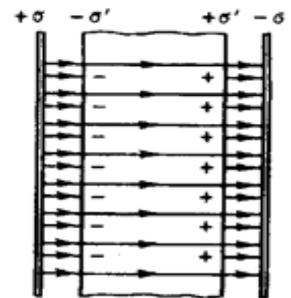


Figure 7.4

where ε is the electric permittivity of the material. It shows how many times the field in vacuum is larger than the field in a dielectric medium.

It should be noted that relation (7.27) is valid only in two cases: a) if a dielectric fills the entire field region; b) if a dielectric occupies a certain region of the field bounded by two equipotential surfaces.

Since the sources of the field are all charges - free and bound, it is impossible to determine the intensity \vec{E} because the distribution of the bound charges in the dielectric is unknown. Then was introduced the electric displacement field \vec{D} as the sum:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}. \quad (7.29)$$

In a homogeneous isotropic dielectric the fields \vec{P} and \vec{E} coincide in direction, therefore, the direction of the vector \vec{D} is the same as that of the \vec{E} :

$$\vec{D} = \varepsilon_0 \vec{E} + \alpha \varepsilon_0 \vec{E} = \varepsilon_0 (1 + \alpha) \vec{E} = \varepsilon_0 \varepsilon \vec{E}. \quad (7.30)$$

In anisotropic dielectrics, the directions of \vec{D} and \vec{E} can be different.

7.5 Gauss' law for electric displacement field. Boundary conditions

The electric displacement vector has a remarkable property, due to which the calculation of the electric field in the dielectric is simplified. This property is revealed by the Gauss' theorem: the total flux of the electric displacement field \vec{D} through an arbitrary closed surface S is equal to the algebraic sum of the free electric charges enclosed by the surface:

$$\oint_S D_n dS = \sum_{i=1}^N q_i. \quad (7.31)$$

At the boundary between two dielectrics with permittivity's ε_1 and ε_2 , if there are no electric charges ($\sigma = 0$), the following boundary conditions are satisfied:

$$E_{1\tau} = E_{2\tau}; \quad \frac{D_{1\tau}}{\varepsilon_1} = \frac{D_{2\tau}}{\varepsilon_2} \quad (7.32)$$

$$D_{1n} = D_{2n}; \quad \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \quad (7.33)$$

7.6 Conductor in an electrostatic field. Conditions of charge equilibrium in a conductor. Electrostatic induction

The charge carriers in a conductor are able to move under the action of an arbitrarily small force. Therefore, for the equilibrium of any charged particle on a conductor, the following conditions must be fulfilled:

a) the electric field is zero everywhere inside the conducting material;

b) The electric field just outside a conductor directs normal to the its surface:

$$\vec{E} = \vec{n}E_n. \quad (7.34)$$

This means that the potential difference of any pair of points both inside the conductor and on its surface is zero $\Delta\varphi=0$. In the absence of current, all points of a conductor have the same potential $\varphi = const$.

When an uncharged conductor is introduced into the electric field, the charge carriers in the conductor begin to move: positive in the direction of the field \vec{E} , negative ones - in the opposite direction. As a result, charges of the opposite sign arise on opposite surfaces of the conductor (Figure 7.5). This charges called *induced charges*. *The phenomenon of redistribution of charges on a conductor under the influence of an external electrostatic field is called the electrostatic induction.*

Earth (ground) is also a conductor. Although there are currents in the earth, they are small, and we can assume that the charges of the earth are close to equilibrium. Therefore, we can assume that all points of the Earth have the same potential.

7.7 Charge distribution in the conductor. Electrostatic protection

If some excess of charge could be placed inside the conductor, the repulsive forces would push them as far apart as possible, causing them to migrate to the surface. The charges moving apart until an equilibrium is achieved. So, *any excess charge on an isolated conductor resides entirely only on its surface.*

If to connect two conductors by metal wire, then both conductors and a wire form a single conductor. If initially there are a potential difference, the field E inside the wire acts the free electrons moves, ie, an electric current would appear in it. This current will continue until the potentials of both conductors become equal.

Thus, the electrostatic field, when an uncharged conductor is introduced into it, changes everywhere, both near the conductor and inside it (figure 7.5).

The field intensity in vacuum just near the surface of a conductor is proportional to charge density σ on the surface:

$$E = \sigma/\epsilon_0. \quad (7.35)$$

If you remove the inner part from a solid conductor, you get a hollow closed conductor. Since there are no charges inside the conductor, the removal of any internal part will not change either the field or the charge distribution. Therefore, the equilibrium distribution of charges in a hollow conductor will remain the same as in a solid conductor - charges will only be on the outer surface.

In this case, the field \vec{E} is equal to zero at any point both inside the cavity and inside its walls. When such a conductor is introduced into an external electric field, induced charges will appear on it. These charges will also be concentrated on the

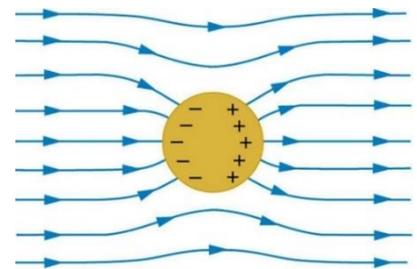


Figure 7.5

outer surface, and the electric field inside the cavity, as in the thickness of the metal, will remain equal to zero. Therefore, the internal cavity in the conductor is shielded from the electric fields of all external charges. This is used in practice for electrostatic protection people and equipment, from the influence of an external electrostatic field.

8 Lecture №8. Energy of electrostatic field. Electric current

The objective of the lecture: to introduce the basic relations for the energy of electrostatic field; introduce the basic concepts and laws of direct current.

8.1 The interaction energy of a system of point charges

The mutual potential energy of the point charges q_1 and q_2 located at a distance r from each other can be considered as the potential energy of the charge q_1 located in the field of charge q_2 , and vice versa:

$$W_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r} = q_1 \varphi_1 = q_2 \varphi_2 = \frac{1}{2} (q_1 \varphi_1 + q_2 \varphi_2), \quad (8.1)$$

where $\varphi_1 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r}$ and $\varphi_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$ respectively, the potentials created by:

- a) charge q_2 at the location of the first charge q_1 ,
- b) charge q_1 at the location of the second charge q_2 .

In the case of a system of n fixed charges, the potential energy of their interaction is equal to:

$$W_{B3} = \frac{1}{2} \sum_{i=1}^n q_i \varphi_i, \quad (8.2)$$

where φ_i is the potential created at the point where the charge q_i is located, with all other charges.

8.2 Energy of a charged conductor and a charged capacitor

The surface of the conductor is equipotential, i.e. in all points the potential is the same. The charge q located on the conductor can be considered as a system of point charges distributed on its surface. Then the energy of the charged conductor is:

$$W = \frac{1}{2} \sum_{i=1}^n \Delta q_i \varphi_i = \frac{1}{2} \varphi \sum_{i=1}^n \Delta q_i = \frac{1}{2} q \varphi.$$

$$W = \frac{1}{2} q \varphi = \frac{q^2}{2C} = \frac{C \varphi^2}{2}. \quad (8.3)$$

Consider a charged capacitor. Let the potential of a positively charged plate be equal to φ_1 , and the potential of the plate on which the charge $-q$ is located be equal to φ_2 . Then:

$$W = \frac{1}{2}(+q)\varphi_1 - \frac{1}{2}(-q)\varphi_2 = \frac{1}{2}q(\varphi_1 - \varphi_2) = \frac{1}{2}qU = \frac{q^2}{2C} = \frac{CU^2}{2}. \quad (8.4)$$

Energy in the case of a flat capacitor:

$$W = \frac{CU^2}{2} = \frac{\varepsilon\varepsilon_0SU^2}{2d} = \frac{\varepsilon\varepsilon_0}{2}\left(\frac{U}{d}\right)^2 Sd = \frac{\varepsilon\varepsilon_0E^2}{2}V, \quad (8.5)$$

where $Sd = V$ is the volume occupied by a uniform electric field.

8.3 Energy and bulk energy density of an electrostatic field

Relation (8.5) relates energy of a capacitor with a field E and its volume V . This means that the energy of the charged capacitor is concentrated in the electric field between its plates. Consequently, energy is concentrated in an electric field. If the field is homogeneous, the energy enclosed in it is distributed with a constant bulk density w equal to:

$$w = \frac{1}{2}\varepsilon\varepsilon_0E^2 = \frac{\vec{E}\cdot\vec{D}}{2}. \quad (8.6)$$

Bulk energy density w - field energy enclosed in unit volume:

$$w = \frac{dW}{dv}. \quad (8.7)$$

Formula (11.12) is also valid in the case of an inhomogeneous field. Knowing the energy density of the electric field at each point, one can find the energy enclosed in the volume V of the electric field:

$$W = \iiint_V w dV = \iiint_V \frac{\varepsilon\varepsilon_0E^2}{2} dV. \quad (8.8)$$

8.4 Electric current

Electric current is an ordered (directed) movement of electric charges. For the emergence and existence of electric current, two conditions must be met:

- a) the presence in this body of free current carriers - charged particles capable of moving within the whole body;
- b) the presence of an electric field inside the body.

The direction of movement of positive charges is conventionally taken as the direction of the current. The current strength I is a physical quantity determined by the electric charge passing through the cross section of the conductor per unit time:

$$I = dq/qt. \quad (8.9)$$

If the current strength and its direction do not change with time, then such a current is called constant.

A physical vector quantity numerically equal to the current per unit surface area perpendicular to the direction of the current is called the current density:

$$j = \frac{dq}{ds_{\perp} dt} = \frac{dI}{ds_{\perp}}. \quad (8.10)$$

The direction of the current density vector \vec{j} corresponds to the direction of the velocity vector of the ordered motion of positive charge carriers.

The current density vector is related to the average velocity vector of ordered motion (drift) $\langle \vec{v} \rangle$ charges:

$$\vec{j} = qn\langle \vec{v} \rangle, \quad (8.11)$$

where n and q are the concentration and charge of the charge carriers. The current flowing through the surface S :

$$I = \int_S j_n dS. \quad (8.12)$$

The continuity equation expresses the law of conservation of charge:

$$-\frac{dq}{dt} = \oint_S j_n dS; \quad (8.13)$$

that is, the current (current flux of the current density vector \vec{j}) flowing through an arbitrary closed surface S is equal to the rate of decrease of the charge inside the volume covered by this surface.

If an electric field is created in the conductor and no measures are taken to maintain it, the movement of current carriers will very quickly lead to potential equalization at the ends of the conductor and the current will stop. In order to maintain the current for a sufficiently long time, it is necessary to continuously remove the charges brought here by current from the end of the conductor with lower potential (for positive current carriers), and continuously bring them to the end with high potential. The forces of the electrostatic field are not able to carry out such a movement of charges. For this, third-party forces are needed.

Third-party forces are characterized by the work they do on charges moving along the chain. A value equal to the work of this forces to move a unit positive charge is called the electromotive force (emf) acting in the circuit or on its section.

Electromotive force, abbreviated *EMF* (denoted \mathcal{E} and measured in Volts), is the electrical action produced by a non-electrical source. A device that converts other

forms of energy into electrical energy, such as a battery (converting chemical energy) or generator (converting mechanical energy), provides an *EMF* as its output.

If an amount of charge dq passes through a source of *emf*, the amount of work the source does will be:

$$\delta A = \mathcal{E} \cdot dq. \quad (8.14)$$

To characterize the “strength” of a source of electric potential energy, we introduce the concept of electromotive force, or *EMF*. The *EMF* of a source of electric potential energy is defined as the amount of electric energy delivered by the source per coulomb of positive charge as this charge passes through the source from the low-potential terminal to the high-potential terminal. Since the *EMF* is energy per unit charge, its units are volts and it is simply called the voltage of the source.

The forces producing the *EMF* - forces of non-electrostatic origin acting on charges from current sources and causing the movement of electric charges inside a direct current source. Third-party forces are doing work on the separation of charges

$$A_{12} = \int_1^2 \vec{F}_{em} d\vec{l} = q_0 \int_1^2 \vec{E}^* d\vec{l}. \quad (8.15)$$

Third-party forces are characterized by the work they do on charges moving along the chain. A value equal to the work of external forces to move a single positive charge is called the electromotive force (*EMF*) acting in the circuit or on its section:

$$\mathcal{E} = \frac{A^*}{q}. \quad (8.16)$$

Ohm's law for an inhomogeneous section of the circuit has the form:

$$I = \frac{\varphi_1 - \varphi_2 + \mathcal{E}_{12}}{R_{12}}. \quad (8.17)$$

A value numerically equal to the work of transferring a unit positive charge by the total field of Coulomb and external forces in a section of a circuit (1 - 2) is called the voltage in this section U_{12} .

Section of a DC circuit that include current sources are called heterogeneous.

When a unit positive charge moves along a certain part of the circuit, both electrostatic (Coulomb) and non-electrostatic forces perform work. The work of electrostatic forces is equal to the potential difference $\Delta\varphi_{12} = \varphi_1 - \varphi_2$ between the initial 1 and final 2 points of the heterogeneous sector. The work of other force is equal, by definition, to the electromotive force \mathcal{E}_{12} acting in this section. Therefore, the full work is:

$$U_{12} = \varphi_1 - \varphi_2 + \mathcal{E}_{12}. \quad (8.18)$$

The generalized Ohm law in differential form relates the current density \vec{j} to the strength \vec{E} of an electrostatic field and the strength \vec{E}^* of the non-electrostatic field of outside forces through electrical conductivity σ :

$$\vec{j} = \sigma(\vec{E} + \vec{E}^*) = \frac{1}{\rho}(\vec{E} + \vec{E}^*); \quad (8.19)$$

where $\rho = 1/\sigma$ is resistivity of conductor.

DC power is:

$$P = UI = I^2R = U^2/R. \quad (8.20)$$

If a current passes through a fixed metal conductor, then the work done by the current completely converted into heat:

$$Q = UIt = I^2Rt = U^2t/R. \quad (8.21)$$

Joule–Lenz law states that the power of heating generated by an electrical conductor is proportional to the product of its resistance and the square of the current. The differential form of the Joule heating equation gives the power per unit volume:

$$w_T = \frac{dP}{dV} = \vec{j} \cdot \vec{E} = \sigma E^2. \quad (8.23)$$

9 Lecture №9. Stationary magnetic field in vacuum and in substance

The objective of the lecture: to familiarize with the basic characteristics and the properties of the magnetic field.

9.1 Magnetic field. The superposition principle. Biot-Savart-Laplace law

In everyday life, magnetic fields are most often encountered as a force created by permanent magnets, which pull on ferromagnetic materials such as iron, cobalt, or nickel, and attract or repel other magnets. Magnetic fields are widely used throughout modern technology, particularly in electrical engineering and electromechanics. The Earth produces its own magnetic field, which is important in navigation, and it shields the Earth's atmosphere from solar wind. Rotating magnetic fields are used in both electric motors and generators.

In 1820, Danish physicist H. Oersted discovered orienting effect of electric current on a magnetic needle. Almost simultaneously the French physicist Ampere discovered that electric currents attracted and repelled each other similar to magnets. According to Ampere's law the force per unit length between two straight parallel current-carrying conductors is:

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}; \quad (9.1)$$

where $\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$ is a magnetic constant, r is distant between two conductors.

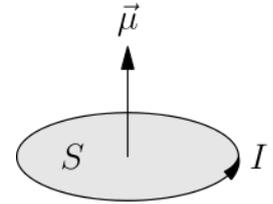


Figure 9.1

It was found that the magnetic interaction is inherent only between moving electric charges and conductors with a current. The magnetic field is special form of matter which carries out the magnetic interaction of currents and moving electric charges. To study the properties of the magnetic field is used with a current carrying loop. Magnetic moment of a current-carrying loop is a current multiplied the area of the loop:

$$\vec{p}_m = IS\vec{n}, \quad (9.2)$$

where \vec{n} is unit normal to the plane of the current carrying loop, the direction of which relates with the current flowing direction by the right screw rule (on figure 9.1 magnetic moment denotes $\vec{\mu}$).

Torque couple of forces acting on current carrying loop is:

$$\vec{M} = [\vec{p}_m, \vec{B}], \quad (9.3)$$

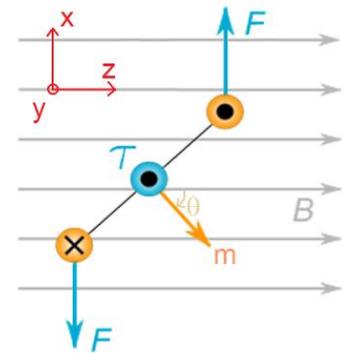


Figure 9.2

where \vec{B} is quantitative characterization of the magnetic field, called magnetic field vector or magnetic flux density. The direction of the magnetic field \vec{B} is taken as the direction of a magnetic arrow freely established in the field.

The torque acting on the frame with the current in the magnetic field acts on it orientationally, trying to establish its magnetic moment along the field direction (figure 9.2). Magnetic field \vec{B} at a given point of a uniform magnetic field is determined numerically by the maximum torque acting on a loop with a magnetic moment equal to one:

$$B = \frac{M_{max}}{p_m}. \quad (9.4)$$

Law of Biot-Savart-Laplace for magnetic field, which creates by an element of a conductor with a current at some point:

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{[Id\vec{l}, \vec{r}]}{r^3}, \quad (9.5)$$

where \vec{r} is the radius-vector from the current element to a given point of the field, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m - magnetic constant.

The direction $d\vec{B}$ is perpendicular to the vectors $d\vec{l}$ and \vec{r} , i.e. perpendicular to the plane in which they lie. This direction is found by the right-hand screw rule: the direction of rotation of the screw head gives direction $d\vec{B}$, if the forward movement of the screw corresponds to the direction of the current in the element $d\vec{l}$.

The modulus of the vector is:

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \alpha}{r^2}, \quad (9.6)$$

where α - the angle between the vectors $Id\vec{l}$ and \vec{r} .

For a magnetic field, as well as for electric, superposition principle is valid: the resulting magnetic induction field generated by several currents, is the vector sum of the magnetic induction fields generated by each current separately:

$$\vec{B} = \sum_{i=1}^N \vec{B}_i. \quad (9.7)$$

Application of the law Biot-Savart-Laplace, together with the principle of superposition of magnetic fields allows you to calculate magnetic field due to current.

Magnetic fields may be represented by continuous lines of force that emerge from north-seeking magnetic poles and enter south-seeking poles. The density of the lines indicate the magnitude of the field, being more concentrated at the poles (where the field is strong) and fanning out and weakening the farther they get from the poles. The direction of the magnetic field at any point is parallel to the direction of nearby field lines.

9.2 Magnetic flux. Gauss theorem

The magnetic flux through a flat surface located in a uniform magnetic field is a scalar product of a magnetic field and area vector of surface:

$$\Phi = \vec{B} \cdot \vec{S} = BS \cos \alpha, \quad (9.8)$$

where α - the angle between the directions of magnetic field \vec{B} and unit normal \vec{n} to the plane surface.

The magnetic flux through a surface located in a non-uniform magnetic field is the surface integral of the normal component of the magnetic field \vec{B} passing through that surface:

$$\Phi = \int_S \vec{B} \cdot d\vec{S} = \int_S B_n dS. \quad (9.9)$$

The SI unit of magnetic flux is the weber (Wb). The magnetic flux is the *net* number of the magnetic field field lines passing through that surface.

Gauss law states that total magnetic flux passing through arbitrary closed surface is zero:

$$\oint_S B_n dS = 0. \quad (9.10)$$

This Gauss theorem for magnetic induction reflects the absence of magnetic charges in nature, so that the magnetic lines have neither beginning nor end, and are closed. The magnetic lines are always closed and cover conductor with current. Vector fields, possessing closed lines, called vortex. The magnetic field is the vortex field.

On the current-carrying conductor in a magnetic field acts a force of Ampere. If the conductor is not fixed, then under the action of Ampere it will move. The magnetic field does work:

$$dA = F_A \cdot dS = I \cdot B \cdot l \cdot dS = I \cdot d\Phi, \quad (9.11)$$

where Φ - magnetic flux through the surface.

Thus, the work on the movement of current-carrying conductor in a magnetic field is numerically equal to the product of the current in the conductor by the magnetic flux swept out by the conductor as it moves.

For a closed loop, in which current is maintained constant, work of the magnetic field during the final movement of the circuit is equal:

$$A_{12} = I\Delta\Phi_{12}, \quad (9.12)$$

where $\Delta\Phi$ - the change in the magnetic flux throw the surface of this closed conducting loop, $\Delta\Phi_{12} = \Phi_2 - \Phi_1$.

9.3 Types of magnetic materials. The magnetization

All material placed in a magnetic field magnetized. The reason for this phenomenon is explained A. Amper putting forward the hypothesis that there is any body microscopic currents caused by the movement of electrons in atoms and molecules. An electron moving in a circular orbit, equivalent to a circular current, so it has an orbital magnetic moment, which module is

$$p_{m.orb} = I \cdot S = evS; \quad (9.13)$$

where I - the equivalent current, ν - the frequency of revolution of an electron in its orbit, S - orbit area.

Subsequently, it was found that in addition to the orbital magnetic moment of the electron has its own (spin) magnetic moment $p_{m.spin}$, which is associated with its own angular momentum, called spin. Spin is an inherent property of the electron, like its charge and mass.

Consequently, the magnetic moment of the electron is the sum of the orbital and spin magnetic moments:

$$\vec{p}_m = \vec{p}_{m.orb} + \vec{p}_{m.spin}. \quad (9.14)$$

Neglecting the magnetic moments of the nuclei can be considered the magnetic moment of the atom as a vector sum of the magnetic moments (orbital and spin) of the electrons within the atom.

The magnetic field in a substance is equal to the sum of an external field and its own field of magnetization:

$$\vec{B} = \vec{B}_{ext} + \vec{B}'_{magn}. \quad (9.15)$$

In an external magnetic field, the electron orbit, oriented relative to the vector \vec{B} in an arbitrary manner, does precession. Precession motion of the electron orbits is equivalent to a circular current generates a magnetic field of an atom. Conducted in this way the magnetic fields of atoms are directed opposite to the external field, and add up to form a magnetic field of a substance that weakens the external magnetic field. This effect is called diamagnetic, and substances magnetized in a magnetic field opposite to the direction of the field, called diamagnetic (Bi, Ag, Au, Cu, C, many organic compounds, etc.). Diamagnetic materials *produce a magnetization that opposes the magnetic field*.

Diamagnetism is present in all substances (rare earths, Pt, Al, etc.), but some of them it is suppressed by the paramagnetic effect. The atoms of paramagnetic substances in the absence of an external magnetic field have a magnetic moment. However, due to thermal motion of the molecules, their magnetic moments are randomly oriented and mutually compensated. When making a paramagnet in an external magnetic field sets the preferred orientation of the magnetic moments of the atoms on the field. Thus, paramagnetic is magnetized, creating its own magnetic field, which coincides with the direction of the external field and amplifies it. This effect is called paramagnetic. Paramagnetic materials *produce a magnetization in the same direction as the applied magnetic field*.

Ferromagnetic these are magnetic materials that have a spontaneous magnetization. They have the following properties:

- Magnetic permeability reaches high values (up to 10^6);

- The dependence of the magnetic permeability μ of the external magnetic field, i.e., the relation between the magnetization vectors and the magnetic field strength is not linear;
- There is magnetic hysteresis. On the figure 9.3 hysteresis loop is a four quadrant $B-H$ graph from where the hysteresis loss, coercive force and re-tentively of magnetic material are obtained;
- The presence of characteristic temperature (Curie point) at which the ferromagnet loses its ferromagnetic properties.

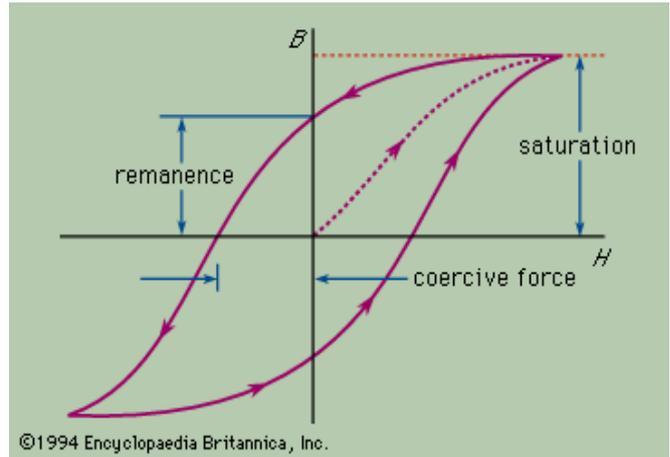


Figure 9.3

9.4 Law of the total current for magnetic field in a material. Magnetic field \vec{H}

For a quantitative description of the magnetized state of the magnetic material is introduced vector quantity – magnetization, which is determined by the magnetic moment per unit volume of the substance:

$$\vec{J} = \frac{\sum_i \vec{P}_{mi}}{\Delta V}. \quad (9.16)$$

The magnetic field in the material is composed of two fields: the external field generated by the field current in vacuum, and the fields generated by currents in the magnetized molecular substance:

$$\vec{B} = \vec{B}_0 + \vec{B}' \quad (9.17)$$

The field produced by molecular currents associated with the magnetization:

$$\vec{B}' = \mu_0 \vec{J}. \quad (9.18)$$

Therefore, an auxiliary magnetic field \vec{H} vector was introduced:

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{J} = \mu_0 (\vec{H} + \vec{J}). \quad (9.19)$$

In weak fields, as experience shows, is directly proportional to the magnetization field strength causing the magnetization, i.e.,

$$\vec{J} = \chi \vec{H}, \quad (9.20)$$

where χ is dimensionless quantity called the magnetic susceptibility of the substance. Magnetic susceptibility is a dimensionless proportionality constant that indicates the degree of magnetization of a material in response to an applied magnetic field. For water $\chi = -9.035 \times 10^{-6}$, for Al $\chi = 2.2 \times 10^{-5}$.

Then the magnetic field \vec{B} in the material can be written as

$$\vec{B} = \mu_0(1 + \chi)\vec{H} = \mu_0\mu\vec{H}, \quad (9.21)$$

where μ is the dimensionless quantity is the magnetic permeability material. Circulation of the magnetic field in the matter:

$$\oint_L \vec{B}_l dl = \mu_0(I + I'), \quad (9.22)$$

where I and I' are respectively conduction current I and micro (molecular) currents I' , covered by an arbitrary closed loop. However, the practical application of (9.17) for the calculation of the magnetic fields is impossible because for this it is necessary to know the distribution of microcurrents I' in magnets.

It is proved that the circulation along an arbitrary closed contour L the vector of the magnetic field \vec{H} in substance is the sum of only conduction currents covered by the contour:

$$\oint_L \vec{H} d\vec{l} = \sum_i I_i. \quad (9.23)$$

It is this property of the strength vector \vec{H} , allowing to calculate the magnetic field in matter, was the reason for the introduction of the auxiliary quantity in the theory of magnetism.

10 Lecture №10. Electromagnetic induction. Fundamentals of Maxwell's theory of electromagnetic field

The objective of the lecture: to consider the phenomenon of electromagnetic induction, its law, and the Maxwell's equations and their physical meaning.

10.1 Electromagnetic induction. The law of electromagnetic induction. Lenz's rule

Occurrence of induced current in a closed conductive loop with the changing of magnetic flux crossed the surface bounded by that contour, known as the electromagnetic induction. Faraday's law states that induced *EMF* equals the negative of the rate of change of magnetic flux penetrating the surface bounded by the contour and not dependent on the cause of the change:

$$\mathcal{E}_i = -\frac{d\Phi}{dt}. \quad (10.1)$$

Lenz's rule: magnetic field due to induction current counteracts the reason it is called.

10.2 Self induction and mutual induction. Self-inductance. Mutual inductance

The electric current flowing in a closed loop creates a magnetic field around itself, the induction of which, by law, Biot-Savart-Laplace, proportional to the current. Coupled with the contour the total magnetic flux, or flux linkage $\Psi=N\Phi$, therefore proportional to the current I in the circuit:

$$\Psi = LI, \quad (10.2)$$

where the proportionality factor L is the inductance of the circuit.

Define inductance as the magnetic flux linkage, divided by the current, as effectively the same value of current flows through each turn of the coil. Inductance L depends on the geometrical shape and dimensions of the conductive circuit, number of the turns as well as magnetic properties of the core. Consider a long solenoid (figure 10.1) of length l , and area of cross section S , which has n turns per unit length, and carries a current I . The longitudinal (i.e., directed along the axis of the solenoid) magnetic field within the solenoid is approximately uniform, and is given by $B = \mu_0 nI$. The total flux through the solenoid wire is:

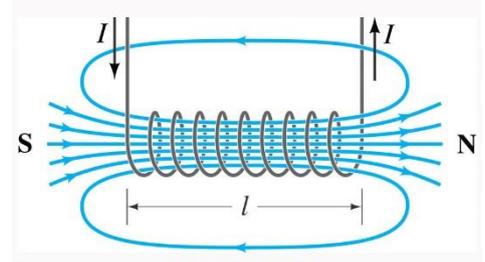


Figure 10.1

$$\Psi=N\Phi = nl\mu_0 nIS = \mu_0 n^2 SI.$$

Thus, the self-inductance of the solenoid is

$$L = \Psi/I = \mu_0 n^2 lS = \mu_0 n^2 V. \quad (10.3)$$

Self-induction is a phenomenon by which a changing electric current produces an induced EMF across the coil itself. EMF self-inductance is:

$$\mathcal{E}_{si} = -\frac{d\Psi}{dt} = -L \frac{dI}{dt} - I \frac{dL}{dt}. \quad (10.4)$$

But the actual equation when the circuit is not deformed and does not change the magnetic permeability (in the absence of ferromagnetic cores) is:

$$\mathcal{E}_{si} = -L \frac{dI}{dt}. \quad (10.5)$$

Mutual induction can define as phenomenon when a current flowing in one coil if it is changing in time induces a voltage in an adjacent another coil. Consider two stationary coils (figure 10.2) located sufficiently close to each other. Let the loop I flows current I_1 . Let the magnetic flux due to current I_1 passing thro the second circuit Ψ_{21} . Then:

$$\Psi_{21} = L_{21}I_1, \quad (10.6)$$

where L_{21} is a mutual inductance of the two adjacent coils.

If the current in the first circuit I_1 is changing in time, then in the second circuit is induced *EMF* of mutual induction \mathcal{E}_{i21} :

$$\mathcal{E}_{i21} = -L_{21} \frac{dI_1}{dt} - I_1 \frac{dL_{21}}{dt}. \quad (10.7)$$

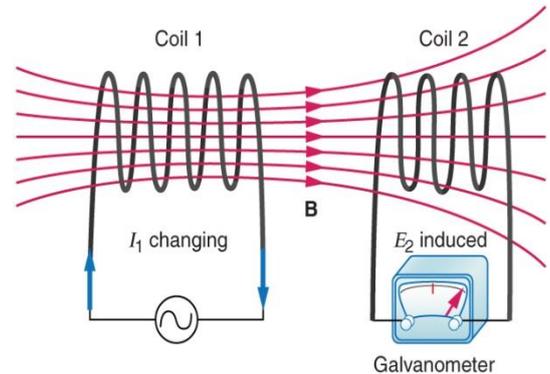


Figure 10.2

If vice versa, current I_2 changes in the secondary circuit, the *EMF* induced in the first circuit, can likewise be written as:

$$\mathcal{E}_{i12} = -L_{12} \frac{dI_2}{dt} - I_2 \frac{dL_{12}}{dt}. \quad (10.8)$$

If both coils are not deforming, their relative positioning is not vary, and the magnetic permeability (in the absence of ferromagnetic cores) does not change then (10.7) may be written as:

$$\mathcal{E}_{i21} = -L_{21} \frac{dI_1}{dt}. \quad (10.9)$$

Mutual inductance of conductive circuits depend on their geometrical shape and size, relative positioning and the magnetic properties of the environment. The calculations are confirmed by experience shows that the mutual inductances of the two circuits are the same $L_{12} = L_{21}$.

Mutual inductance is the basic operating principal of the transformer, motors, generators etc.

10.3 Maxwell interpretation of electromagnetic induction. The vortex electric field. The first Maxwell equation

When a conductor moving in a constant magnetic field, the induced current caused by the magnetic component of the Lorentz force. What force excites induced current in the stationary conductor is placed in an alternating magnetic field? The answer was given by Maxwell. According to Maxwell, any change with time in the

magnetic field induce in the surrounding space the electric field. The circulation of the electric field vector on any closed and fixed loop is:

$$\oint_l E_l dl = - \frac{\partial \Phi}{\partial t}. \quad (10.10)$$

According to (10.10) the circulation of an electric field vector is in general not zero. This means that the induced by varying in time magnetic field, electric field is not potential; the field lines are closed and it is a vortex (as opposed to the electrostatic field).

Between Maxwell and Faraday understanding of the phenomenon of electromagnetic induction there is a significant difference. According to Faraday, electromagnetic induction is excites an electric current. To observe it, you must have a closed conductive circuit. In essence Maxwell electromagnetic induction is excites the vortex electric field, rather than the current. Electromagnetic induction can be observed when in general there is no any conductors. The appearance of the induced current in a closed conductor when making the latest in an alternating magnetic field is only one manifestation of the electric field resulting from the change in the magnetic field. But electric field can produce other actions, such as to polarize the dielectric breakdown of the capacitor to cause, accelerate and slow down the charged particles. It may cause an electric current in a conductor unlocked.

Maxwell's formulation of the law of induction is more general than the wording of Faraday. It is one of the most important generalizations of electrodynamics.

Mathematically, the law of induction in the sense expressed by Maxwell where the integral is taken in a closed circuit, which may be held in the insulator, and not necessarily in a conductor, as Faraday:

$$\oint_l E_l dl = - \frac{\partial}{\partial t} \iint_S \vec{B} d\vec{S} = - \iint_S \frac{\partial \vec{B}}{\partial t} d\vec{S}, \quad (10.11)$$

where S is the arbitrary surface spanning the contour.

10.4 Displacement current. Maxwell's second equation

Ordinary electric currents, called conduction currents, whether steady or varying, produce an accompanying magnetic field in the vicinity of the current. The British physicist J.C. Maxwell in the 19th century predicted that a magnetic field also must be associated with a changing electric field even in the absence of a conduction current, a theory that was subsequently verified experimentally. As magnetic fields had long been associated with currents, the predicted magnetic field also was thought of as stemming from another kind of current. Maxwell gave it the name displacement current, which was proportional to the rate of change of the displacement field. Maxwell called displacement current density the value of

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t}. \quad (10.12)$$

As electric charges do not flow through the insulation from one plate of a capacitor to the other, there is no conduction current; instead, a displacement current is said to be present to account for the continuity of the magnetic effects. In fact, the calculated size of the displacement current between the plates of a capacitor being charged and discharged in an alternating-current circuit is equal to the size of the conduction current in the wires leading to and from the capacitor. Displacement currents play a central role in the propagation of electromagnetic radiation such as light and radio waves, through empty space. A traveling, varying magnetic field is everywhere associated with a periodically changing electric field that may be conceived in terms of a displacement current. Maxwell called the sum of:

$$\vec{J} + \vec{J}_D = \vec{J}_{total} \quad (10.13)$$

the total current density. Conditions closure must be recorded for the total current:

$$\text{div } \vec{J}_{total} = 0. \quad (10.14)$$

This means that the total current is always solenoidal and its lines are closed. So, Maxwell's second equation has integral form:

$$\oint_L \vec{H} d\vec{\ell} = \iint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) dS. \quad (10.15)$$

The electric displacement \vec{D} is defined as:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P},$$

where ε_0 is the electric constant;

\vec{E} is the electric field intensity;

\vec{P} is the polarization of the medium.

Differentiating this equation with respect to time defines the *displacement current density*, which therefore has two components in a dielectric:

$$\vec{J}_D = \varepsilon_0 \frac{d\vec{E}}{dt} + \frac{d\vec{P}}{dt}. \quad (10.16)$$

The first term on the right hand side is present in material media and in free space. It doesn't necessarily come from any actual movement of charge, but it does have an associated magnetic field, just as a current does due to charge motion. Some authors apply the name *displacement current* to the first term by itself.

The second term on the right hand side, called polarization current density, comes from the change in polarization of the individual molecules of the dielectric material. Polarization results when, under the influence of an applied electric field, the charges in molecules have moved from a position of exact cancellation.

10.5 The system of Maxwell's equations

Supplementing the basic facts of electromagnetism establishment magnetic action bias currents, Maxwell was able to write the system of fundamental equations of electrodynamics. There are four field equations. The integral form, they look like:

$$\oint_L \vec{E}_\ell d\ell = -\iint_S \frac{\partial \vec{B}}{\partial t} d\vec{S}; \quad (10.17)$$

$$\oint_L \vec{H} d\vec{\ell} = \iint_S \left(\vec{j} + \frac{\partial \vec{D}}{\partial t} \right) dS; \quad (10.18)$$

$$\iint_S \vec{D} d\vec{S} = \iiint_V \rho dV; \quad (10.19)$$

$$\iint_S \vec{B} d\vec{S} = 0. \quad (10.20)$$

Physical equations for homogenous and isotropic materials with constant dielectric ε and magnetic μ permittivity are:

$$\vec{D} = \varepsilon \varepsilon_0 \vec{E}; \quad \vec{B} = \mu \mu_0 \vec{H}; \quad \vec{j} = \sigma \vec{E}.$$

Maxwell's equations reveal that the electric field is generated either by electric charges or by varying in time magnetic field. Magnetic fields can also be excited by moving electric charges (electric currents) or by alternating electric field. Maxwell's equations in integral form are true in cases where there is a discontinuity surface, on which the properties of the medium and the electric and magnetic fields are changed abruptly. Therefore, in this form of the Maxwell equations are more common than in differential form, which means that all values in space and time vary continuously.

Maxwell's equations were obtained by generalization of experimental facts. They should be regarded as a basic axiom of classical electrodynamics.

10.6 Electromagnetic field. Relativity of electric and magnetic fields

Maxwell's equations (10.17) – (10.20) shows the close relationship of the electric and magnetic fields, which leads to the conclusion about the existence of a single electromagnetic field. By the electromagnetic field apply the principle of

relativity, as the vacuum propagation of electromagnetic waves in all reference frames occurs at the same speed. Also, the principle of relativity implies that the separate examination of the electric and magnetic fields has a relative meaning. In other words, the separation of the electromagnetic field into its component electric and magnetic fields depends on the choice of the reference frame, that is, with respect.

11 Lecture №11. Electric oscillations and waves

The objective of the lecture: to consider the definitions and general characteristics of various types of oscillations in oscillatory circuit.

11.1 Free harmonically oscillation in an oscillatory circuit

The oscillation is the repetitive variation in time of some measure about a central value according to a periodic or almost periodic law. If the oscillation occur under the influence of external periodic action it is called *forced oscillation*. The oscillation of a system with its own natural frequency ω_0 and under *no external influence* other than the impulse that initiated the motion are called *free oscillations*. Oscillatory processes are found everywhere in nature and technology. In astronomy the planets periodically revolve around the Sun, variable stars such as Cepheid's periodically change their brightness, the movement of the Moon causes tides, various periodic processes occur inside living organisms, etc.

Electromagnetic oscillations are repeated changes in electrical and magnetic quantities: charge, current, voltage, electric and magnetic fields. Such oscillations arise, for example, in an oscillatory circuit, closed circuit containing a capacitance C and an inductor L .

Consider an ideal oscillatory circuit that does not have an active resistance R . If you charge the capacitor from the DC power network U_0 by setting the key K to "1" and then turning the key K to "2" the capacitor will start discharging through the inductor and an increasing

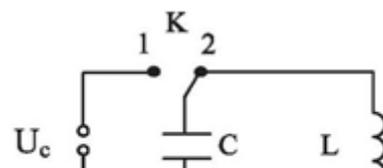


Figure 11.1

current will appear in the circuit I . In this case an *EMF* of self-induction $\mathcal{E}_{si} = -L \frac{di}{dt}$ appears in the coil. In the ideal contour the *EMF* \mathcal{E}_{si} of self induction is equal to the voltage between their plates:

$$u_c = -\frac{q}{C}, \quad (11.1)$$

where q is the charge of the negative charged plate.

So, we receive the equation:

$$\frac{q}{C} = -L \frac{di}{dt}, \quad (11.2)$$

where i is the current in the circuit;

q is the charge on the plates of the capacitor.

The derivative of charge is current, so that gives us a second order differential equation:

$$\frac{q}{C} = -L \frac{d^2 q}{dt^2}. \quad (11.3)$$

After transferring all the terms to the left side and dividing by the inductance L we obtain the differential equation of free oscillations in the oscillatory LC -circuit:

$$\frac{d^2 q}{dt^2} + \frac{q}{LC} = 0; \quad \text{or} \quad \ddot{q} + \omega_0^2 q = 0, \quad (11.4)$$

where $\omega_0 = \frac{1}{\sqrt{LC}}$ is natural angular frequency of the LC -circuit.

Equation (11.4) is the linear homogeneous differential equation of the second order. Its solution is a harmonic function:

$$q = q_m \cos(\omega_0 t + \varphi_0), \quad (11.5)$$

where q_m the initial and peak value of a charge on the plates of the capacitor. The voltage u_c and current i in the circuit change under the harmonic law too:

$$u_c = U_m \cos \omega_0 t, \quad (11.6)$$

where $U_m = q_m/C$ is the amplitude of voltage on a capacitor. The derivative of charge is current, so we receive:

$$i = I_m \cos\left(\omega_0 t + \frac{\pi}{2}\right), \quad (11.7)$$

where $I_m = \omega_0 q_m$ is the amplitude of the current in an inductor.

The period of free oscillations is determined by Thomson's formula:

$$T = 2\pi\sqrt{LC}. \quad (11.8)$$

The energy W_E of the electric field of the capacitor and the energy W_M of the magnetic field of the coil change periodically with time:

$$W_E = (CU_m^2/2) \cdot \cos^2(\omega_0 t); \quad (11.9)$$

$$W_M = (LI_m^2/2) \cdot \sin^2(\omega_0 t). \quad (11.10)$$

The total energy W of electromagnetic oscillations consists of these two energies. Since there are no losses in the ideal circuit associated with the release of heat the total energy of free oscillations is conserved:

$$W = W_E + W_M = Cu^2/2 + Li^2/2 = CU_m^2/2 = LI_m^2/2 = const. \quad (11.11)$$

11.2 The free damped oscillations

Under normal conditions all conductors have an *active resistance*. Therefore free oscillations in a real circuit are damped. In figure 11.2 the active resistance of the conductor is R .

According to the generalized Ohm's law:

$$iR = -\frac{q}{c} - L \frac{di}{dt}. \quad (11.11)$$

Due to definition $i = \frac{dq}{dt}$; after transferring all the terms to the left side and dividing by the coil inductance L we obtain the differential equation of free damped oscillations in a RLC -circuit:

$$\frac{d^2q}{dt^2} + \frac{R}{L} \cdot \frac{dq}{dt} + \frac{q}{LC} = 0, \quad (11.12)$$

Where $\beta = \frac{R}{2L}$ is damping factor.

Then the differential equation of oscillation in RLC -circuit takes the form:

$$\frac{d^2q}{dt^2} + 2\beta \frac{dq}{dt} + \omega_0^2 q = 0; \quad (11.13)$$

or the form:

$$\ddot{q} + 2\beta\dot{q} + \omega_0^2 q = 0. \quad (11.14)$$

When the condition $R < 2\sqrt{LC}$ (weak damping) is satisfied, equations (11.13) and (11.14) describes damped oscillations (figure 11.3):

$$q(t) = q_0 e^{-\beta t} \cos(\omega t + \alpha), \quad (11.15)$$

where $\omega = \sqrt{\omega_0^2 - \beta^2}$ is the cyclic frequency and it is less than the natural frequency ω_0 of free undamped oscillations;

q_0 is the amplitude of charge at the initial time;

α - the initial phase of the damped oscillations.

Conditional period of damped oscillations is:

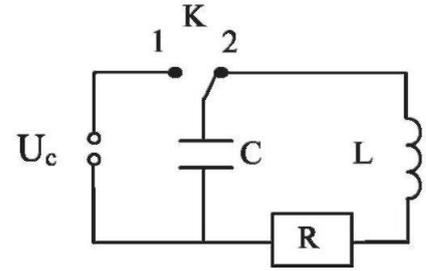


Figure 11.2

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}. \quad (11.15)$$

The ratio of the amplitudes of two consecutive oscillations corresponding to the moments of time differing by a period:

$$\frac{A(t)}{A(t+T)} = e^{\beta T}$$

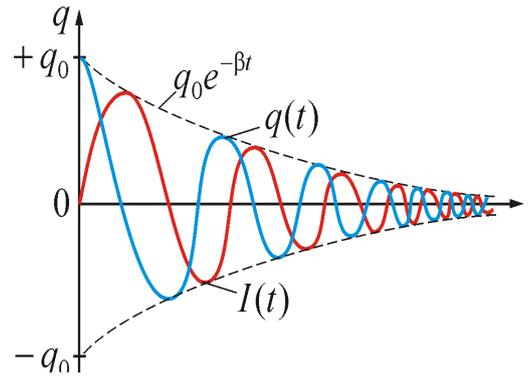


Figure 11.3

is called a *damping decrement*. Its logarithm is a logarithmic decrement of damping:

$$\theta = \ln \frac{A(t)}{A(t+T)} = \beta T = \frac{T}{\tau} = \frac{1}{N_e}, \quad (11.15)$$

where N_e is the number of oscillations that occur during a decrease in the amplitude at e times;

τ is the relaxation time.

The logarithmic damping decrement is a constant for a given oscillating system. To characterize the oscillatory system one uses the concept of quality factor Q that for small values of the logarithmic decrement is:

$$Q = \frac{\pi}{\theta} = \pi N_e = \frac{\pi}{\beta T} = \frac{\omega}{2\beta}, \quad (11.16)$$

where T and ω are the period and angular frequency of damped oscillations respectively.

For a series RLC -circuit the quality factor is:

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}. \quad (11.17)$$

11.3 The alternative current as the forced oscillations

Forced oscillations occur when a periodic force that is external to the oscillating system drives an oscillating system.

The steady-state oscillation of the current in the circuit is called alternative current (AC). To receive AC in an electric oscillating circuit is necessary to apply an alternating voltage from the mains or generator

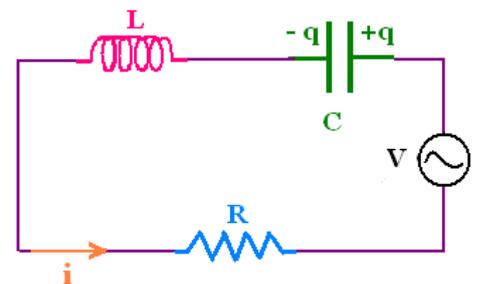


Figure 11.4

- a source of external variable EMF (figure 11.4). Let the applied voltage varies with time harmonically:

$$u(t) = U_m \cos \omega t. \quad (11.18)$$

In this case, if the condition quasi stationarity $T \gg l/c$ is performed, the differential equation of a forced oscillations in the circuit has the form:

$$L \frac{di}{dt} + iR + \frac{q}{C} = U_m \cos \omega t, \quad (11.19)$$

after dividing by the L :

$$\frac{d^2q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{1}{LC} q = \frac{U_m}{L} \cos \omega t. \quad (11.20)$$

The steady-state solution of this inhomogeneous differential equation is:

$$q(t) = q_m \cos(\omega t + \psi), \quad (11.21)$$

where $q_m = I_m/\omega = CU_{cm}$ is the charge amplitude;

ψ is the phase shift the charge oscillation from the external EMF , i.e., charge oscillation on the capacitor is always behind in phase from oscillation of applied voltage.

The current i is the derivative of the charge, then:

$$i(t) = I_m \cos(\omega t - \varphi), \quad (11.22)$$

where $I_m = \omega q_m$ is the current amplitude;

$\varphi = (\psi - \pi/2)$ is the phase shift between current and applied voltage.

To find I_m and φ we represent the initial equation (11.19) in the form of the sum of the instantaneous values of the voltages on the active resistance, the capacitor and the inductance:

$$u_L + u_C + u_R = u(t); \quad (11.23)$$

$$u_R = iR = RI_m \cos(\omega t - \varphi); \quad (11.24)$$

$$u_C = I_m \omega \cos(\omega t - \varphi - \pi/2); \quad (11.25)$$

$$u_L = L \frac{di}{dt} = I_m \omega L \cos(\omega t - \varphi + \pi/2). \quad (11.26)$$

From ratios (11.24) – (11.26), the expressions for the amplitude (peak) values of voltages on each of the RLC -circuit element follows:

$$U_R = I_m R; \quad (11.27)$$

$$U_c = I_m \frac{1}{\omega C}; \quad (11.28)$$

$$U_L = I_m \omega L. \quad (11.29)$$

From ratios (11.24) – (11.26) it is clear that u_R is in phase with current i ; u_c is behind in phase of the current by $\pi/2$ and u_L is ahead in phase of current by $\pi/2$.

In physics and engineering, a *phasor* (a portmanteau of *phase vector*), is a vector representing a sinusoidal function whose amplitude A , angular frequency ω , and initial phase φ are time-invariant. The sum of multiple phasors produces another phasor. That is because the sum of sinusoids with the same frequency is also a sinusoid with that frequency. A phasor continually rotating, but in use phasor diagrams are static. Imagine that the phasors are rotating in an anti-clockwise (counter clockwise) direction. Every phasor in the diagram will have the same angular velocity because they represent sine oscillations of identical frequency. The length of the each phasor arm is directly related to the amplitude of the oscillation it represents, and the angle between the phasors is the same as the angle of phase shift between respective voltage oscillation and current oscillation in the circuit.

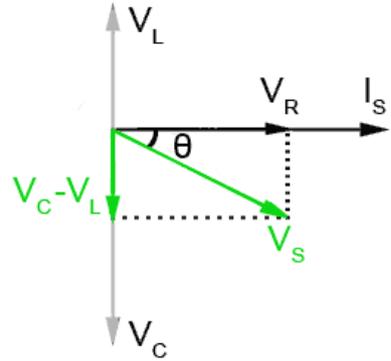


Figure 11.5

Let's construct a phasor diagram for the voltage amplitudes according to equation (11.23) and considering the relations (11.27) – (11.29). At first the current axis should be drawn, since in a serial circuit the current is the same everywhere, Then, the phasors of the corresponding amplitudes of voltage on the circuit elements should be drawn taking into account the phase relations. In final it is obtained the Figure 11.5 (in this figure amplitude of voltage denote as V , and phase shift between applied voltage and current denote as θ). By the rule of vector addition we find their sum, which according to (11.23) is equal to the amplitude of the applied voltage U_m :

$$U_m^2 = U_R^2 + (U_L - U_C)^2 \quad (11.30)$$

It is easy to see that for the amplitude value of the current and the phase shift the following expressions are obtained:

$$I_m = \frac{U_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}; \quad (11.31)$$

$$tg \varphi = \frac{\omega L - 1/\omega C}{R}. \quad (11.32)$$

According to (11.28), the peak value of the current is directly proportional to the peak value of the applied voltage. This ratio is considered as Ohm's law for alternating current (for series connection). In the general case of an arbitrary connection the law is written in the form:

$$I_m = \frac{U_m}{Z}, \quad (11.33)$$

where Z - impedance of the circuit to alternating current, which is dependent on the circuit parameters R , L , C compounds of the elements and the frequency ω of the applied voltage.

11.4 Power in AC circuits. Power factor. The root mean square values of the alternating current and voltage

The instantaneous value of the power of the alternating current in the circuit is equal to the product of the instantaneous values of the voltage and current:

$$N(t) = UI = U_m I_m \cos \omega t \cdot \cos(\omega t - \varphi). \quad (11.34)$$

Of practical interest is the average for the period T power value:

$$\langle P \rangle = \frac{1}{2} I_m U_m \langle \cos(2\omega t - \varphi) + \cos \varphi \rangle = \frac{1}{2} I_m U_m \cos \varphi. \quad (11.35)$$

The same power of developing DC quantities I and U called the root mean square (or effective) values of voltage and current. All ammeter and voltmeter for AC circuits are graduated it at current values I and U .

For a sinusoidal voltage is true the relations:

$$U_{rms} = \frac{U_{peak}}{\sqrt{2}}; \quad I_{rms} = \frac{I_{peak}}{\sqrt{2}}. \quad (11.36)$$

The average power consumed by the AC circuit:

$$\langle P \rangle = IU \cos \varphi, \quad (11.37)$$

where $\cos \varphi$ is power factor:

$$\cos \varphi = \frac{R}{Z}. \quad (11.38)$$

A purely resistive has power factor 1, while a circuit with only inductance and capacitance has pf= 0. In general, the pf depends on ω ; in the series RLC - circuit, for example, it's equal to 1 at resonance, but lower at other frequencies.

11.5 Resonance. Resonance curves

The sharp increase in the amplitude of the forced vibration when the frequency ω of the driving force to the natural frequency ω_0 of the oscillating system is called resonance. This value is called the resonant frequency ω_r , it is:

$$\omega_p = \sqrt{\omega_0^2 - 2\beta^2} . \quad (11.38)$$

Displacement amplitude in mechanical oscillator at resonance are equal to:

$$A_p = \frac{F_0}{2m\beta\sqrt{\omega_0^2 - \beta^2}} . \quad (11.39)$$

On the figure 11.6 are shown the resonance curves for the different values of damping factor β .

When the peak voltages on the capacitor and the inductor are equal each to other and opposite in phase, the amplitude value of the current in the series resonant circuit reaches the maximum possible value, and the phase shift becomes zero. This phenomenon is called resonance of current. The resonant frequency for the current in series *RLC*-circuit coincides with the natural frequency of the circuit:

$$\omega_p = \omega_0 = \frac{1}{\sqrt{LC}} . \quad (11.40)$$

11.6 Electromagnetic waves and their properties

As mentioned previously Maxwell's theory not only predicted the existence of electromagnetic waves (EM-waves) but also indicated all their basic properties.

1) The propagation speed in vacuum is:

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} . \quad (11.41)$$

In a nonconducting neutral non-ferromagnetic medium is

$$v = \frac{c}{\sqrt{\epsilon\mu}} . \quad (11.42)$$

2) The vectors \vec{E} , \vec{B} and \vec{v} are mutually perpendicular and form a right-hand triple, which is an internal property of the EM wave (figure 11.7).

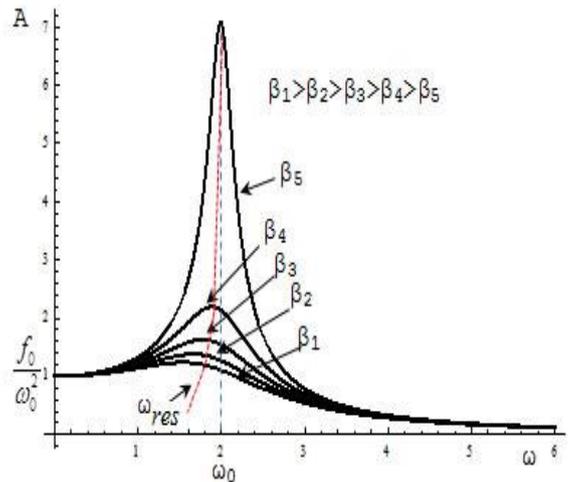


Figure 11.6

3) The vectors \vec{E} and \vec{B} always oscillate in the same phase, and there is a bond between them at any point:

$$\sqrt{\varepsilon_0 \varepsilon} E = \sqrt{\mu_0 \mu} H. \quad (11.43)$$

4) In the absence of ferroelectrics and ferromagnets in a medium the volume energy density of the EM-field is

$$w = \frac{\vec{E}\vec{D}}{2} + \frac{\vec{B}\vec{H}}{2}.$$

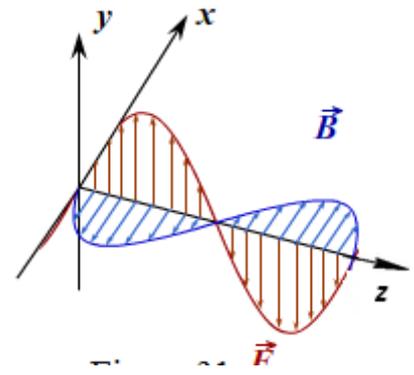


Figure 11.7

The equations of the plane electromagnetic wave traveling along the axis Ox :

$$\vec{E} = \vec{j} E_m \cos(\omega t - kx + \varphi_0), \quad (11.45)$$

$$\vec{H} = \vec{k} H_m \cos(\omega t - kx + \varphi_0). \quad (11.46)$$

The Poynting vector \vec{S} which is of the electromagnetic energy flux density is:

$$\vec{S} = [\vec{E}\vec{H}]. \quad (11.47)$$

According to classical electrodynamics EM-waves are excited by electric charges moving with acceleration a . If at least one of the electric dipole charges begins to oscillate, then *the dipole begins to emit EM waves*.

The wave amplitude depends on distance r from the dipole as:

$$E_m \sim H_m \sim \frac{1}{r} \sin \theta,$$

where θ is an angle between an axis of a dipole and a position vector \vec{r} ; so the intensity of a wave is:

$$I \sim \frac{1}{r^2} \sin^2 \theta. \quad (11.48)$$

The power of the dipole radiation is proportional to the square of the second derivative of the electric moment of the dipole $\vec{p}_e = q \cdot \vec{l}$:

$$N \sim \ddot{\vec{p}}_e.$$

When the dipole oscillates at a frequency ω then the power:

$$N \sim q^2 \omega^4. \quad (11.49)$$

The average radiation power is proportional to ω^4 , i.e.,

$$\langle N \rangle \sim \omega^4. \quad (11.50)$$

Therefore at a low frequency, for example, in AC transmission line, the radiation is insignificant. When transmitting EM signals over long distances high frequencies modulated with useful low-frequency information are used.

12 Lecture №12. Wave–particle duality of electromagnetic radiation and particles of substance

The objective of the lecture: to consider the notion of particle-wave duality of electromagnetic radiation and microparticles of matter on the examples of their manifestation.

12.1 The quantum nature of electromagnetic radiation. Thermal radiation. Quantum hypothesis and Planck's formula

Visible light is electromagnetic waves in the wavelength range of $380 \div 760$ nm, but in some phenomena, for example, the photoelectric effect, it manifests itself as a flux of the special particles (photons). This is the corpuscular-wave duality of electromagnetic radiation adopted in modern physics. The initial reason was the impossibility of explaining the laws of thermal irradiation in the last quarter of the 19th century.

Radiation by bodies of electromagnetic waves, can be carried out due to various types of energy. The emission of electromagnetic waves due to the internal energy of bodies is *thermal radiation*. When we heat a body to high temperature, it gives off a glow. For instance, when we heat a bar of iron to 1200 or 1300 K, it glows in a bright orange or yellow color. Bodies at room temperature also emit thermal radiation, but the glow is infrared, and not visible to the eye. All other types of luminescence, excited due to any type of energy, except internal - luminescence: chemiluminescence, electroluminescence, cathodoluminescence, photoluminescence. Thermal radiation is also one of the fundamental mechanisms of heat transfer.

Thermal radiation is the electromagnetic radiation emitted by bodies due to their internal energy at any temperature above the absolute zero. This is the only type of radiation capable of being in equilibrium with the body emitting it. All other types of radiation are nonequilibrium. Thermal radiation has a continuous range of frequencies (wavelengths).

Let's consider the main characteristics of thermal radiation. *Luminosity* - the total energy radiated per unit surface area of a body across all wavelengths per unit time (or the radiant emittance) is:

$$R_T = \frac{dW}{dS \cdot dt} = \frac{d\Phi}{dt}, \quad (12.1)$$

where $d\Phi/dS$ is an energy flux density, i.e. the amount of energy emitting outside through the surface unit area per unit time.

A spectral radiance (or emissive ability) of a body is:

$$r_{\omega,T} = \frac{dR_{\omega,T}}{d\omega}. \quad (12.2)$$

The absorptivity (or absorptance) of the surface of a material is the ratio of the spectral radiant flux absorbed by that surface $d\Phi'_{\omega T}$ to the spectral radiant flux received by that surface $d\Phi_{\omega T}$ at a given frequency and temperature:

$$\alpha_{\omega T} = \frac{d\Phi'_{\omega T}}{d\Phi_{\omega T}}. \quad (12.3)$$

For a *blackbody* $\alpha_{\omega,T} \equiv 1$. Such substances are absent in nature but we have the its model: a small hole in opaque closed cavity. Any ray passing into the interior practically does not come out because of multiple reflections inside and partial absorption at each reflection.

The first law of thermal radiation had been discovered by Kirchhoff in 1859: the ratio of emissive ability (or, spectral radiance) to absorptivity does not depend on the nature of the bodies, for all bodies it is the same *universal function of frequency (wavelength) and temperature* $f(\omega, T)$:

$$\left(\frac{r_{\omega T}}{\alpha_{\omega T}}\right)_1 = \left(\frac{r_{\omega T}}{\alpha_{\omega T}}\right)_2 = \left(\frac{r_{\omega T}}{\alpha_{\omega T}}\right)_3 = \dots = f(\omega, T). \quad (12.4)$$

It is easy to see that the universal function $f(\omega, T)$ is nothing but the spectral radiance of a blackbody.

If a radiation-emitting object meets the physical characteristics of a black body in thermodynamic equilibrium, the radiation is called blackbody radiation. Blackbody radiation has a characteristic, continuous frequency spectrum that depends only on the body's temperature.

The *Stefan-Boltzmann law* states the luminosity of a blackbody with its absolute temperature:

$$R = \int_0^{\infty} f(\omega, T) d\omega = \sigma T^4, \quad (12.5)$$

where $\sigma = 5.67 \cdot 10^{-8} \frac{W}{K^4 m^2}$ is the Stefan-Boltzmann constant.

Wien's displacement law states that the blackbody radiation curve for different temperatures will peak (figure 12.1) at different wavelengths that are inversely proportional to the temperature. Wien's displacement law states that the spectral radiance of blackbody radiation per unit wavelength, peaks at the wavelength λ_{\max} given by:

$$\lambda_m = \frac{b}{T}, \quad (12.6)$$

where T is the absolute temperature of the blackbody;

$b = 2.90 \times 10^{-3} \text{ m}\cdot\text{K}$ is a *Wien's displacement constant*.

In theoretical studies, it is more convenient to use the frequency function $f(\omega, T)$ to characterize the spectral composition of equilibrium thermal radiation, and in experimental studies, it is more convenient to use the wavelength function $\varphi(\lambda, T)$. Both functions are related to each other by the ratio:

$$f(\omega, T) = \frac{2\pi c}{\omega^2} \varphi(\lambda, T) = \frac{\lambda^2}{2\pi c} \varphi(\lambda, T). \quad (12.4)$$

At the end of the 19th century, physicists were unable to explain why the observed spectrum of blackbody radiation, which by then had been accurately measured, diverged significantly at higher frequencies from that predicted by existing theories. In 1900, Max Planck heuristically derived a formula for the observed spectrum by assuming that a hypothetical electrically charged oscillator in a cavity that contained black-body radiation could only change its energy in a minimal increment:

$$\mathcal{E} = h\nu, \quad (12.5)$$

that was proportional to the frequency of its associated electromagnetic wave. This resolved the problem of the divergence of the theoretical Rayleigh–Jeans curve from the observed in experimental curves at different temperatures.— it is shown on Figure 12.1, predicted classical physics. This discovery was a pioneering insight of modern physics and is of fundamental importance to quantum theory development.

The Planck formula for the universal Kirchhoff function has the form:

$$f(\omega, T) = \frac{\hbar \omega^3}{4\pi^2 c^2} \frac{1}{\exp\left[\frac{\hbar \omega}{kT} - 1\right]}, \quad (12.6)$$

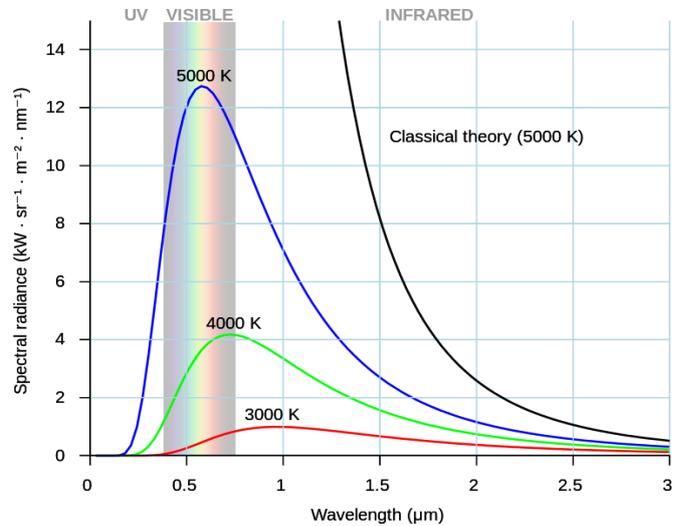


Figure 12.1.

where $\hbar = \frac{h}{2\pi} = 1.05 \cdot 10^{-34}$ J/s is reduced Plank's constant.

Plank's formula completely agrees with experimental data in all interval of frequencies from zero to infinity.

12.2 Compton effect. Photoelectric effect. Photon

A Compton scattering X-ray examining different substances, found in the scattered radiation beams along with the original wavelength λ also contains a longer wavelength rays λ' , and it was found that the difference of wavelength and the nature of the substance does not depend on:

$$\lambda' - \lambda = \Delta\lambda = \lambda_c(1 - \cos \theta), \quad (12.7)$$

where θ is the angle between the direction of the scattered light with the direction of the primary beam.

Maxwell's theory could not explain these facts, since, according to classical electrodynamics the scattered radiation - a secondary electromagnetic waves emitted by the vibrating electrons substance. The electrons perform forced oscillations under the influence of an electric field of the incident electromagnetic wave. Therefore, the frequency (and wavelength) of the scattered radiation, according to these ideas, must match the original frequency.

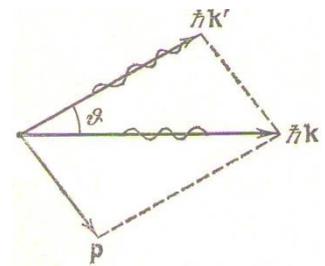


Figure 12.2

All the features of Compton effect explained by considering the scattering as a process of elastic collision of X-ray photons with nearly free electrons of atoms. If the original resting free electron incident photons with energy ε and momentum \vec{p} , after scattering by electrons, its energy ε' reduced, since, according to the laws of conservation of energy and momentum (Figure 12.2), part of the energy transferred to the electron "recoil":

$$\varepsilon + m_0c^2 = \varepsilon' + m_0c^2 + T; \quad (12.8)$$

$$\vec{p}_\gamma = \vec{p}'_\gamma + \vec{p}_e. \quad (12.9)$$

It follows from the equations that the Compton constant is $\lambda_c = \frac{2\pi\hbar}{mc} = 2.43$ nm.

The *photoelectric effect* is the emission of electrons when light hits a material. Electrons emitted in this manner can be called *photoelectrons*.

In 1905, A. Einstein provided an explanation of the photoelectric effect, an experiment that the wave theory of light failed to explain. He did so by postulating the existence of photons,

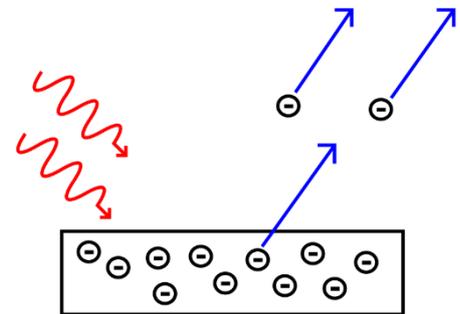


Figure 12.3

quanta of light energy with particulate qualities. On the figure 12.3 shown that incoming phonons on the left strike a metal plate (bottom), and eject electrons, depicted as flying off to the right.

A photon is a quantum of an electromagnetic field, an elementary particle that can exist only when flying at the speed of light in a vacuum c and has no mass, that is, a massless particle.

12.3 The relationship between the wave and corpuscular properties of electromagnetic radiation

Light shows the properties of the wave in the phenomena of diffraction and interference at a scale comparable to the wavelength of light. For example, even single photons passing through a double slit, creating an interference pattern on the screen, defined by Maxwell's equations. The wave properties of light plays a decisive role in the laws of his interference, diffraction, polarization, and corpuscular - in the interaction of light with matter processes. Laws of equilibrium thermal radiation, the photoelectric effect and the Compton effect can be successfully interpreted only on the basis of ideas about light as a stream of discrete photons.

The larger the wavelength, the smaller the momentum and energy of the photon and the harder it is to detect corpuscular properties of light. For example, external photoelectric effect occurs only at photon energies greater than or equal to the electron work function of a substance. The smaller the wavelength of electromagnetic radiation, the greater the energy and momentum of photons, and the harder it is to detect the wave properties of light. For example, X-rays diffracted at only a very "thin" grating - the crystal lattice of the solid. However, experiments have shown that the photon is not a short pulse of electromagnetic radiation, for example, it can not be split into several beams by optical dividers.

12.4 The hypothesis of de Broglie. Wave-particle duality of the microparticles of a substance

Wave-particle duality is the concept that every particle or quantum entity may be described as either a particle or a wave.

Wave-particle duality was formulated for the first time in 1924 by L. de Broglie. De Broglie's idea means that the duality is not only a feature of one optical phenomena, but has universal value. He supposed that substance particles together with corpuscular properties have wave properties as well. De Broglie transferred the same rules of transition from one physical picture to another both true for light (electromagnetic radiation) and particles.

The hypothesis of de Broglie is a hypothesis *of universalism of wave-particle duality* according to that not only photons, but also electrons and other microparticles of substance at the same time have corpuscular properties and wave too. Therefore, if the particle has corpuscular characteristics - energy W and an momentum p , then

the corresponding wave characteristics of a particle frequency ω_B and wavelength λ_B are connected with corpuscular by ratios:

$$\omega_B = \frac{W}{\hbar}; \quad (12.10)$$

$$\lambda_B = \frac{h}{p}. \quad (12.11)$$

The associated waves with freely moving particles are called *de Broglie waves*. So, to a moving electron (or any other particle) with not really high energy ($v \ll c$) corresponds a wave process, the wavelength of which is:

$$\lambda_B = \frac{h}{mv}, \quad (12.12)$$

where m and v are mass and speed of a particle.

The Davisson–Germer experiment was a 1923–27 experiment, in which electrons, scattered by the surface of a crystal of nickel metal, displayed a diffraction pattern. This confirmed the hypothesis, advanced by de Broglie, of wave-particle duality, and was an experimental milestone in the creation of quantum mechanics. Now you can yourself verify the presence the wave properties of atoms by performing the experiment on the diffraction of copper ions in our laboratory.

Introduced first in 1927 W. Heisenberg postulated his uncertainty principle, which states:

$$\Delta x \cdot \Delta p_x \geq \hbar, \quad (12.13)$$

where Δx and Δp_x are the uncertainties of the particle's position x and linear momentum p_x respectively.

This principal states that the more precisely the position of some particle is determined, the less precisely its momentum can be known, and vice versa.

The modern explanation of the uncertainty principle: just as it is nonsensical to discuss the precise location of a wave on a string, particles do not have perfectly precise positions; likewise, just as it is nonsensical to discuss the wavelength of a "pulse" wave traveling down a string, particles do not have perfectly precise momenta that corresponds to the inverse of wavelength. Moreover, when position is relatively well defined, the wave is pulse-like and has a very ill-defined wavelength, and thus momentum. Conversely, when momentum, and thus wavelength, is relatively well defined, the wave looks long and sinusoidal, and therefore it has a very ill-defined position. For energy – time interval it is:

$$\Delta E \cdot \Delta t \geq \hbar, \quad (12.14)$$

where ΔE is the uncertainty of the energy of the state of a microparticle in this state for the time Δt , or the uncertainty of the energy of a micro-object in a certain process of duration Δt .

Statistical description of the microparticles movement allows to organically combine their corpuscular properties with the wave. According to statistical interpretation, intensity of waves of de Broglie in any place of space is proportional to probability to find a particle in this place.

13 Lecture №13. Quantum states

The objective of the lecture: to introduce the basic concepts of quantum mechanics and give an idea of its methods.

13.1 Setting the state in quantum mechanics. Wave function and its statistical meaning

The major elements of the fundamental mechanics are *the concept of a state* and *the equation describing dynamics of a particle state*. So, in the classical mechanics, the state of a particle at a given time is specified by the coordinates x, y, z and momentum projections p_x, p_y, p_z . The basic equation is the second Newton's law. In the physics of the micro world, such determination of the state of micro particle is not applicable due to their wave properties. Because of the state of a microparticle can be described only by some function having wave properties.

The state of a micro particle in quantum mechanics is defined by a complex function of coordinates and time, which is called the wave function $\Psi(x, y, z, t)$. The wave function is a field in the mathematical sense; since it is complex, the waves described by this function *are unobservable*.

According to N. Born (1926), the square of the module of the Ψ -function determines the probability dP that the particle will be detected within the volume dV near the point with coordinates x, y, z :

$$dP = A|\Psi|^2 dV = A\Psi^*\Psi dV. \quad (13.1)$$

Since the sum of the probabilities of all possible locations of the particle must be 1, that is, the particle will be somewhere, we must also require that the wave function satisfy:

$$\iiint_{-\infty}^{\infty} \Psi^*\Psi dx dy dz = 1. \quad (13.2)$$

This requirement is known as *the normalization condition*.

In accordance with its meaning, Ψ -function must satisfy *the standard conditions*: be unique, continuous and finite, in addition, it must have continuous and finite derivatives.

The concept of a wave function is a fundamental postulate of quantum mechanics; the wave function defines the state of the system at each spatial position, and time.

13.2 Time-dependent and time-independent Schrödinger equations

The time-dependent Schrödinger equation, which gives a description of a system evolving with time, has the form:

$$-\frac{\hbar^2}{2m}\Delta\Psi + U\Psi = i\hbar\frac{\partial\Psi}{\partial t}, \quad (13.3)$$

where $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant;

$\Delta = \nabla^2$ is the Laplace operator or Laplacian;

m is the microparticle's mass;

$U = U(x, y, z, t)$ is the potential function of a microparticle;

$i = \sqrt{-1}$ – the imaginary unit.

Equation (13.3) is the basic equation of nonrelativistic quantum mechanics.

A special role in quantum theory is played by stationary states — the states in which all observable physical quantities does not change over time. Stationary states are realized if the potential function explicitly depends only on the spatial coordinates of the particle $U = U(x, y, z)$. In stationary states, the general solutions always are:

$$\Psi(x, y, z, t) = \psi(x, y, z)e^{-i\frac{E}{\hbar}t}, \quad (13.4)$$

where $\psi(x, y, z)$ is a function of all the spatial coordinates of a particle;

E is a constant equal to total energy of a particle, i.e., the sum of its kinetic and potential energies.

For the stationary states, the time-independent Schrödinger equation has form:

$$\Delta\psi + \frac{2m}{\hbar^2}(E - U)\psi = 0, \quad (13.5)$$

where $U = U(x, y, z)$ – is the potential energy of a particle.

It follows from the meaning of the function that quantum mechanics is of a statistical nature. The Schrödinger equation allows you to find the ψ -function of a given state and, therefore, determine the probability of a particle being at different regions of a space.

Thus, Schrödinger's equation in quantum mechanics is the analogue of Newton law in classical mechanics.

13.3 Eigenvalues and eigenfunctions of the problem

The Schrödinger equation includes, as a parameter, the total energy of the particle E . In the theory of differential equations, it is proved that equations of the form (13.5) have solutions satisfying the standard conditions, not for any values of the energy E , but only for some selected values. These selected values are called the *eigen-*

values of the corresponding quantity, i.e. energy. Solutions corresponding to the eigenvalues of E are called the *eigenfunctions* of the problem. The set of eigenvalues is called the magnitude spectrum. If this set forms a discrete sequence, the spectrum is called discrete. If the eigenvalues form a continuous sequence, the spectrum is called continuous.

13.4 Solution of the Schrödinger equation for the simplest quantum systems. Particle in a rectangular potential well. Quantum tunneling

Let us find the eigenvalues of energy E and the corresponding eigenfunctions $\psi(x)$ for a particle located in an infinitely deep one-dimensional potential well. Suppose a particle can only move along the X -axis. Let the motion be limited by walls impenetrable to the particle: $x = 0$ and $x = l$. In this case, the potential energy is zero inside the well at $0 < x < l$ and is infinity at $x < 0$ and $x > l$ (on the figure 13.1 potential energy designed as $V(x)$). The Schrödinger equation for such a task is simplified:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\psi = 0. \quad (13.6)$$

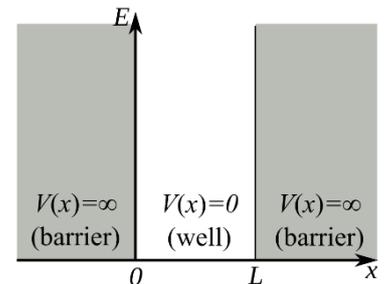


Figure 13.1

Outside the potential well, where the potential energy is infinitely large, the probability of detecting a particle is equal to zero, which means that the ψ -function is also equal to zero. In accordance with standard conditions, the wave function must be continuous. Therefore, at the boundaries of the potential well, it must satisfy the condition:

$$\psi(0) = \psi(l) = 0.$$

From equation (13.6) and the conditions imposed on the ψ -function, quantization of energy directly follows, i.e. the eigenvalues of the particle energy are discrete:

$$E_n = \frac{\pi^2 \hbar^2}{2ml^2} n^2, \quad (13.7)$$

where n - this is an integer parameter $n = 1, 2, 3, \dots$

The energy of a particle in an "infinite potential well" takes only certain discrete values. This is true for a potential pit of any shape. The quantized energy values are called energy levels, and the number n , which defines the energy levels of a particle, is called the principal quantum number.

Eigenfunctions look like standing waves on a string:

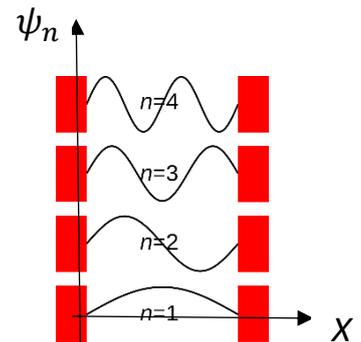


Figure 13.2

$$\psi_n(x) = \sqrt{\frac{2}{l}} \sin \frac{\pi n x}{l}. \quad (13.8)$$

Eigenfunctions for the first 4 states in a one-dimensional particle in a box are shown in figure 13.2. In the next figure are shown the probability density functions for the same states. A consideration of the probability density of particle detection at different distances from the walls of the pit shows that behavior of the microparticle indicates that the ideas about the trajectories in quantum mechanics are untenable.

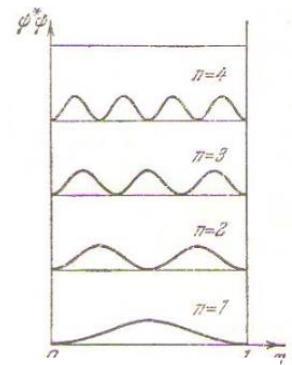


Figure 13.3

Quantum tunneling is the quantum mechanical phenomenon where a subatomic particle passes through a potential barrier. Quantum tunneling is not predicted by the laws of classical mechanics where surmounting a potential barrier requires enough potential energy. In other words, tunneling is the case when microparticle overcome a potential barrier though its total energy E is less than the height of the barrier U . The tunnel effect is a phenomenon of exclusively quantum nature, impossible in classical mechanics and even contradicting it.

Quantum tunneling or tunneling is the quantum mechanical phenomenon where a subatomic particle passes through a potential barrier. Quantum tunneling is not predicted by the laws of classical mechanics where surmounting a potential barrier requires enough potential energy. In other words, tunneling effect is the case when the microparticle overcomes a potential barrier though its total energy E (which remains unchanged during tunneling) is less than the height of the barrier U . The tunnel effect is a phenomenon of exclusively quantum nature, impossible in classical mechanics and even contradicting it.

The transparency D of a potential barrier is the probability of a microparticle passing through it depends on width l of the barrier, the height $(U-E)$ of the barrier above the level of the total energy and the mass m :

$$D = \exp \left[-\frac{2l}{\hbar} \sqrt{2m(U - E)} \right]. \quad (13.9)$$

In the figure 13.4 is a graph of the particle wave function $\psi(x)$ in the case of the quantum tunneling through the barrier. A particle coming from the left does not have enough energy to climb the barrier. However, it can sometimes tunnel to other side; after which the energy of the tunneled particle is the same, but the probability amplitude is decreased.

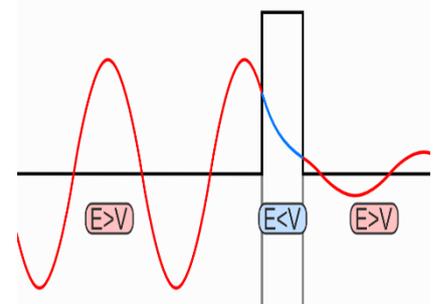


Figure 13.4

Tunneling occurs with barriers of thickness around 1–3 nm and smaller, but is the cause of some important macroscopic physical phenomena. For instance, tunnelling is a source of current leakage in very-large scale-integration (VLSI) electronics and results in the substantial power drain and heating effects that plague high-speed and mobile technology; it is considered the lower limit on how small computer chips can be made. Tunneling is a fundamental technique used to program the floating gates of flash memory, which is one

of the most significant inventions that have shaped consumer electronics in the last two decades. Cold emission of electrons from metals, tunneling diodes, alpha decay of nuclei, spontaneous fission of nuclei, etc. - there are some examples of manifestations of the quantum tunneling.

13.5 The hydrogen atom. Energy spectrum of the hydrogen atom. Spatial quantization

This problem in quantum mechanics reduces to the problem of the motion of an electron in the Coulomb field of the nucleus. The potential energy of the interaction of an electron with a nucleus having a charge Ze (for a hydrogen atom $Z = 1$):

$$U(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}, \quad (13.10)$$

where r is the distance between the electron and the nucleus. $U(r)$ decreases indefinitely with decreasing r (as the electron approaches the nucleus).

With account the expression (13.10) the stationary Schrödinger equation takes the following form:

$$\Delta\psi + \frac{2m}{\hbar^2} \left(E + \frac{Ze^2}{4\pi\epsilon_0 r} \right) \psi = 0, \quad (13.11)$$

where m is the electron mass;

E is the total energy of an electron in an atom.

The field in which the electron moves is centrally symmetric. Therefore we need to use the spherical coordinate system: r, θ, φ . It can be shown that equation (33.2) in a spherical coordinate system has the required (that is, single-valued, finite and continuous) solutions in the following cases:

- for any positive values of E ;
- for discrete negative energy values equals to:

$$E_n = -\frac{me^4}{8h^2\epsilon_0^2} \frac{Z^2}{n^2}, \quad (13.12)$$

where n is the *principal* quantum number, $n = 1, 2, 3, \dots$

The lowest level corresponding to the minimum possible energy is *the ground one*, all the rest ($n = 2, 3, \dots$) are *excited levels*. At negative energies the motion of an electron is bound – it is inside a hyperbolic "potential well". As the principal quantum number n grows the energy levels are more closely located and at $n \rightarrow \infty$ $E_n \rightarrow 0$. At $E > 0$ the motion of the electron is free, this region of the continuous spectrum corresponds to *the ionized atom*. The ionization energy of the hydrogen atom is:

$$E_i = -E_1 = \frac{me^4}{8h^2\epsilon_0^2} = 13,55 \text{ eV}. \quad (13.13)$$

In quantum mechanics it is proved that the Schrödinger equation (13.9) is satisfied by the *eigenfunctions* determined by three quantum numbers: *the principal n , the orbital one l , and the magnetic one m_l .*

The principal quantum number n determines the energy levels of the electron in the atom and can take any integer values starting with unity $n = 1, 2, 3, \dots$

It follows from the solution of the Schrödinger equation that the angular momentum (mechanical orbital momentum) of an electron is quantized:

$$L_l = \hbar\sqrt{l(l+1)}, \quad (13.14)$$

where l is the orbital quantum number, which for a given n takes values:

$$l = 0, 1, 2, \dots, n - 1; \quad (13.15)$$

and determines *the orbit angular momentum of an electron* in an atom.

From the solution of the Schrödinger equation it also follows that the angular momentum vector \vec{L}_l of an electron can have only such orientations in space for which its projection on the Z direction of the external magnetic field assumes quantized values that are multiples of \hbar :

$$L_{l_z} = \hbar m_l, \quad (13.16)$$

where m_l is the magnetic quantum number, which for a given l can take the values:

$$m_l = 0, \pm 1, \pm 2, \dots, \pm l, \quad (13.17)$$

i.e., of all $(2l + 1)$ values. Thus, the magnetic quantum number determines *the projection of the angular momentum of an electron on a given direction* and the orbit angular momentum vector \vec{L}_l of an electron in an atom can have in the space $(2l + 1)$ orientations. The number of different states corresponding to a given n is:

$$\sum_{l=0}^{n-1} (2l + 1) = n^2. \quad (13.18)$$

The quantum numbers n and l characterize the size and shape of the electron cloud in space. Quantum numbers n , l and m_l allow more complete description of the emission (absorption) spectrum of the hydrogen atom obtained in Bohr's theory.

Quantum mechanics introduces *selection rules* that limit the number of possible transitions of electrons in an atom associated with the emission and absorption of light.

The change in the orbital quantum number satisfies condition:

$$\Delta l = \pm 1. \quad (13.19)$$

The change in the magnetic quantum number satisfies condition:

$$\Delta m_l = 0, \pm 1. \quad (13.20)$$

13.6 Spin of electron

To explain the *fine structure* of spectral lines as well as a number of other difficulties in atomic physics American physicists Uhlenbeck and Goudsmit (1925) assumed that the electron has its own indestructible mechanical momentum – *spin* not related to the motion of an electron in space. *The spin* of an electron (and all other microparticles) is a quantum quantity, it does not have a classical analogue; it is an internal inherent property of the electron similar to its charge and mass.

If the electron is assigned its own mechanical moment of the pulse (spin) L_s then its own magnetic moment \vec{p}_{ms} corresponds to it. Spin is quantized by law:

$$L_s = \sqrt{s(s+1)}, \quad (13.21)$$

where s is the *spin* quantum number.

By analogy with the orbital angular momentum the projection L_{sz} of the spin is quantized so that the vector \vec{L}_s can take $2s + 1$ orientations. Since only two orientations were observed in the Stern and Gerlach experiments (1922) then $2s + 1 = 2$ whence $s = 1/2$. The projection of the spin on the direction of the external magnetic field being a quantized quantity is determined by:

$$L_{sz} = \hbar m_s, \quad (13.22)$$

where m_s is the magnetic spin quantum number. It can have only two values:

$$m_s = \pm \frac{1}{2}. \quad (13.23)$$

Thus experimental data led to the need to characterize electrons (and microparticles in general) with an additional internal degree of freedom. Therefore in order to fully describe the state of an electron in an atom it is necessary to specify in addition to the principal, orbital and magnetic quantum numbers also the *magnetic spin* quantum number.

13.7 Pauli principle. The distribution of electrons in an atom by states

Summarizing the experimental data Pauli (1925) formulated *the principle* that in a system of identical fermions any two of them can not be in the same state unilaterally.

In other words, in an atom *there can not be two electrons in a state with the same set of four quantum numbers*:

principal quantum number n ($n = 1, 2, 3, \dots$);

orbital quantum number l ($l = 0, 1, 2, \dots, n - 1$);

magnetic quantum number m_l ($m_l = -l, -l + 1, \dots, -1, 0, +1, \dots, +l$);

magnetic spin quantum number m_s ($m_s = +1/2, -1/2$).

Thus, the Pauli principle asserts that two electrons bound in the same atom differ in the values of at least one quantum number. The maximum number of electrons in states determined by a given quantum number n is:

$$Z(n) = \sum_{l=0}^{n-1} 2(2l+1) = 2n^2. \quad (13.24)$$

With the help of the Pauli principle one can understand the periodic repetition of the properties of atoms.

14 Lecture №14. Elements of quantum statistics and the band theory of solids

The objective of the lecture: to give an idea of quantum statistics and the band theory of solids.

14.1 Phase space. Density of states

In quantum mechanics, the state of a system is determined by a wave function of a probabilistic nature. The concept of a trajectory for microparticles is absent. If the wave functions of two electrons overlap, then the fundamental possibility of distinguishing between these identical particles disappears. If we interchange the electrons, then it is impossible to experimentally distinguish between the states of the system. When two states are indistinguishable, then this is one and the same state. According to *the principle of indistinguishability of identical particles* in a system of identical particles, only those states are realized that do not change when two particles are rearranged. Or, interchanging two quantum particles does not lead to a new microstate.

In quantum statistics, systems consisting of a huge number of particles are investigated using the laws of quantum mechanics, the basis of which are the wave-corpulence duality of particles of matter and the principle of indistinguishability of identical particles.

To describe the state of a system of particles, is introduced phase space. The coordinate axes of the six-dimensional phase space select the usual X, Y, Z axes and the momentum projections p_x, p_y, p_z . The state of the micro particle in the phase space, taking into account the Heisenberg uncertainty relation, corresponds not to a point, but to a phase cell of volume $\Delta\Gamma_i$. In quantum statistics, the problem is posed of the distribution of particles over the cells of the phase space the volume element of which, taking into account the Heisenberg uncertainty relation, is equal to:

$$\Delta\Gamma = \Delta x \Delta y \Delta z \Delta p_x \Delta p_y \Delta p_z = h^3; \quad (14.1)$$

and also, the problem of determining the average values of physical quantities that characterize the macroscopic state of the system. Particles, the number of which is equal to ΔN_i in the volume $\Delta\Gamma_i$, can be distributed in various ways between Δg_i states with energy E_i . Then the number of quantum states in the volume $\Delta\Gamma_i$ with energies from E_i to $E_i + \Delta E_i$ is:

$$\Delta g_i = \frac{\Delta\Gamma_i}{h^3}. \quad (14.2)$$

The average value of any function is determined using the distribution function, which also allows you to find the probability of the given state of the system.

14.2 The concept of quantum statistics of Bose-Einstein and Fermi-Dirac. Bosons and fermions

In quantum mechanics, two types of particles are distinguished: bosons - particles with integer or zero spin (in units of \hbar), which do not apply the Pauli principle and obey the Bose-Einstein distribution (for example, some nuclei, photons, phonons), and fermions - particles with a half-integer spin (electrons, protons, neutrons, etc.). Fermions are described by quantum Fermi-Dirac statistics and obey the Pauli principle, according to which in a system of identical fermions there cannot be two (or more) particles at the same state.

The Bose-Einstein distribution function f_B expresses the average “density of population” of the bosons of states with a given energy or their average number in one state:

$$f_{B-E} = \frac{\Delta N(E_i)}{\Delta g_i}. \quad (14.3)$$

The energy distribution of Bose-Einstein for the bosons has the form:

$$f_{B-E} = \frac{1}{\exp\left[\frac{E_i - \mu}{kT}\right] - 1}, \quad (14.4)$$

where k is Boltzmann constant;

T is thermodynamic temperature;

μ is the chemical potential equal to the work performed under isobaric-isothermal conditions at increase in the number of particles in the system by one.

The chemical potential of the bosons is $\mu \leq 0$, otherwise the average number of bosons in this state becomes negative, which makes no sense. The bosons do not obey the Pauli principle, and the probability P of the occurrence of a boson in the state, in which there are already n particles, is proportional to n .

The Fermi-Dirac distribution function is determined similarly:

$$f_{F-D} = \frac{1}{\exp\left[\frac{E_i - \mu}{kT}\right] + 1}. \quad (14.5)$$

Here μ , in contrast to (14.4) can be positive.

Thus, there are important differences between the statistical behavior of bosons and fermions. Roughly speaking, bosons have a tendency to clump into the same quantum state, which underlies phenomena such as the laser, Bose–Einstein condensation, and superfluidity. Fermions, on the other hand, are forbidden from sharing quantum states, giving rise to systems such as the Fermi gas. This is known as the Pauli Exclusion Principle, and is responsible for much of chemistry, since the electrons in an atom (fermions) successively fill the many states within shells rather than all lying in the same lowest energy state.

A system of particles is called degenerate if its properties are described by quantum laws. Degeneration becomes essential for Bose or Fermi gases at low temperatures and high densities. An important example of a degenerate Fermi gas is electron gas in metals.

14.3 Stationary states of electrons in metals

The energy distribution of electrons in a metal is described by the Fermi-Dirac distribution function:

$$f_{\phi} = \frac{1}{\exp\left(\frac{E - \mu}{kT}\right) + 1}. \quad (14.6)$$

At absolute zero ($T = 0 \text{ K}$) for $E < \mu$, we have $f_F = 1$ and for $E > \mu$, $f_F = 0$.

The graph of the Fermi-Dirac distribution function at $T = 0 \text{ K}$ and $T > 0$ is shown in figure 14.1. It is clear that μ represents the maximum electron energy at $T = 0$. It is called the Fermi energy: $\mu = E_F$. The highest energy level occupied by electrons in a metal at $T = 0$ is called the Fermi level.

The law of distribution of conduction electrons in a metal at $T = 0 \text{ K}$ has the form:

$$dn_0(E) = \frac{4\pi(2m)^{3/2} E^{1/2} dE}{h^3}. \quad (14.7)$$

The Fermi energy at $T=0$ is:

$$E_{\phi} = \frac{h^2}{2m} \left(\frac{3n_0}{8\pi} \right)^{2/3}. \quad (14.8)$$

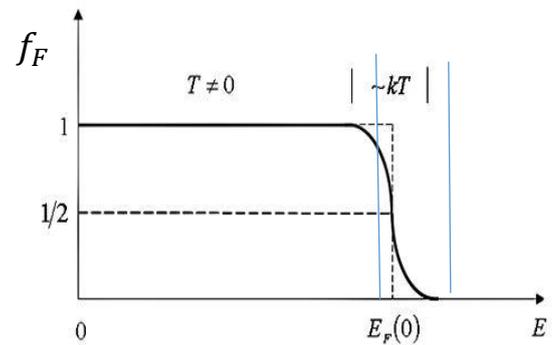


Figure 14.1

The average energy of conduction electrons in a metal at $T=0$ is:

$$\langle E \rangle = \frac{3}{5} E_F. \quad (14.9)$$

14.4 The band structure of the energy spectrum of electrons in crystals. Conductors, semiconductors and dielectrics

According to band theory, the periodic electric field existing in solids substantially changes the energy states of electrons. Instead of energy levels characteristic of an isolated atom, in a crystal consisting of N interacting atoms, energy zones are formed containing N levels separated by an interval of the order of 10^{-23} eV. Such permitted energy zones are separated by forbidden zones (band gap). Figure 14.2 shows the level splitting as a function of the distance r between atoms. Only levels of valence electrons and overlying levels not occupied by electrons are noticeably split.

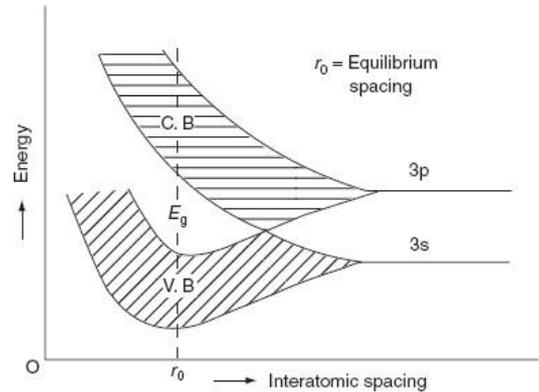


Figure 14.2

The existence of energy bands allows us to explain the existence of metals, semiconductors, and dielectrics. The resolved band arising from the level of valence electrons in the ground state of the atom is called the valence band (VB).

Three cases are possible depending on the degree of filling of this zone. In case (a) this zone is not completely filled with electrons. This crystal is a metal. The same will happen when overlapping neighboring allowed zones.

In the second case, in figure 14.3 (b), the valence band is completely filled with electrons and separated from the nearest allowed band (conduction band) by a small band gap ΔE (on the order of tenths of an eV). Such a crystal is a semiconductor.

If the width ΔE of the band gap is large (of the order of several eV), as in case (c), the crystal is an insulator.

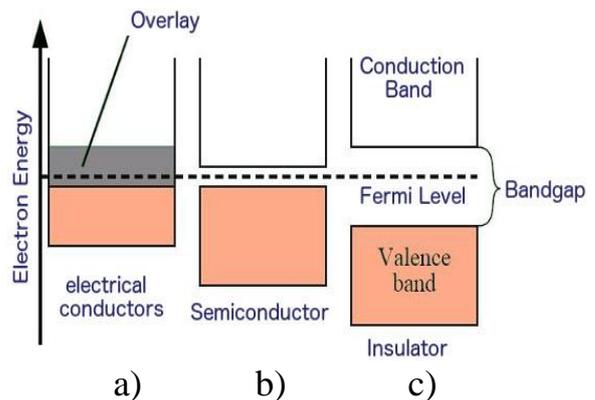


Figure 14.3

14.5 Semiconductors. Intrinsic and impurity conductivity of semiconductors

Semiconductors are substances whose specific resistance varies over a wide range from 10^{-5} to 10^8 Ohm·m and decreases very quickly with increasing temperature. Semiconductors include many chemical elements (germanium Ge, silicon Si, selenium Se, tellurium Te, arsenic As and others), a huge number of alloys and chemical compounds (gallium arsenide, etc.). The most widely used semiconductors are silicon (Si) and germanium (Ge).

Distinguish between intrinsic and impurity semiconductors. Own are chemically pure semiconductors. In them, at $T = 0$ K, all levels of the valence band (VB) are filled with electrons and there are no electrons in the conduction band (CB). The electric field cannot transfer them from the valence to the conduction band; therefore, intrinsic semiconductors at $T = 0$ K behave like dielectrics. At $T > 0$ K, as a result of thermal generation, a part of the electrons passes from the upper levels of the VB to the lower levels of the CB (figure 14.4 a). Due to the formation of vacant levels in the VB, the behavior of the VB electrons can be represented as the motion of positively charged quasiparticles, called holes.

The level distribution of electrons obeys the Fermi-Dirac distribution. For intrinsic semiconductors, the Fermi level is located in the middle of the band gap and the Fermi energy counted from the top of the valence band is $E_F = \Delta E_g/2$.

The specific conductivity γ of intrinsic semiconductors depends on temperature according to the exponential law:

$$\gamma = \gamma_0 \exp\left(-\frac{\Delta E_g}{2kT}\right), \quad (14.10)$$

where ΔE_g is the band gap, γ_0 is a constant.

Impurity conductivity is divided into electronic (or n-type) and hole (p-type) conductivity. To obtain an n-type semiconductor, for example, silicon Si - an element of the IV group, a donor impurity is introduced, i.e. group V element - phosphorus P, arsenic As, etc. The Si atom has in its structure 4 neighboring atoms with which, giving away one electron, it forms covalent bonds. The fifth electron of the impurity atom remains "superfluous." The energy levels of such electrons are located in band gap (figure 14.4 b) below the bottom of the CB, for the transfer of electrons into which a small energy is required, for As in Si $\Delta E_d = 0.054$ eV, obtained, for example, by thermal excitation.

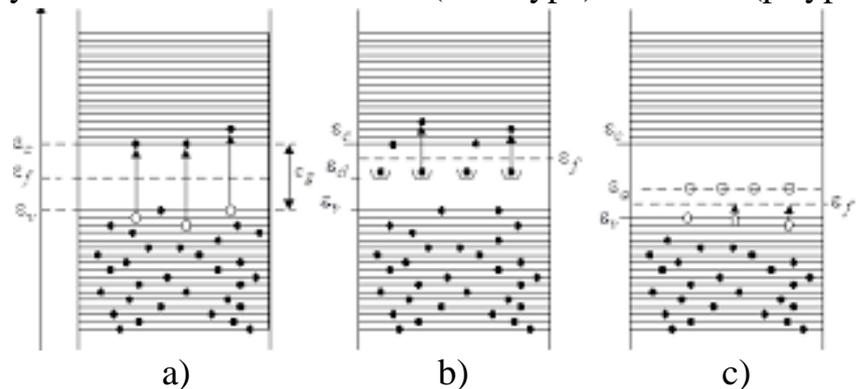


Figure 14.4

When replacing the Si atom with a trivalent atom of an acceptor impurity (boron B, aluminum Al, etc.), a deficiency of one electron arises for the formation of saturated covalent bonds. The missing electron can be borrowed from a neighboring Si atom, in which a positive hole appears in this case. Sequential filling of holes with neighboring electrons is equivalent to the movement of holes and leads to the conductivity of the semiconductor. Acceptor levels arise in the BG above the VB ceiling (for B in Si $\Delta E_h = 0.08$ eV), the transition of electrons from the VB to acceptor levels leads to the appearance (generation) of holes in the VB (figure 14.4 c). The reverse

transition corresponds to the breaking of one of the four covalent bonds of the impurity atom with its neighbors and the recombination of the resulting electron and hole.

With increasing temperature, the concentration of impurity carriers quickly reaches saturation, i.e. impurity conductivity dominates at low T ; the contribution of intrinsic conductivity increases with temperature. Thus, the conductivity of the semiconductor at high T becomes mixed.

14.6 The quantum theory of electrical conductivity of metals

The quantum theory of electrical conductivity, based on quantum mechanics and quantum Fermi-Dirac statistics, has revised the issue of electrical conductivity of metals. According to wave-particle duality, the motion of an electron is associated with a wave process. An ideal crystal lattice (there are stationary particles in its nodes and there are no periodicity violations) “electronic waves” do not scatter. This corresponds to the fact that the metal was not supposed to provide any electric resistance.

In a real crystal lattice, there are always defects, which can be, for example, impurities, vacancies; as well as thermal fluctuations. In a real crystal lattice, “electron waves” are scattered by these inhomogeneities, which is the cause of the electrical resistance of metals.

According to quantum theory, the electrical conductivity of a metal is:

$$\gamma = \frac{ne^2\langle l_F \rangle}{m\langle u_F \rangle}, \quad (14.11)$$

resembling a classic formula, but having excellent physical content. Here $\langle l_F \rangle$ is the mean free path of an electron with an energy equal to the Fermi energy, and $\langle u_F \rangle$ is the average thermal velocity of such electrons.

According to the classical theory, $\langle u \rangle \sim \sqrt{T}$, therefore, it could not explain the true dependence on temperature. In quantum theory, the average speed $\langle u_F \rangle$ of electrons is practically independent of temperature, since the Fermi level remains almost unchanged with temperature. However, with increasing temperature, the scattering of “electron waves” by thermal lattice vibrations (by phonons) increases, which corresponds to a decrease in the mean free path of electrons. In the temperature range $\langle l_F \rangle \sim T^{-1}$, therefore, taking into account the independence of $\langle u_F \rangle$ on temperature, we find that the resistivity of metals ($\rho \sim 1/\sigma$) in accordance with the experimental data increases proportionally to T . Thus, the quantum theory of electrical conductivity metals eliminated this difficulty of the classical theory. Its results are fully consistent with experimental data.

15 Lecture №15. The atomic nucleus. Classification of elementary particles

The objective of the lecture: give an idea of the structure of the atomic nucleus and the properties of nuclear forces and the classification of elementary particles.

15.1 Structure of the atomic nucleus and its main characteristics

The nucleus of the simplest atom - the hydrogen atom - consists of one elementary particle, called the proton. The nuclei of all other atoms consist of two types of elementary particles - *protons and neutrons*. These particles are called *nucleons*. *Proton* (p) has a charge of $+e$ and a mass of $m_p = 938.28$ MeV. For comparison, the electron mass is $m_e = 0.511$ MeV. The spin of proton equals $s = 1/2$ and its own magnetic moment is $\mu_p = +2,79\mu_N$ where $\mu_N = 5,05 \cdot 10^{-27}$ J/Tl is the nuclear magneton.

Neutron (n) was discovered in 1932 by Chadwick. The electric charge of this particle is zero, and the mass is $m_n = 937.58$ MeV. The difference between the masses of neutron and proton is about leaves 1.3 MeV. The neutron has a spin equal to $s = 1/2$ and (despite the absence of an electric charge) by its own magnetic moment $\mu_n = -1.91\mu_N$.

In a free state the neutron is *unstable (radioactive)* - it spontaneously decays, turning into a proton and emitting an electron ($-e$) and another particle called antineutrino $\tilde{\nu}$: $n \rightarrow p + e^- + \tilde{\nu}$. The half-life (i.e., the time during which half of the original number of neutrons decays) is approximately 12 minutes.

One of the most important characteristics of the atomic nucleus is *the charge number*. It is equal to the number of protons that make up the nucleus and determines *its charge* which is equal to $+Ze$. Z defines the ordinal number of the chemical element in the Mendeleev's periodic table. Therefore, it is also called the atomic number of the nucleus. The number of nucleons (that is, the total number of protons and neutrons) in the nucleus is denoted as A and is called the mass number of the nucleus. *The number of neutrons* in the nucleus is $N = A - Z$.

Nuclei is denoted as A_ZX where X is the chemical symbol of a given element. Mass number is placed on the left at the top, an atomic number is on the lower left. Nuclei with the same Z but different A are called *isotopes*. Most chemical elements have several stable isotopes. For example, hydrogen has three isotopes: ordinary hydrogen 1_1H or protium; heavy hydrogen 2_1H or deuterium; tritium 3_1H . Protium and deuterium are stable, tritium is radioactive.

There are about 1500 *nuclei* with different Z or A , or both. About a fifth of these nuclei are stable, the rest are radioactive. Many nuclei were obtained by artificial means with the help of nuclear reactions. In nature, there are elements with an atomic number from 1 to 92. Plutonium (Pu , $Z = 94$) after obtaining it by artificial means was found in insignificant quantities in a natural mineral - uraninite. The remaining transuranium (i.e., after uranic) elements (from 93 to 118) were obtained artificially through various nuclear reactions.

In the first approximation, the nucleus can be regarded as a ball whose *radius* is fairly accurately determined by formula:

$$r = (1,3 \div 1,7) \cdot 10^{-15} \text{ m.} \quad (15.1)$$

It follows from the formula that the volume of the nucleus is proportional to the number of nucleons in the nucleus. Thus, *the density of matter in all nuclei is approximately the same.*

The spins of the nucleons are added to the net spin of the nucleus. The spin of the nucleon is $s = 1/2$. Therefore, the quantum number of the nuclear spin will be half-integer for an odd number of nucleons and integer or zero for an even number. For all even-even nuclei (i.e., a nucleus with an even number of protons and an even number of neutrons), the spin is zero.

15.2 Mass defect and the binding energy of the nucleus

Nucleons in the nucleus are firmly held by nuclear forces. In order to remove the nucleon from the nucleus it is necessary to do a great work, that is, to transfer a considerable energy to the nucleus. The binding energy of an atomic nucleus characterizes the intensity of nucleon interaction in the nucleus and is equal to the maximum energy that must be expended in order to separate the nucleus into separate noninteracting nucleons without informing them of the kinetic energy. Each nucleus has its own binding energy. The more this energy, the more stable the atomic nucleus. The nucleus mass is less than the sum of the masses of its constituent nucleons and this *mass defect* is determined by the following formula

$$\Delta m = |Zm_p + (A - Z)m_n| - m_N. \quad (15.2)$$

Precisely the mass of the nucleus can be determined with the help of mass spectrometers. Applying the law of interrelation of mass and energy, we get:

$$E_b = \Delta m \cdot c^2. \quad (15.3)$$

The specific binding energy is the binding energy of the nucleus per 1 nucleon:

$$E_{sp} = \frac{E_b}{A}. \quad (15.4)$$

In figure 15.1 is a graph of the experimentally established dependence of binding energy on the mass number. The curve in the figure has a weak maximum. The greatest specific binding energy has elements with mass numbers from 50 to 60 (iron and elements close to it). The nuclei of these elements are *most stable*.

It can be seen from the graph that the reaction of fission of heavy

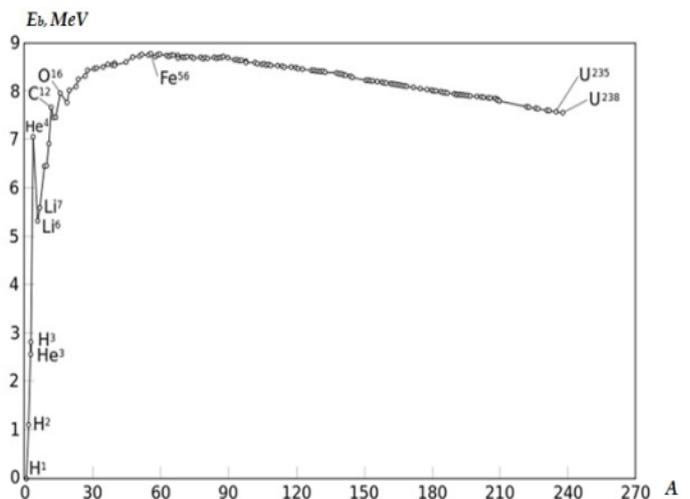


Figure 15.1

nuclei into the nuclei of the elements of the middle part of Mendeleev's periodic table, as well as the reaction of the synthesis of light nuclei (hydrogen, helium) into heavier nuclei, are energetically favorable reactions since they are accompanied by the formation of more stable nuclei (with larger E_{sp}) and, consequently, flow with the release of energy ($E > 0$).

15.3 Nuclear forces. Nucleus models

Between the nucleons in the nucleus specific forces exist which considerably exceed the Coulomb repulsive forces between the protons. They are called *nuclear forces*. Nuclear forces belong to the class of so-called *strong interactions*. *The basic properties of nuclear forces* are the next.

- 1) Nuclear forces are *attractive* forces.
- 2) Nuclear forces are *short-range* forces.
- 3) Nuclear forces are characterized by *charge independence*: the nuclear forces acting between two protons, or a proton and a neutron, are the same in magnitude, i.e. nuclear forces are not electrical in nature.
- 4) Nuclear forces are characterized by *saturation*, i.e., each nucleon in the nucleus interacts only with a limited number of nucleons closest to it.
- 5) Nuclear forces depend on the *mutual orientation of interacting nucleons' spins*.
- 6) Nuclear forces *are not central*.

Liquid drop model of the nucleus was its first model. It is based on an analogy between the behavior of nucleons in the nucleus and the behavior of molecules in a drop of liquid. Thus, in both cases the forces are *short-ranged* and characterized by *saturation*. Under given external conditions *the constant density of its substance* is characteristic. *The volume of the drop and the volume of the nucleus are proportional to the number of particles*. *The essential difference* between a nucleus and a liquid drop in this model is that it treats the nucleus as a drop of an electrically charged incompressible fluid that obeys the laws of quantum mechanics. The drop model of the nucleus explained the mechanism of nuclear fission reactions of nuclei, but could not explain the increased stability of nuclei containing magic numbers of protons and neutrons.

Shell model of the nucleus assumes the distribution of nucleons in the nucleus over discrete energy levels filled by the Pauli principle and relates the stability of nuclei with the filling of these levels. It is believed that *the nucleus with fully filled shells are most stable*. The shell model of the nucleus made it possible to explain the spins and magnetic moments of the nuclei, the different stability of atomic nuclei and also for the description of light and medium nuclei, as well as for nuclei in the ground state. Also there were generalized model of the nucleus.

15.4 Transformation of atomic nuclei. Laws of radioactive decay

Radioactivity is the ability of atomic nuclei to transform into other nuclei with the emission of a particle spectrum. If nuclei transformation occurs spontaneously, then radioactivity is called *natural*. If decay is carried out artificially the radioactivity is *artificial*. Radioactivity was discovered by the Becquerel in 1896 who observed the emission of penetrating radiation by uranium.

The main types of radioactive radiation are α -, β - and γ -radiation. There are other types associated with the emission of protons or positrons, as well as spontaneous fission of nuclei, *K*-capture.

The theory of radioactive decay is based on the assumption that the nuclei undergo radioactive transformations independently of each other. Then the number of nuclei dN decaying in a small time interval dt is proportional to the number of available nuclei N and time dt :

$$dN = -\lambda N dt, \quad (15.4)$$

where λ is the *radioactive decay constant* and characterizes the decay activity of the element.

Equation (15.4) shows that the number of radioactive atoms decreases with time at the decay process. After integrating, we obtain

$$N = N_0 \cdot e^{-\lambda t}. \quad (15.5)$$

This is Rutherford's law of radioactive decay. Here N_0 is the initial number of radioactive nuclei; N is the number of non-decayed nuclei by the time t .

The lifetime of radioactive nuclei is usually characterized by a half-life $T_{1/2}$, that is, an interval of time for which the number of radioactive nuclei will be halved. Based on this definition, it is easy to find the relationship between the half-life and the decay constant:

$$N_0/2 = N_0 e^{-\lambda T_{1/2}}; \quad (15.6)$$

$$T_{1/2} = \ln 2/\lambda = 0.693/\lambda. \quad (15.7)$$

The average lifetime of the radioactive nucleus $\tau = 1/\lambda$.

Thus, the average lifetime of the radioactive nucleus is the inverse to *radioactive decay constant*.

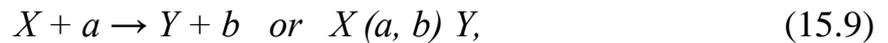
Activity of the nuclide (the general name of atomic nuclei, differing in the number of protons and neutrons) of the radioactive source is the average number of decays per second:

$$A = \left| \frac{dN}{dt} \right| = \lambda N. \quad (15.8)$$

Unit of activity in SI is Becquerel: 1 Bq is activity of nuclide at which one disintegrations per second occurs. In nuclear physics, a non-systemic unit of nuclide activity of a radioactive source which is called Curie (Ci): $1 \text{ Ci} = 3,7 \cdot 10^{10} \text{ Bq}$.

15.5 Nuclear reactions. The problem of energy sources

A nuclear reaction is the process of strong interaction of an atomic nucleus with an elementary particle or other nucleus, leading to a transformation of the nucleus (or nuclei). Symbolically, the nuclear reaction is written in the form:



where X and Y are the initial and final nuclei; a and b are the incident and emitted particle(s).

The amount of released energy is called the reaction energy:

$$Q = [(m_1 + m_2) - (m_3 + m_4)]c^2, \quad (15.10)$$

where the first mass sum is taken for the initial nucleus and the particle and the second mass sum for the final nucleus and the particle. If $(m_1 + m_2) > (m_3 + m_4)$, the energy is released and the reaction is called *exothermic*, otherwise - the energy is absorbed and the reaction is called *endothermic*.

In 1936 *N. Bohr* established that the reactions caused by not very fast particles proceed in two stages. When a particle a is captured by the X nucleus, an intermediate nucleus (or compound nucleus) C is formed which in the second stage emits a particle b :



Some reactions take place without the formation of an intermediate nucleus, they are called direct nuclear interactions.

A special place in nuclear reactions is occupied by the fission reactions of heavy nuclei under the action of neutrons, accompanied by the emission of two or three secondary neutrons. At the same time fission fragments of similar mass are formed. Most neutrons are emitted almost instantaneously ($t < 10^{-14}$ s, they are called *instantaneous ones*), and a small part (about 0.7%), called *delayed neutrons*, is emitted after a period of 0.05 s to 60 s. In these reactions an energy of about 1.1 MeV / nucleon is released. The secondary neutrons emitted during fission can cause new fission events, which makes it possible to implement a *fission chain reaction*. The latter is characterized by a coefficient of multiplication k of neutrons, which should be greater than one for the development of the chain reaction. Chain reactions are divided into managed and unmanaged. The first are carried out in nuclear reactors, the latter - in the explosion when *the critical mass* is reached.

The reactions of the synthesis of atomic nuclei are characterized by the fact that in them the energy released per nucleon is much greater than in the fission reactions of heavy nuclei. For example, when *dividing the ${}_{92}\text{U}^{238}$ nucleus*, about *200 MeV or 0.84 MeV/nucleon* is released, whereas *in the synthesis of deuterium and tritium this value is approximately 3.5 MeV/nucleon*.

The reactions of the synthesis of light nuclei into heavier nuclei, occurring at ultrahigh temperatures ($T > 10^7$ K) are called *thermonuclear reactions*. Managed thermonuclear fusion opens up mankind the possibility of access to *an almost inexhaustible source of energy*.

15.6 Elementary particles. Leptons, hadrons. Quarks

In physics an elementary particle is a particle with no sub structure, thus not composed of other particles. Now the goal is to consider the fundamental principles that define our understanding of nature through the study of elementary particles. This approach led to the model of the development of the Universe at the earliest stages (the Big Bang Model, Gamow, 1948). The same physical representations are necessary for understanding both a very small world (microcosm) and a very large world (the Universe).

How did the Universe come about and how does it work? It consists of particles, between which there are four types of fundamental interactions: gravitational, electromagnetic, strong and weak.

Gravitational attraction acts between all particles. Gravity, in spite of its weakness, unites the basic structures of the Universe. It holds the stars together and all the planets on their orbits.

The electromagnetic force keeps the electrons in atoms, connects atoms into molecules, liquids and solids. A photon is a massless particle - quantum of electromagnetic radiation.

The strong (nuclear) interaction acts between hadrons - baryons and mesons. Baryons are hadrons containing an odd number of quarks. Most well known baryons such as the proton and neutron have three quarks. It leads to the existence of a large number of atomic nuclei, and consequently, to atoms and chemical elements. Mesons are hadrons containing an even number of quarks (at least 2). Most well known mesons are composed of a quark-antiquark pair. There are six types, known as flavors, of quarks: u - up, d - down, s - strange, c - charm, b - bottom and t - top (table 15.1). Protons and neutrons consist of quarks of two types u and d : $p = uud$, $n = ddu$. It is believed that quarks can not exist in a free state: they are "in confinement" inside the hadron.

Electrons, muons, taons and corresponding them neutrinos belong to the class of particles called *leptons*. In particle physics a lepton is an elementary particle of half-integer spin that does not undergo strong interactions. There are 12 leptons in total (particles and antiparticles) (table 15.1).

The weak interaction explains the decay of nuclei, initiates the process of burning stars, creating the probability of the formation of chemical elements.

The four mentioned forces provide the interaction between elementary (or fundamental) particles. The carriers of the four fundamental interactions are elementary particles, which are called fundamental bosons. Four bosons (with spin 1) were observed experimentally, in addition to the corresponding antiparticles: a photon (γ), a gluon (g), a neutral weak boson (Z^0), and charged weak bosons (W^\pm), there are antiparticles for each of them. Quantum of the gravitational field is a graviton - a neutral massless particle with spin 2, in the existence of which theoretical physicists are sure and which is difficult to detect experimentally.

Table 15.1 Standard model of elementary particles

Three generation of matter (fermions)		1-st	2-nd	3-d	Q charge in unit e	Interactions/ force carriers (bozons)	
Quarks	Upper	u	c	T	+2/3	g (gluon)	H
	Lower	d	s	b	-1/3	γ (photon)	higgs - bozon
Leptons	charged	e	μ	τ	-1	Z – z- bozon	
	neutrino	ν_e	ν_μ	ν_τ	0	W– w-bozon	

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PHYSICS

Lecture notes for students of specialty
5B071800 – Electric power engineering

Editor Qurmanbekova M.D.
Standardization specialist Mukhametsarieva G.I.

Signed to publication
Edition 3 copies
Volume 6.2 quires

Format 60×84 1/16
Typographical paper № 1
Order____. Price 3100 KZT

Copying-duplicating Bureau of non-profit joint stock company
“Gumarbek Daukeev Almaty University of power engineering and telecommu-
nications”126, Baitursynov str., Almaty, 050013