Non-Profit Joint Stock Company

ALMATY UNIVERSITY OF POWER ENGINEERING AND TELECOMMUNICATIONS

The department of Mathematical modeling and software

PROBABILITY THEORY AND MATHEMATICAL STATISTICS

Methodological Guidelines and Tasks for carring out
the calculation- graphical works for students of specialty
5B070200 - Automation and management

Almaty 2017

Compilers: Astrakhantseva L. N., Baisalova M.Zh., Probability theory and mathematical statistics. Methodological Guidelines and Tasks for carring out the calculation- graphical works for students of specialty 5B070200 - Automation and management. – Almaty, 2017 – 62 p.

Methodological Guidelines and Tasks for carrying out the calculation-graphical works contain exemplary calculations №1, 2 of course “Probability theory and mathematical statistics” for students of specialty 5B070200 - Automation and management.

The basic theoretical questions of the program and the solution of an exemplary embodiment are given.

Tables 20, figures 10, bibl. 6.

Reviewer: candidate of sciences (PhD) in Physics and Mathematics, R.E.Kim

Printed according to the Publishing plan of Non-Profit Joint Stock Company “Almaty University of Power Engineering and Telecommunications” for 2017

© Non-Profit Joint Stock Company
“Almaty University of Power Engineering and Telecommunications”, 2017

Introduction

Probability theory studies regularities inherent to mass random phenomena. It provides theoretical base for mathematical statistics which deals with wide circle of practical problems.

That's why, methodological guidelines contain calculation-graphical works by two parts. In the first part, we consider main issues of probability theory, in the second part – issues of mathematical statistics.

In each part, theoretical issues, tasks and exemplary embodiment solution are given.

Number of student variant is defined by group list. Calculation-graphical work should be readable and should be made in notebook.

1 Calculation graphical work №1. Elements of probability theory.

Aims: introducing to notions of random event, its probability, basic theorems of probability theory; study distribution laws and numerical characteristics of discrete and continuous random variables.

1.1 Theoretical issues

1. Subject of probability theory. Random events. Space of elementary events. Algebra of events.
2. Statistical, geometrical and classical definition of probability.
3. Probability addition and multiplication theorems.
4. Total probability formula. Bayes formula. Bernoulli formula.
5. Local and integral Laplace theorem. Tests repetition. Poisson formula.6.
6. Discrete and continuous random variables. Distribution laws of discrete random variable.
7. Integral distribution function. Distribution density function.
8. Numerical characteristics of random variable. Mathematical expectation, variance and root-square-mean deviation of discrete and continuous random variables.
9. Binomial distribution, Poisson distribution. Uniform and exponential distribution, reliability function.
10. Normal distribution.
11. Notion of limit theorems. Big numbers laws, central limit theorem.

1.2 Calculation tasks

1. At distribution center there are three types of products: n_1 products of the 1-st type, n_2 - second type, n_3 - third type ($\sum_i^3 n_i = n$). Find:

a) relative frequency of the 1-st type products;

b) probability, that all m chosen products will be the 1-st type products;

c) probability, that among m chosen products there will be m_1 the 1-st type products;

d) probability, that among m chosen products there will be m_1 the 1-st type products, m_2 - second type, m_3 - third type products ($\sum_{i}^{3} m_i = m$);

e) probability, that among m chosen products there will be at least one the 1-st type product.

№	n	n_1	n_2	n_3	m	m_1	m_2	m_3
1.1	70	20	26	24	5	2	1	2
1.2	75	40	20	15	8	4	1	3
1.3	85	35	30	20	5	2	1	2
1.4	90	20	40	30	7	2	2	3
1.5	87	30	45	12	8	3	2	3
1.6	100	25	55	20	15	8	3	4
1.7	90	40	24	26	9	4	3	2
1.8	95	28	42	25	10	3	5	2
1.9	85	30	15	40	7	2	2	3
1.10	90	17	33	40	6	1	3	2
1.11	85	31	25	29	5	2	2	1
1.12	75	28	32	15	5	1	2	2
1.13	100	30	41	29	9	3	4	2
1.14	80	32	28	20	7	3	2	2
1.15	85	24	26	35	5	1	3	1
1.16	100	41	29	30	10	5	3	2
1.17	90	29	21	40	12	6	4	2
1.18	85	25	35	25	7	2	2	3
1.19	80	18	42	20	5	1	2	2
1.20	95	43	27	25	9	3	4	2
1.21	70	22	28	20	9	2	4	3
1.22	80	30	21	29	7	3	1	1
1.23	90	42	20	28	6	1	3	2
1.24	75	24	26	25	8	2	4	2
1.25	100	37	33	30	10	2	3	5
1.26	90	26	34	30	8	3	2	3
1.27	80	31	29	20	5	1	2	2
1.28	95	29	31	35	8	3	2	3
1.29	96	34	26	36	7	4	1	2
1.30	89	25	35	29	5	1	2	2

2. Three heavy guns shoot at a target independently of each other. Probability of target round equals p_1, p_2, p_3 respectively for the first, second and third guns. Find probability that:

a) all three guns will hit the target;

b) only one gun will hit the target;

c) two guns will hit the target, one – will not;

d) at least one gun will hit the target.

№	p_1	p_2	p_3	№	p_1	p_2	p_3	№	p_1	p_2	p_3
2.1	0.9	0.6	0.5	2.11	0.5	0.9	0.4	2.21	0.5	0.7	0.9
2.2	0.8	0.7	0.6	2.12	0.7	0.8	0.5	2.22	0.6	0.5	0.8
2.3	0.7	0.5	0.8	2.13	0.5	0.7	0.6	2.23	0.7	0.9	0.7
2.4	0.6	0.9	0.8	2.14	0.4	0.6	0.7	2.24	0.8	0.4	0.6
2.5	0.5	0.7	0.9	2.15	0.5	0.5	0.8	2.25	0.9	0.5	0.5
2.6	0.9	0.6	0.8	2.16	0.6	0.9	0.5	2.26	0.4	0.6	0.8
2.7	0.8	0.5	0.7	2.17	0.7	0.8	0.6	2.27	0.5	0.7	0.9
2.8	0.5	0.8	0.6	2.18	0.8	0.5	0.7	2.28	0.6	0.8	0.7
2.9	0.6	0.9	0.5	2.19	0.9	0.6	0.8	2.29	0.7	0.9	0.5
2.10	0.7	0.9	0.4	2.20	0.9	0.4	0.9	2.30	0.8	0.9	0.4

3. Three kits of lamps arrived to a warehouse. In the first kit - n_1 lamps, in the second - n_2, and n_3 lamps are in the third kit ($\sum_i^3 n_i = 1000$). In the first kit there were m_1% of defective lamps, m_2% - in the second and m_3% defective lamps - in the third kit. It is requested:

a) to find probability that chosen at random lamp will be defective;

b) chosen at random lamp is defective. Find probability that it is from i – th kit (i =1,2,3).

№	n_1	n_2	m_1	m_2	m_3	i	№	n_1	n_2	m_1	m_2	m_3	i
3.1	520	220	5	8	7	1	3.16	100	250	7	8	5	1
3.2	270	410	10	5	9	2	3.17	430	180	5	4	7	2
3.3	250	140	8	7	4	2	3.18	170	540	6	5	8	3
3.4	190	380	5	9	30	1	3.19	650	120	10	9	8	2
3.5	290	610	6	3	3	2	3.20	400	180	7	10	5	1
3.6	270	430	10	6	4	2	3.21	120	380	10	6	9	2
3.7	280	360	7	10	9	1	3.22	270	340	9	5	4	3
3.8	520	110	5	7	10	1	3.23	430	120	10	7	6	2
3.9	240	290	9	8	4	3	3.24	360	120	5	10	8	1
3.10	310	410	7	2	5	3	3.25	420	210	8	7	6	1
3.11	520	110	3	6	7	2	3.26	370	130	10	6	5	2
3.12	280	310	9	8	4	2	3.27	410	200	5	10	8	3

3.13	400	320	4	5	8	1	3.28	280	510	10	6	5	3
3.14	350	240	9	8	7	1	3.29	710	120	2	10	4	3
3.15	190	520	5	2	4	3	3.30	460	240	5	9	7	1

4. Probability of *A* event occurrence in each test is equal to *p*. Find probability that event *A* in *n* tests will happen:

a) exactly k_1 times;

b) not less than k_1 times;

c) not more than k_2 times;

d) at least one time (for odd variants, where $n = 10$);

e) from k_1 till k_2 times (for even variants, where $n = 100$).

№	k_1	k_2	p	№	k_1	k_2	p	№	k_1	k_2	p
4.1	3	5	0.6	4.11	2	5	0.4	4.21	6	8	0.7
4.2	62	82	0.6	4.12	80	95	0.4	4.22	70	80	0.7
4.3	5	7	0.8	4.13	5	8	0.8	4.23	4	7	0.6
4.4	55	75	0.8	4.14	60	90	0.6	4.24	65	80	0.75
4.5	4	8	0.8	4.15	2	8	0.7	4.25	7	9	0.75
4.6	40	60	0.8	4.16	80	90	0.8	4.26	78	92	0.75
4.7	3	7	0.3	4.17	5	9	0.8	4.27	2	6	0.7
4.8	50	80	0.3	4.18	70	95	0.8	4.28	30	85	0.7
4.9	4	6	0.3	4.19	3	6	0.7	4.29	4	9	0.7
4.10	45	75	0.4	4.20	50	60	0.7	4.30	80	95	0.6

5. Discrete random variable *X* is given by distribution series. Find:

a) its distribution function *F*(*x*), construct graph of *F(x)*;

b) mathematical expectation, variance, standard deviation, mode;

c) probability, that *X* will hit at interval (*a;b*).

	X	x_1	x_2	x_3	x_4	x_5	x_6	a	b
	P	p_1	p_2	p_3	p_4	p_5	p_6		
5.1	X	0	1	2	4	6	9	-2	7
	P	0.05	0.15	0.3	0.25	0.15	0.1		
5.2	X	-3	-2	-1	0	2	4	-1	3
	P	0.15	0.3	0.02	0.14	0.18	0.21		
5.3	X	1	2	3	5	7	8	-3	6
	P	0.3	0.14	0.16	0.1	0.2	0.1		
5.4	X	-4	-3	-2	0	1	2	0	1
	P	0.2	0.08	0.23	0.27	0.12	0.1		
5.5	X	1	2	4	5	7	9	3	8
	P	0.19	0.21	0.06	0.14	0.12	0.28		
6.6	X	-1	0	2	3	5	7	-4	4

	P	0.26	0.14	0.07	0.2	0.03	0.3		
5.7	X	-2	-1	0	3	5	7	1	6
	P	0.18	0.09	0.01	0.2	0.22	0.3		
5.8	X	1	2	4	5	6	8	0	6
	P	0.3	0.17	0.13	0.1	0.2	0.1		
5.9	X	1	2	3	4	7	9	5	8
	P	0.11	0.29	0.06	0.14	0.17	0.23		
5.10	X	0	1	2	3	7	9	4	8
	P	0.06	0.14	0.3	0.25	0.15	0.1		
5.11	X	-3	-2	0	1	2	4	-1	3
	P	0.15	0.3	0.01	0.14	0.19	0.21		
5.12	X	-1	0	3	5	7	8	1	6
	P	0.25	0.14	0.16	0.1	0.2	0.15		
5.13	X	-4	-3	-2	0	2	4	-1	3
	P	0.2	0.07	0.24	0.26	0.13	0.1		
5.14	X	-3	-1	0	3	4	7	-2	6
	P	0.12	0.09	0.01	0.2	0.28	0.3		
5.15	X	-1	0	1	3	7	8	2	6
	P	0.26	0.14	0.15	0.2	0.1	0.15		
5.16	X	-2	-1	0	1	2	7	-3	5
	P	0.17	0.09	0.01	0.3	0.23	0.2		
5.17	X	1	2	3	5	6	7	0	4
	P	0.1	0.14	0.16	0.1	0.2	0.3		
5.18	X	-3	-1	0	3	5	6	-2	4
	P	0.16	0.09	0.01	0.3	0.24	0.2		
5.19	X	1	2	5	6	7	8	3	6
	P	0.2	0.15	0.15	0.1	0.3	0.1		
5.20	X	-1	0	2	4	7	8	1	5
	P	0.23	0.18	0.12	0.2	0.1	0.17		
5.21	X	1	2	4	5	6	8	0	7
	P	0.3	0.14	0.16	0.03	0.2	0.17		
5.22	X	-4	-3	-1	0	1	3	-2	2
	P	0.2	0.03	0.24	0.26	0.17	0.1		
5.23	X	1	2	3	4	7	9	0	8
	P	0.17	0.23	0.09	0.11	0.12	0.28		
5.24	X	0	1	3	5	7	8	2	6
	P	0.2	0.14	0.16	0.12	0.3	0.08		
5.25	X	-5	-3	-2	0	1	3	-4	2
	P	0.2	0.06	0.21	0.29	0.14	0.1		
5.26	X	1	2	3	5	8	9	4	7
	P	0.18	0.22	0.05	0.15	0.12	0.28		
5.27	X	1	3	4	5	7	8	2	6
	P	0.3	0.16	0.14	0.01	0.2	0.19		

5.28	X	-5	-3	-1	0	1	3	-4	2
	P	0.1	0.03	0.14	0.36	0.17	0.2		
5.29	X	0	2	3	4	6	8	1	7
	P	0.26	0.14	0.05	0.15	0.12	0.28		
5.30	X	-1	0	2	3	7	8	1	6
	P	0.21	0.16	0.14	0.1	0.2	0.19		

6. Continuous random variable X is given by distribution density function $f(x)$. Find:

a) its distribution function $F(x)$;

b) mathematical expectation, variance, standard deviation, mode, median;

c) probability, that X will hit in interval $(a;b)$.

Construct graphs of $F(x)$ and $f(x)$.

№	$f(x)$	a	b	№	$f(x)$	a	b
6.1	$\begin{cases} 0, x \le 0, x > 4 \\ \frac{x}{8}, 0 < x \le 4 \end{cases}$	1	3	6.16	$\begin{cases} 0, \ x \le 0, x > 3 \\ \frac{2}{3}\left(1 - \frac{x}{3}\right), \ 0 < x \le 3 \end{cases}$	-1	2
6.2	$\begin{cases} 0, x \le -3, x > -2 \\ \frac{6}{x^2}, -3 < x \le -2 \end{cases}$	-2,5	0	6.17	$\begin{cases} 0, x \le 0, x > \frac{\pi}{6} \\ 4\sin 2x, 0 < x \le \frac{\pi}{6} \end{cases}$	0	$\frac{\pi}{12}$
6.3	$\begin{cases} 0, x \le -\frac{\pi}{2}, x > \frac{\pi}{2} \\ 0{,}5\cos x, -\frac{\pi}{2} < x \le \frac{\pi}{2} \end{cases}$	0	$\frac{\pi}{4}$	6.18	$\begin{cases} 0, x \le 1, x > 2 \\ \frac{2}{x^2}, 1 < x \le 2 \end{cases}$	0	1,5
6.4	$\begin{cases} 0, x \le 0, x > 1 \\ \frac{4}{\pi(1+x^2)}, 0 < x \le 1 \end{cases}$	0	$\frac{\sqrt{3}}{3}$	6.19	$\begin{cases} 0, \ x \le 2, x > 3 \\ \frac{2x}{5}, \ 2 < x \le 3 \end{cases}$	1	2,5
6.5	$\begin{cases} 0, \ x \le 0, x > 3/4 \\ \frac{2}{\pi\sqrt{1-x^2}}, \ 0 < x \le 3/4 \end{cases}$	0	$\frac{1}{2}$	6.20	$\begin{cases} 0, x \le 0, x > \frac{1}{\sqrt{3}} \\ \frac{6}{\pi(1+x^2)}, 0 < x \le \frac{1}{\sqrt{3}} \end{cases}$	0,1	1
6.6	$\begin{cases} 0, x \le 0, x > \pi \\ 0{,}5\sin x, 0 < x \le \pi \end{cases}$	0	$\frac{\pi}{2}$	6.21	$\begin{cases} 0, \ x \le -1, x > 2 \\ \frac{1}{9}(x+1)^2, \ -1 < x \le 2 \end{cases}$	0	1
6.7	$\begin{cases} 0, x \le 0, x > 2 \\ \frac{x+2}{6}, 0 < x \le 2 \end{cases}$	1	2	6.22	$\begin{cases} 0, x \le 0, x \ge \frac{1}{2} \\ \frac{6}{\pi\sqrt{1-x^2}}, 0 < x < \frac{1}{2} \end{cases}$	$\frac{1}{4}$	1
6.8	$\begin{cases} 0, \ x \le 4, x > 5 \\ \frac{2x}{9}, \ 4 < x \le 5 \end{cases}$	3	4,5	6.23	$\begin{cases} 0, x \le 3, x > 5 \\ \frac{7{,}5}{x^2}, 3 < x \le 5 \end{cases}$	2	4

6.9	$\begin{cases} 0, x \le \frac{\pi}{2}, x > \frac{5\pi}{6} \\ -2\cos x, \frac{\pi}{2} < x \le \frac{5\pi}{6} \end{cases}$	0	$\frac{2\pi}{3}$	6.24	$\begin{cases} 0, x \le 0, x > \frac{\pi}{6} \\ 6\sin 3x, 0 < x \le \frac{\pi}{6} \end{cases}$	0	$\frac{\pi}{12}$
6.10	$\begin{cases} 0, x \le 1, x > 2 \\ 3(x-1)^2, 1 < x \le 2 \end{cases}$	1,5	2	6.25	$\begin{cases} 0, x \le 1, x > 2 \\ 2x - 2, 1 < x \le 2 \end{cases}$	0	1,5
6.11	$\begin{cases} 0, x \le 0, x > \frac{\pi}{4} \\ 2\cos 2x, 0 < x \le \frac{\pi}{4} \end{cases}$	$\frac{\pi}{8}$	$\frac{\pi}{4}$	6.26	$\begin{cases} 0, x \le -2, x > 2 \\ \frac{1}{2\pi}\sqrt{4-x^2}, -2 < x \le 2 \end{cases}$	0	1
6.12	$\begin{cases} 0, x \le 0, x > 4 \\ \frac{1}{2}(1-\frac{x}{4}), 0 < x \le 4 \end{cases}$	1	3	6.27	$\begin{cases} 0, x \le 0, x > 5 \\ \frac{2}{5}(1-\frac{x}{5}), 0 < x \le 5 \end{cases}$	1	4
6.13	$\begin{cases} 0, x \le 0, x > 2 \\ \frac{x+1}{4}, 0 < x \le 2 \end{cases}$	-1	1	6.28	$\begin{cases} 0, x \le 0, x > \frac{\pi}{6} \\ 3\cos 3x, 0 < x \le \frac{\pi}{6} \end{cases}$	$\frac{\pi}{12}$	$\frac{\pi}{9}$
6.14	$\begin{cases} 0, x \le 0, x > 1 \\ 3x^2, 0 < x \le 1 \end{cases}$	0,2	1,2	6.29	$\begin{cases} 0, x \le -3, x > 3 \\ \frac{1}{2\pi}\sqrt{9-x^2}, -3 < x \le 3 \end{cases}$	0	2
6.15	$\begin{cases} 0, x \le 0, x > \frac{\pi}{3} \\ 2\sin x, 0 < x \le \frac{\pi}{3} \end{cases}$	0	$\frac{\pi}{6}$	6.30	$\begin{cases} 0, x \le 1, x > 4 \\ \frac{2x}{15}, 1 < x \le 4 \end{cases}$	2	3

7. Reliability tests of N products are made. Probability of non-passing the tests by a product is equal to m%. Work out the distribution law for number of products, which did not pass the tests (random variable X). Find mathematical expectation, variance and root-mean-square deviation of this random variable.

№	N	m	№	N	m	№	N	m
7.1	3	10	7.11	4	15	7.21	3	11
7.2	2	12	7.12	5	13	7.22	2	16
7.3	4	20	7.13	3	14	7.23	4	29
7.4	5	25	7.14	2	20	7.24	5	10
7.5	3	30	7.15	4	27	7.25	3	17
7.6	2	10	7.16	5	20	7.26	2	21
7.7	4	15	7.17	3	19	7.27	4	22
7.8	5	17	7.18	2	23	7.28	5	24
7.9	3	12	7.19	4	11	7.29	3	18
7.10	2	15	7.20	5	28	7.30	2	22

8. The device consists of N elements. Probability of failure of an element during one year of operation is p and it doesn't depend on status of other elements. It is required to:

a) work out distribution law of fault elements;

b) find the probability of failure of not less than m elements per year.

№	N	m	p	№	N	m	p	№	N	m	p
8.1	2000	4	0,001	8.11	1500	6	0,005	8.21	1000	6	0,005
8.2	1000	5	0,007	8.12	4000	2	0,006	8.22	4500	2	0,003
8.3	3000	7	0,004	8.13	8000	2	0,001	8.23	2000	4	0,001
8.4	2000	5	0,002	8.14	6500	6	0,002	8.24	1000	5	0,007
8.5	1000	6	0,005	8.15	3000	2	0,005	8.25	3000	7	0,004
8.6	5000	2	0,001	8.16	1500	3	0,002	8.26	2000	5	0,002
8.7	2000	4	0,001	8.17	2000	4	0,001	8.27	1000	6	0,005
8.8	1500	5	0,008	8.18	1000	5	0,007	8.28	6500	8	0,007
8.9	3500	7	0,004	8.19	3500	1	0,002	8.29	7000	6	0,002
8.10	2000	2	0,003	8.20	2000	5	0,001	8.30	5500	9	0,004

9a. Variants 1-15.

Scale division value of measurement device is equal to a. Device indications are rounded off till nearest integer scale division. Random variable X – is error of indication rounding. Find:

a) distribution density function $f(x)$ of X;

b) distribution function $F(x)$;

c) mathematical expectation, variance;

d) probability, that during the measurement the error less (more) than m will be made.

Construct graphs of $F(x)$ and $f(x)$.

9b. Variants 16 – 30.

Subway trains are moving strictly on schedule. Traffic interval is a minutes. Random variable X - is waiting time for train. Find:

a) its distribution density function $f(x)$;

b) distribution function $F(x)$;

c) mathematical expectation, variance;

d) probability, that a passenger, who has just come to the station, will wait for train less (more) than m minutes.

Construct graphs of $F(x)$ and $f(x)$.

№	a	m	№	a	m	№	a	m
9.1	0,2	0,04	9.11	0,3	0,08	9.21	19	8
9.2	0,3	0,02	9.12	0,6	0,01	9.22	20	5
9.3	0,1	0,06	9.13	0,9	0,06	9.23	25	5
9.4	0,5	0,01	9.14	0,5	0,05	9.24	9	3

9.5	0,6	0,05	9.15	0,8	0,07	9.25	14	7
9.6	0,9	0,02	9.16	5	3	9.26	18	9
9.7	0,1	0,08	9.17	10	4	9.27	24	8
9.8	0,7	0,01	9.18	15	5	9.28	6	3
9.9	0,4	0,06	9.19	6	2	9.29	12	6
9.10	0,5	0,07	9.20	20	10	9.30	16	8

10. Time of non-failure operation of element (random variable *T*) has exponential distribution with parameter λ, where λ - is intensity of faults in time unit. Find:

a) distribution density function *f(t);*

b) distribution function *F(t)*, indicate its probabilistic sense;

c) reliability function *R(t)*, indicate its probabilistic sense;

d) mathematical expectation, variance;

e) probability, that for time *t*, the element will fail, and probability, that for time *t*, the element will not fail.

Construct graphs of *F(t), R(t)* and *f(t)*.

№	λ	*t*	№	λ	*t*	№	λ	*t*
10.1	1	5	10.11	2	5	10.21	3	8
10.2	2	10	10.12	3	10	10.22	4	4
10.3	3	6	10.13	4	6	10.23	6	3
10.4	4	8	10.14	6	8	10.24	7	2
10.5	6	4	10.15	7	4	10.25	8	1
10.6	7	3	10.16	8	3	10.26	9	10
10.7	8	2	10.17	9	2	10.27	10	6
10.8	9	1	10.18	10	1	10.28	1	7
10.9	10	7	10.19	1	10	10.29	2	8
10.10	1	9	10.20	2	6	10.30	3	2

11. Random deviation of product size from standard size (random variable *X*) obeys the normal distribution law with parameters *a* and σ. Find:

a) distribution density function *f*(*x*);

b) distribution function *F*(*x*);

c) mathematical expectation, variance;

d) probability of hitting in interval $(\alpha;\beta)$;

e) probability, that deviation from standard size is not more than δ by absolute value.

Construct graphs of *F*(*t*) and *f*(*t*).

№	*a*	σ	α	β	δ	№	*a*	σ	α	β	δ
11.1	10	1	8	14	2	11.16	10	2	9	14	2
11.2	12	2	7	14	3	11.17	12	4	5	14	3

11.3	14	3	10	15	5	11.18	14	1	9	15	5
11.4	11	5	9	12	3	11.19	11	6	8	12	3
11.5	13	2	6	13	2	11.20	13	4	6	17	2
11.6	12	3	7	15	4	11.21	12	9	8	15	4
11.7	10	2	8	17	2	11.22	10	3	6	17	2
11.8	12	4	6	14	6	11.23	12	5	6	13	6
11.9	14	6	11	19	5	11.24	14	2	12	19	5
11.10	15	5	8	12	3	11.25	15	3	4	12	3
11.11	17	4	6	14	2	11.26	17	1	5	14	2
11.12	12	5	7	18	4	11.27	12	4	9	18	4
11.13	18	5	6	12	3	11.28	11	3	4	12	3
11.14	10	4	6	15	2	11.29	17	2	5	19	5
11.15	12	3	5	18	4	11.30	13	5	6	18	3

1.3 Solution of exemplary embodiment

1. At distribution center there are three types of details: 40 details of the 1-st type, 50- second type, 30 - third type (120 details in total). Find:

a) relative frequency of the 1-st type details;

b) probability, that all 20 chosen details will be the 1-st type details;

c) probability, that among 20 chosen details there will be 9 the 1-st type details;

d) probability, that among 20 chosen details there will be 9 the 1-st type details, 6 - second type, 5 - third type details;

e) probability, that among 20 chosen details there will be at least one the 1-st type detail.

Solution:

a) relative frequency of event A (denotes $P^*(A)$) is ratio of number of m tests where event A has occurred to the total number n of tests: $P^*(A) = m/n$.

Let event A – is choice of the 1-st type details then $P^*(A) = 40/120 = 1/3$.

In other items we use classical definition of event A probability: $P(A) = m/n$, where m – number of tests, favorable to occurrence of event A, n – is total number of tests;

b) let event A – all 20 details taken at random are the 1-st type details. Total number of elementary events is equal to number of different ways: how to take 20 details from 120 details, i.e. $n = C_{120}^{20}$; number of favorable events is equal to number of different ways: how to take from 40 the 1-st type details 20 details, i.e. $m = C_{40}^{20}$. Thus, $P(A) = m/n = C_{40}^{20}/C_{120}^{20} = 4{,}679 \times 10^{-12}$;

c) let event A – is that among 20 details taken at random, 9 will be the 1-st type details. As it is said above, $n = C_{120}^{20}$; number m of favorable to event A of the space of elementary events, we obtain combining each of C_{40}^{9} combinations (C_{40}^{9} is equal to number of different ways to choose 9 the 1-st type details from 40 the 1-st

type details) with each of C_{80}^{11} combinations (C_{80}^{11} is equal to number of different ways to choose 11 second and third types details from 80 second and third types details), i.e. $m = C_{40}^{9} \cdot C_{80}^{11}$. Thus, $P(A) = m/n = C_{40}^{9} \cdot C_{80}^{11} / C_{120}^{20} = 0{,}097$.

d) let event A – is choice of 9 the 1-st type details, 6 – second type and 5 – third type details among 20 at random chosen details. To solve the task we again use classical definition of event A probability: $P(A) = m/n$, where n is number of all possible ways how to choose 20 details from available 120 details, i.e. $n = C_{120}^{20}$. Number m of elementary events favourable to event A is found by above indicated way, i.e. $m = C_{40}^{9} \cdot C_{50}^{4} \cdot C_{30}^{5}$. So, $P(A) = m/n = 0{,}021$.

e) let event A is that among 20 chosen details, there will be at least one the 1-st type detail. Then opposite event $\bar{A}$ is that among 20 chosen details, there won't be any the 1-st type detail. As we have done in item b), let's find probability of this event by formula $P(\bar{A}) = m/n = C_{80}^{20} / C_{120}^{20} = 1{,}2 \times 10^{-4}$. Then probability of event A equals $P(A) = 1 - P(\bar{A}) = 1 - 1{,}2 \times 10^{-4} \approx 1$, i.e. this event is almost certain event.

Calculating number of combinations we used function *combin* in MathCAD. Below, we give copy of file, where *combin*(Q,R) is entered as user's function C(Q,R), which allows to get values of combinations with arbitrary Q and R.

$C(40,9) = 2.734 \times 10^{8}$

$C(Q,R) := combin(Q,R)$

$C(80,11) = 1.048 \times 10^{13}$

$C(120,20) = 2.946 \times 10^{22}$

$C(40,20) = 1.378 \times 10^{11}$

$\dfrac{C(80,20)}{C(120,20)} = 1.2 \times 10^{-4}$

$\dfrac{C(40,9) \cdot C(80,11)}{C(120,20)} = 0.097$

$\dfrac{C(40,9) \cdot C(50,6) \cdot C(30,5)}{C(120,20)} = 0.021$

2. Three heavy guns shoot at a target independently of each other. Probability of target round equals 0,75; 0,8, 0,9 respectively for the first, second and third guns. Find probability that:

a) all three guns will hit the target;

b) only one gun will hit the target;

c) two guns will hit the target, one – will not;

d) at least one gun will hit the target.

Solution: let event A_1 – is that the first gun hit the target, A_2 – is that the second gun hit the target and A_3 – is that the third gun hit the target. By condition of the task $P(A_1)$=0,75, $P(A_2)$=9,8, $P(A_3)$=0,9;

a) let event A – is that all three guns hit the target, then $A = A_1A_2A_3$. Since A_1, A_2, A_3 are independent events, then $P(A) = P(A_1A_2A_3) = P(A_1)P(A_2)P(A_3) = = 0{,}75 \cdot 0{,}8 \cdot 0{,}9 = 0{,}54$;

b) let event B – is that only one gun hit the target, then $B = A_1\overline{A}_2\overline{A}_3 + \overline{A}_1A_2\overline{A}_3 + \overline{A}_1\overline{A}_2A_3$, where $\overline{A}_1, \overline{A}_2, \overline{A}_3$ are events, opposite to events A_1, A_2, A_3, i.e. miss of the first, second and third gun respectively. Since $P(\overline{A}_1) = 1 - P(A_1) = 1 - 0{,}75 = 0{,}25$, $P(\overline{A}_2) = 1 - P(A_2) = 1 - 0{,}8 = 0{,}2$, $P(\overline{A}_3) = 1 - P(A_3) = = 1 - 0{,}9 = 0{,}1$ and since these events are incompatible events, then $P(B) = P(A_1\overline{A}_2\overline{A}_3) + P(\overline{A}_1A_2\overline{A}_3) + P(\overline{A}_1\overline{A}_2A_3) = 0{,}75 \cdot 0{,}2 \cdot 0{,}1 + 0{,}25 \cdot 0{,}8 \cdot 0{,}1 + 0{,}25 \cdot 0{,}2 \cdot 0{,}9 = =0{,}08$;

c) event C – is that two guns will hit the target but one will miss, is worked out similarly as it was done in item b), i.e. $C = A_1A_2\overline{A}_3 + \overline{A}_1A_2A_3 + A_1\overline{A}_2A_3$. Its probability is found by the same way: $P(C) = 0{,}75 \cdot 0{,}8 \cdot 0{,}1 + 0{,}25 \cdot 0{,}8 \cdot 0{,}9 + 0{,}75 \cdot 0{,}2 \cdot 0{,}9 = 0{,}3456$;

d) let event D – is that at least one gun will hit the target. Let's consider opposite event $\overline{D}$ - all three guns will miss. Since $\overline{D} = \overline{A}_1\overline{A}_2\overline{A}_3$, then

$$P(D) = 1 - P(\overline{D}) = 1 - P(\overline{A}_1\overline{A}_2\overline{A}_3) = 1 - 0{,}25 \cdot 0{,}2 \cdot 0{,}1 = 0{,}995.$$

3. Three kits of lamps arrived to a warehouse. In the first kit - 100 lamps, in the second - 300, and 600 lamps in the third kit ($\sum_i^3 n_i = 1000$). In the first kit there were 5% of defective lamps, 4% - in the second and 6% defective lamps in the third kit. It is requested:

a) to find probability that chosen at random lamp will be defective;

b) chosen at random lamp is defective. Find probability that it is from 2–nd kit.

Solution. Let event A – is that taken at random lamp will be defective; and events B_1, B_2, B_3 – are that the chosen lamp is from the first, second and third kit respectively (these events are called the hypotheses).

a) probability of event A is found by the total probability formula:

$P(A) = P(B_1)P(A/B_1) + P(B_2)P(A/B_2) + P(B_3)P(A/B_3)$, where $P(A/B_i)$ – are conditional probabilities, that chosen lamp is from i-th kit (i=1,2,3). By conditions of the task, we have: $P(B_1) = 100/1000 = 0{,}1$; $P(B_2) = 300/1000 = 0{,}3$; $P(B_3) = 600/1000 = 0{,}6$; $P(A/B_1) = 0{,}05$; $P(A/B_2) = 0{,}04$; $P(A/B_3) = 0{,}06$.

So, $P(A) = 0.1 \cdot 0.05 + 0{,}3 \cdot 0{,}04 + 0{,}6 \cdot 0{,}06 = 0{,}053$;

b) in this item it is required to find conditional probability $P(B_2/A)$. Let's use Bayes formula: $P(B_i / A) = \dfrac{P(B_i / A)P(A/B_i)}{\sum_{k=1}^{n} P(B_k)P(A/B_k)}$, $i = 1,2,\ldots,n$.

In our case $P(B_2/A) = \dfrac{P(B_2/A)P(A/B_2)}{\sum_{k=1}^{3} P(B_k)P(A/B_k)} = \dfrac{0{,}3 \cdot 0{,}04}{0{,}053} = 0{,}226.$

4. Probability of *A* event occurrence in each test is equal to 0,8. Find probability that event *A* in *n* tests will happen:

a) exactly k_1 times (event *A*);

b) not less than k_1 times (event *B*);

c) not more than k_2 times (event *C*);

d) at least one time (for odd variants, where *n* = 10) (event *D*);

e) from k_1 till k_2 times (for even variants, where *n* = 100) (event *E*).

Solution: in this task we use formulas, which contain probability, that in *n* tests the event *A* will occur *k* times. The probability designates as $P_n(k)$. Depending from task conditions, there are different ways for task solution:

1) Let *n* =10, k_1=9, k_2=2 (for odd variants). Here *n* is not big, so the probability of event *A* can be found by Bernoulli formula: $P_n(k) = C_n^k p^k q^{n-k}$, where $q = 1 - p$, $(k = 0,1,2,..n)$. Probabilities of events *B* and *C* can be found as sums of probabilities: $P_n(k) + P_n(k+1) + ... + P_n(n)$- is probability, that event will happen not less than *k* times in *n* independent tests, i.e. either *k*, or *k*+1, … , or *n* times; $P_n(0) + P_n(1) + ... + P_n(k)$- is probability, that event will happen not more than *k* times in *n* independent tests, i.e. either 0, or 1, or 2, ... , or *k* times. These probabilities are called the cumulative probabilities. Thus,

a) $P(A) = P_{10}(9) = C(10,9) \cdot 0.8^9 \cdot 0.2^1 = 0.268$;

b) $P(B) = P_{10}(9) + P_{10}(10) = 1{,}246 \times 10^{-9}$;

c) $P(C) = P_{10}(0) + P_{10}(1) + P_{10}(2) = 0{,}96$;

d) let's consider event $\overline{D}$ opposite to event *D*. $\overline{D}$ - is event, that event *A* never occurred in series of 10 independent tests. Then, $P(D) = 1 - P(\overline{D}) = 1 - P_{10}(0) \approx 0{,}566$;

Below, you find copy of file from MathCAD with calculations.

$C(Q,R) := combin(Q,R)$

$C(10,9) \cdot 0.8^9 \cdot 0.2^1 = 0.268$

$C(10,9) \cdot 0.08^9 \cdot 0.92^1 + C(10,10) \cdot 0.08^{10} \cdot 0.92^0 = 1.246 \times 10^{-9}$

$C(10,0) \cdot 0.08^0 \cdot 0.92^{10} + C(10,1) \cdot 0.08^1 \cdot 0.92^9 + C(10,2) \cdot 0.08^2 \cdot 0.92^8 = 0.96$

$1 - C(10,0) \cdot 0.08^0 \cdot 0.92^{10} = 0.566$

2) Let n =100, k_1=70, k_2=80 (for even variants). Since number of independent tests *n* are big, then probability $P_n(k)$ that event *A* will occur *k* times in n tests, is found by the local Moivre – Laplace theorem and it approximately is equal to $P_n(k) \cong \frac{1}{\sqrt{npq}}\varphi(x)$, where $x = \frac{k-np}{\sqrt{npq}}$, $0 < p < 1$, $\varphi(x) = \frac{1}{\sqrt{2\pi}}\exp(-x^2/2)$ (values of this function can be found from special tables or by means of built-in function *dnorm* in MathCAD system).

To define probabilities of events *B*, *C* and *E* we will use integral Moivre – Laplace theorem: probability $P_n(k_1,k_2)$, that number *k* of occurrence of some event A will be between k_1 and k_2 is approximately equal to $P_n(k_1,k_2) \approx \Phi(x_2) - \Phi(x_1)$, where $x_2 = \frac{k_2-np}{\sqrt{npq}}$, $x_1 = \frac{k_1-np}{\sqrt{npq}}$, $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_0^x \exp(-t^2/2)dt$ - is Laplace function (or probabilities integral). Its values are found from special tables or by means of built-in function *pnorm* in MathCAD system.

a) $P(A) = P_{100}(80) \cong \frac{1}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}}\varphi(2{,}5) = 0{,}018/4 = 0{,}0045$; $x = \frac{70-100\cdot 0{,}8}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}} = -2{,}5$;

b) $P(B) = P_{100}(k \ge 80) = P_{100}(80{,}100) \approx \Phi(x_3) - \Phi(x_2) = 0{,}5$; $x_2 = 0, x_3 = 5$;

c) $P(C) = P_{100}(k \le 70) = P_{100}(0{,}70) \approx \Phi(x_1) - \Phi(x_4) = 0{,}006$; $x_1 = -2{,}5, x_4 = -20$;

d) $P(E) = P_{100}(70{,}80) \approx \Phi(x_2) - \Phi(x_1) = 0{,}494$, $x_2 = 0$, $x_1 = -2{,}5$.

Below, you find copy of the file, where calculations are made in MathCAD system.

$n := 100$ $k1 := 70$ $k2 := 80$

$p := 0.8$ $q := 1 - p$

$x1 := \frac{k1 - n\cdot p}{\sqrt{n\cdot p\cdot q}}$ $x2 := \frac{k2 - n\cdot p}{\sqrt{n\cdot p\cdot q}}$ $x3 := \frac{n - n\cdot p}{\sqrt{n\cdot p\cdot q}}$ $x4 := \frac{0 - n\cdot p}{\sqrt{n\cdot p\cdot q}}$

$x1 = -2.5$ $x2 = 0$

$pnorm(x2,0,1) - pnorm(x1,0,1) = 0.494$

$dnorm(x1,0,1) = 0.018$

$pnorm(x3,0,1) - pnorm(x2,0,1) = 0.5$

$pnorm(x1,0,1) - pnorm(x4,0,1) = 6.21 \times 10^{-3}$

Or other variant

$\Phi(x) := pnorm(x,0,1) - 0.5$

$P(k1,k2) := \Phi(x2) - \Phi(x1)$

$\Phi(x1) = -0.494$ $\Phi(x2) = 0$

$P(k1,k2) = 0.494$ $x3 = 5$ $x4 = -20$

$\Phi(x3) = 0.5$ $\Phi(x4) = -0.5$

$\Phi(x3) - \Phi(x2) = 0.5$ $\Phi(x1) - \Phi(x4) = 6.21 \times 10^{-3}$

3) There is another formula to define probabilities $P_n(k)$, which can be used if n is big, but p is small, and product $\lambda = n \cdot p$ - is not a big number. It is Poisson formula $P_n(k) \approx \lambda^k \cdot e^{-\lambda} / k!$.

Let n=1000, k=6, p=0,003, $\lambda = 1000 \cdot 0{,}003 = 3$, So,

$P(A) = P_{1000}(6) = 3^6 \cdot e^{-3} / 6! = 0{,}05$.

For calculation, you can use table of values of function $p(k, \lambda) = \lambda^k \cdot e^{-\lambda} / k!$, given in some text books. Or you can use function *dpois* in MathCAD system. Below, we give copy of the file with calculations in MathCAD.

$p(k, \lambda) := dpois(k, \lambda)$ $p(6, 3) = 0.05$

5. Discrete random variable *X* is given by distribution series.

X	0	10	20	30	40	50
P	0,05	0,15	0,3	0,25	0,2	0,05

Find:

a) its distribution function *F*(*x*), construct graph of *F(x)*;

b) mathematical expectation, variance, standard deviation, mode;

c) probability, that *X* will hit at interval (15;45).

Solution:

a) distribution function *F(x)* (integral distribution function) of random variable *X* defines probability of the event, that $X<x$. For discrete random variable it is found by formula $F(x) = P(X < x) = \sum_{x_i < x} p_i = \sum_{x_i < x} P(X = x_i)$, where summation is made by all *i*, for which $x_i < x$.

So,

1) If $x \le 0$, then $F(x) = P(X < 0) = 0$.

2) If $0 < x \le 10$, then $F(x) = P(X = 0) = 0{,}05$;

3) If $10 < x \le 20$, then $F(x) = P(X = 0) + P(X = 10) = 0{,}05 + 0{,}15 = 0{,}2$.

4) If $20 < x \le 30$, then

$$F(x) = P(X = 0) + P(X = 10) + P(X = 20) = 0{,}2 + 0{,}3 = 0{,}5.$$

5) If $30 < x \le 40$, then

$$F(x) = P(X = 0) + P(X = 10) + P(X = 20) + P(X = 30) = 0{,}5 + 0{,}25 = 0{,}75.$$

6) If $40 < x \le 50$, then

$$F(x) = P(X = 0) + P(X = 10) + P(X = 20) + P(X = 30) +$$
$$+ P(X = 40) = 0{,}75 + 0{,}2 = 0{,}95.$$

7) If $x > 50$, then $F(x) = P(X = 0) + P(X = 10) + P(X = 20) + P(X = 30) +$ $+ P(X = 40) + P(X = 50) = 0{,}95 + 0{,}05 = 1$.

Thus,
$$F(x) = \begin{cases} 0, \ if \ \ x \le 0 \\ 0{,}05, \ if \ \ 0 < x \le 10 \\ 0{,}2, \ if \ \ 10 < x \le 20 \\ 0{,}5, \ if \ \ 20 < x \le 30 \\ 0{,}75, \ if \ 30 < x \le 40 \\ 0{,}95, \ if \ \ 40 < x \le 50 \\ 1, \ if \ \ x > 50 \end{cases}.$$

Graph is constructed in MathCAD system (see below);

b) let's find numerical characteristics. For discrete random variable, mathematical expectation is equal to sum of products of its all possible values by probabilities of these values: $M(X) = \sum_i x_i p_i$. So,

$$M(X) = 0 \cdot 0{,}15 + 10 \cdot 0{,}15 + 20 \cdot 0{,}3 + 30 \cdot 0{,}25 + 40 \cdot 0{,}2 + 50 \cdot 0{,}05 = 25{,}5.$$

Variance of random variable X can be found by formula $D(X) = M[X - M(X)]^2$, or by formula $D(X) = M(X^2) - [M(X)]^2$. For discrete random variable these formulas can be re-written as follows: $D(X) = \sum_i (x_i - M(X))^2 \cdot p_i$ or $D(X) = \sum_i x_i{}^2 p_i - (\sum_i x_i p_i)^2$. Root-mean-square deviation (or standard deviation) is equal to $\sigma(x) = \sqrt{D(x)}$; mode of the discrete random variable (marked as M_0) – is its value, accepted with maximal probability; probability of hitting of X at interval $(a;b)$ is found by formula $P(a;b) = F(b) - F(a)$. In our task these variables are equal to:

$D(x)$=154,75; $\sigma(x) = \sqrt{154{,}75} = 12{,}44$; M_0=20;

$P(15;45) = F(45) - F(15)$=0,75.

Below, we give copy of the file with calculations in MathCAD system. Calculation of variance is given for both formulas.

ORIGIN= 1

$s := (0 \ \ 10 \ \ 20 \ \ 30 \ \ 40 \ \ 50)$

$M := s^T \cdot p^T$

$p := (0.05 \ \ 0.15 \ \ 0.3 \ \ 0.25 \ \ 0.2 \ \ 0.05)$

$M = 25.5$

$s0 := s^T - M$

$s0 = \begin{pmatrix} -25.5 \\ -15.5 \\ -5.5 \\ 4.5 \\ 14.5 \\ 24.5 \end{pmatrix}$

$D := \left[\overrightarrow{(s0 \cdot s0)}\right]^T \cdot p^T$

$D = 154.75$

$s2 := \begin{pmatrix} 0^2 & 10^2 & 20^2 & 30^2 & 40^2 & 50^2 \end{pmatrix}$

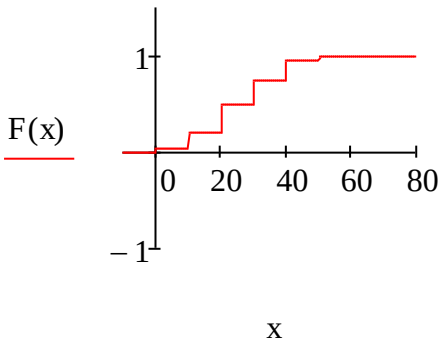
$D1 := s2^T \cdot p^T - M^2$

$D1 = 154.75$

$\sigma := \sqrt{D} = 12.44$

$$F(x) := \begin{cases} 0 & \text{if } x \le 0 \\ 0.05 & \text{if } 0 < x \le 10 \\ 0.2 & \text{if } 10 < x \le 20 \\ 0.5 & \text{if } 20 < x \le 30 \\ 0.75 & \text{if } 30 < x \le 40 \\ 0.95 & \text{if } 40 < x \le 50 \\ 1 & \text{if } x > 50 \end{cases}$$

$F(45) - F(15) = 0.75$



6. Continuous random variable *X* is given by distribution density function *f(x)*.

$$f(x) = \begin{cases} 0, & if \quad x \le 0 \\ \frac{2}{9}(3x - x^2), & if \quad 0 < x \le 3. \\ 0, & if \quad x > 3 \end{cases}$$

Find:

a) its distribution function *F(x)*;

b) mathematical expectation, variance, standard deviation, mode, median;

c) probability, that *X* will hit in interval (1;4).

Construct graphs of *F(x)* and *f(x)*.

Solution:

a) distribution function can be found by formula $F(x)=\int_{-\infty}^{x} f(x)dx$. So, if $x\le 0$, then $f(x)=0$, then $F(x)=\int_{-\infty}^{0} 0\,dx=0$;

if $0<x\le 3$, then $F(x)=\int_{-\infty}^{0} 0\,dx+\int_{0}^{x}\frac{2}{9}(3x-x^2)dx=-\frac{x^2(2x-9)}{27}$;

if $x>3$, then $F(x)=\int_{-\infty}^{0} 0\,dx+\int_{0}^{3}\frac{2}{9}(3x-x^2)dx+\int_{3}^{x} 0\,dx=1$.

Thus, desired distribution functions is as follows:

$$F(x)=\begin{cases} 0, \ if \ x\le 0 \\ -\frac{x^2(2x-9)}{27}, \ if \ 0<x\le 3 \\ 1, if \ x>3 \end{cases};$$

b) numerical characteristics of continuous random variables are found by formulas: mathematical expectation – $M(x)=\int_{-\infty(a)}^{\infty(b)} xf(x)dx$, variance – $D(x)=\int_{-\infty(a)}^{\infty(b)}(x-M(x))^2 f(x)dx$ or $D(x)=\int_{-\infty(a)}^{\infty(b)} x^2 f(x)dx-\left[M(x)\right]^2$ (limits of integrals depend on case, if possible values of random variable belong to all axe *Ox* or to interval (*a;b*)); root-mean-square deviation – $\sigma(x)=\sqrt{D(x)}$; mode of the random variable X is it's such a value M_o when distribution density is maximal; median of continuous random variable *X* is it's such a value M_e for which it is equally probable, whether random variable is less or more than M_e, i.e. $P(X<M_e)=P(X>M_e)=0{,}5$.

Thus, in our task:

$M(x)=\int_{0}^{3} 2/9x(3x-x^2)dx=1{,}5;$

$D(x)=\int_{0}^{3} x^2\cdot 2/9(3x-x^2)dx-1{,}5^2=0{,}45;$

$\sigma(X)=\sqrt{0{,}45}=0{,}671.$

To find mode we should find maximum of function $f(x)=2/9(3x-x^2)$ at segment [0; 3]. For this, we find derivative and equal it to zero: $f'(x)=2/3-4x/9$, $f'(x)=0$ when x=3/2, it is extreme point. Let's check it for extremum: $f'(1)>0$, $f'(2)<0$, so, moving through point x=3/2 the sign of derivative changed from plus to minus, then x=3/2 is point of maximum, so M_o=3/2. Note, that if *f(x)* is linear function, i.e. its graph is a straight line, then the maximum is located at ends of the segment and M_o is found by graph of *f(x)*.

We find median from condition $P(X < M_e) = 0{,}5$, where $P(X < M_e) = P(-\infty < X < M_e) = P(0 < X < M_e)$.

Since $P(0 < X < M_e) = \int_0^{M_e} 2/9(3x - x^2)dx = -M_e^{\,2}(2M_e - 9)/27$, then, solving equation $-M_e^{\,2}(2M_e - 9)/27 = 0{,}5$, we obtain three roots, and only one $M_e = 1{,}5$ is appropriate root.

Below, we give copy of file with calculation in MathCAD system.

$$f(x) := \frac{2}{9}\cdot\left(3\cdot x - x^2\right) \qquad \sqrt{\frac{9}{20}} = 0.671$$

$$\int_0^3 x\cdot f(x)\,dx \to \frac{3}{2} = 1.5 \qquad \int_0^3 x^2\cdot f(x)\,dx - \left(\frac{3}{2}\right)^2 \to \frac{9}{20} = 0.45$$

$$\frac{d}{dx}f(x) \to \frac{2}{3} - \frac{4\cdot x}{9} \qquad f1(x) := \frac{d}{dx}f(x)$$

$$\int_0^y f(x)\,dx \to -\frac{y^2\cdot(2\cdot y - 9)}{27} \qquad f1(x)\ \text{solve} \to \frac{3}{2}$$

$$f1(1) = 0.222 \qquad f1(2) = -0.222$$

$$-\frac{y^2\cdot(2\cdot y - 9)}{27} - 0.5\ \text{solve} \to \begin{pmatrix} 4.0980762113533159403 \\ -1.0980762113533159403 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 4.098 \\ -1.098 \\ 1.5 \end{pmatrix}$$

c) probability that X will hit at the interval (1;4) is equal to:

$$P(1 < X < 4) = P(1 < X < 3) + P(3 < X < 4) = \int_1^3 2/9(3x - x^2)dx + \int_3^4 0dx = 0{,}741$$

or $P(1 < X < 4) = F(4) - F(1) = 1 - \left(-\frac{1\cdot(2-9)}{27}\right) = \frac{20}{27} = 0{,}74.$

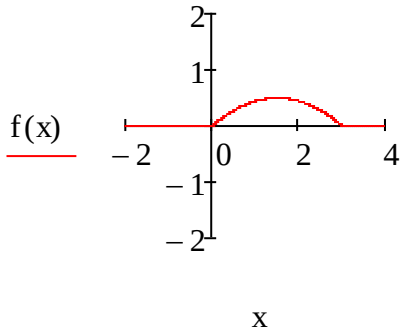
Below, we give copy of file with calculations in MathCAD.

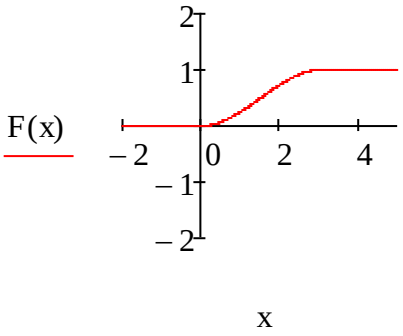
$$\int_1^3 f(x)\,dx \to \frac{20}{27} = 0.741 \qquad f2(x) := -\frac{x^2\cdot(2\cdot x - 9)}{27}$$

$$f2(1) = 0.259 \qquad 1 - 0.259 = 0.741$$

Graphs of functions $F(x)$ and $f(x)$ we construct in MathCAD system:

$$f(x) := \begin{cases} 0 & \text{if } x \le 0 \\ \left[\frac{2}{9}\cdot\left(3\cdot x - x^2\right)\right] & \text{if } 0 < x \le 3 \\ 0 & \text{if } x > 3 \end{cases} \qquad F(x) := \begin{cases} 0 & \text{if } x \le 0 \\ \frac{x^2\cdot(9 - 2\cdot x)}{27} & \text{if } 0 < x \le 3 \\ 1 & \text{if } x > 3 \end{cases}$$





7. Reliability tests of 6 products are made. Probability of non-passing the tests by each product is equal to 25%. Work out the distribution law for number of products, which did not pass the tests (random variable X). Find mathematical expectation, variance and root-mean-square deviation of this random variable.

Solution: discrete random variable X – is number of products which did not pass the tests. Its possible values: $x_1 = 0$ (all products passed the tests), $x_2 = 1$ (one product did not pass the tests) etc., $x_7 = 6$ (six products did not pass the tests). Possible values are independent and occurrence probability is equal for each of them. So, random variable X is distributed by binomial law: $P(X = k) = P_n(k) = C_n^k p^k q^{n-k}$, where $k = 0,1,2,\ldots,6$, $q = 1 - p$, $n = 6$.

So, $P(X = 0) = P_6(0) = C_6^0 0{,}25^0 0{,}75^6 = 0{,}178$,

$P(X = 1) = P_6(1) = C_6^1 0{,}25^1 0{,}75^5 = 0{,}356$; $P(X = 2) = P_6(2) = C_6^2 0{,}25^2 0{,}75^4 = 0{,}297$,

$P(X = 3) = P_6(3) = C_6^3 0{,}25^3 0{,}75^3 = 0{,}132$; $P(X = 4) = P_6(4) = C_6^4 0{,}25^4 0{,}75^2 = 0{,}033$,

$P(X = 5) = P_6(5) = C_6^5 0{,}25^5 0{,}75^1 = 0{,}004$; $P(X = 6) = P_6(6) = C_6^6 0{,}25^6 0{,}75^0 = 0{,}0002$.

Then desired distribution law is as follows:

X	0	1	2	3	4	5	6
p	0,178	0,356	0,297	0,132	0,033	0,004	0,0002

Numerical characteristics of binomial distribution can be defined by standard formulas for discrete random variables:

$M(X) = \sum_i x_i p_i$, $D(X) = \sum_i (x_i - M(X))^2 \cdot p_i$ or $D(X) = \sum_i x_i^{\,2} p_i - (\sum_i x_i p_i)^2$. But, it will be easier to use properties of mathematical expectation and variance, when X

is number of event occurrence at *n* tests: $M(X)=np$, $D(X)=npq$. So, in our case $M(X)=6\cdot 0{,}25=1{,}5$, $D(X)=6\cdot 0{,}25\cdot 0{,}75=1{,}125$. $\sigma(x)=\sqrt{D(X)}\approx 1{,}06$.

Below, we give copy of file with calculations in MathCAD.

$C(Q,R) := \text{combin}(Q,R)$

$C(6,0)\cdot 0.25^{0}\cdot 0.75^{6} = 0.178$

$C(6,1)\cdot 0.25^{1}\cdot 0.75^{5} = 0.356$

$C(6,2)\cdot 0.25^{2}\cdot 0.75^{4} = 0.297$

$C(6,3)\cdot 0.25^{3}\cdot 0.75^{3} = 0.132$

$C(6,4)\cdot 0.25^{4}\cdot 0.75^{2} = 0.033$

$C(6,5)\cdot 0.25^{5}\cdot 0.75^{1} = 4.395 \times 10^{-3}$

$C(6,6)\cdot 0.25^{6}\cdot 0.75^{0} = 2.441 \times 10^{-4}$

Analyses of the binomial distribution is comfortable to perform in MathCAD system by using special functions with root word *binom* (*dbinom*, *pbinom*, *qbinom*, *rbinom*). For example, function *dbinom*(*k*,*n*,*p*) gives values of probabilities, etc.

8. Equipment consists of 1000 elements. One element failure probability for one year of operation doesn't depend on status of other elements and equals 0,001. Find:

- distribution law for number of failure elements;
- probability, that not less than 2 elements will fail per year.

Solution:

a) discrete random variable X – is number of failed elements and it is distributed by Poisson's law (it is limited distribution law for binomial distribtution law, when event occurrence probability *p* in each test is small but number of tests *n* is big)): $P(X=k)=P_n(k)\approx\frac{\lambda^k}{k!}e^{-\lambda}$, where $\lambda=np=1000\cdot 0{,}001=1$, $k=0,1,2,\ldots,1000$, $n=1000$.

Thus, $P(X=0)=P_{1000}(0)\approx\frac{1^0}{0!}e^{-1}=0{,}368$; $P(X=1)=P_{1000}(1)\approx\frac{1^0}{1!}e^{-1}=0{,}368$;

$P(X=2)=P_{1000}(2)\approx\frac{1^2}{2!}e^{-1}=0{,}184$;

$P(X=3)=P_{1000}(3)\approx\frac{1^3}{3!}e^{-1}0{,}061$, и т.д. $P(X=10)=P_{1000}(10)\approx\frac{1^{10}}{10!}e^{-1}=0{,}0000001$

etc. Desired distribution law:

X	0	1	2	3	…	10	…	1000
p	0,368	0,368	0,184	0,061	…	0,0000001	…	0

Below you find copy of file with calculation from MathCAD.

$p(k,\lambda) := dpois(k,\lambda)$

$p(0,1) = 0.368$ $\quad$ $p(4,1) = 0.015$

$p(1,1) = 0.368$ $\quad$ $p(5,1) = 3.066 \times 10^{-3}$

$p(2,1) = 0.184$ $\quad$ $p(10,1) = 1.014 \times 10^{-7}$

$p(3,1) = 0.061$ $\quad$ $p(1000,1) = 0$

b) not less than two elements failure probability is calculated by formula:

$P(X \geq 2) = \sum_{k=2}^{1000} P_{1000}(k)$ или $P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$ = 1 - 0,368 - 0,368 = 0,264.

In MathCAD system Poisson distribution law is corresponded by special functions with root word pois (dpois, ppois, qpois, rpois). For example, function dpois(k,n,p) gives values of probabilities etc.

9a. Scale division value of measurement device is equal to 0,2. Device indications are rounded off till nearest integer scale division. Random variable X – is error of indication rounding. Find:

a) distribution density function *f(x)* of *X*;

b) distribution function *F(x)*;

c) mathematical expectation, variance;

d) probability, that during the measurement the error less (more) than 0,04 will be made.

Construct graphs of *F(x)* and *f(x)*.

Solution: random variable X – is error of indication rounding and it is distribution uniformly between two integer scale divisions; $b - a = 0{,}2$ – is length of interval where all possible values of X is included. Density of uniform distribution is found by formula $f(x) = \begin{cases} \dfrac{1}{b-a}, \ if \ \ x \in (a,b) \\ 0, \ \ if \ \ x \notin (a,b) \end{cases}$,

Distribution function – $F(x) = \begin{cases} 0, \ \ if \ x \leq a \\ \dfrac{x-a}{b-a}, \ if \ a < x \leq b \\ 1, \ if \ x > b \end{cases}$.

Mathematical expectation and variance – $M(X) = \dfrac{a+b}{2}$, $D(X) = \dfrac{(b-a)^2}{12}$.

Probability of hitting into interval (α, β) – $P(\alpha < X < \beta) = \dfrac{\beta - \alpha}{b-a}$.

So, in our task:

a) $f(x)=\begin{cases}\dfrac{1}{0,2}=5, \text{ if } x\in(0;0,2)\\ 0, \text{ if } x\notin(0;0,2)\end{cases}$;

b) $F(x)=\begin{cases}0, \text{ if } x\le 0\\ \dfrac{x}{0,2}=5x, \text{ if } 0<x\le 0,2\\ 1, \text{ if } x>0,2\end{cases}$;

c) $M(X)=\dfrac{0,2+0}{2}=0,1;\ \ D(X)=\dfrac{(0,2-0)^2}{12}=0,003$;

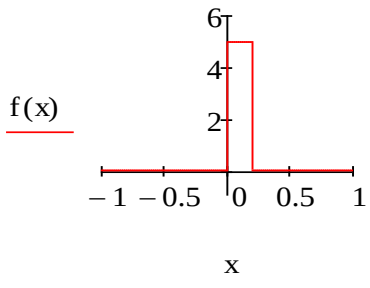
d) it is clear, that during the measurement the error will be less than 0,04, if it will hit into interval (0;0,04) or into interval (0,16;0,2) (event A), i.e. this event probability is equal to

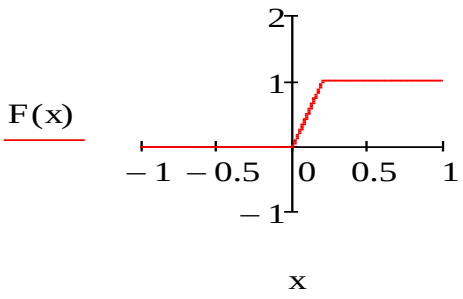
$$P(A)=P(0<X<0,04)+\ P(0,16<X<0,2)=\frac{0,04-0}{0,2}+\frac{0,2-0,16}{0,2}=0,4;$$

during measurement the error will be more than 0,04 if it will hit into interval (0,04;0,16) (event B), i.e. this event probability is equal to $P(B)=P(0,04<X<0,16)=\dfrac{0,16-0,04}{0,2}=0,6$ or $P(B)=1-P(A)$.

Let's construct graphs of $F(x)$ and $f(x)$ in MathCAD system:

$$f(x):=\begin{cases}0 & \text{if } x\le 0\\ 5 & \text{if } 0<x\le 0.2\\ 0 & \text{if } x>0.2\end{cases}\qquad F(x):=\begin{cases}0 & \text{if } x\le 0\\ (5\cdot x) & \text{if } 0<x\le 0.2\\ 1 & \text{if } x>0.2\end{cases}$$





9b. Subway trains are moving strictly on schedule. Traffic interval is 5 minutes. Random variable X - is waiting time for train. Find:

a) its distribution density function $f(x)$;

b) distribution function $F(x)$;

c) mathematical expectation, variance;

d) probability, that a passenger, who has just come to the station, will wait for train less (more) than 3 minutes.

Construct graphs of $F(x)$ and $f(x)$.

Solution: random variable X – is train waiting time; it is distributed uniformly between two train consecutive arrivals; $b-a=5$ – is length of interval where all possible values of X are concluded. All formulas for uniform distribution see in previous task 9a.

In our task:

a) $f(x)=\begin{cases}\frac{1}{5}=0,2, \ if \ \ x\in(0;5)\\ 0, \ \ if \ \ x\notin(0;5)\end{cases}$;

b) $F(x)=\begin{cases}0, \ \ if \ x\le 0\\ \frac{x}{5}=0,2x, \ if \ 0<x\le 5\\ 1, \ if \ x>5\end{cases}$;

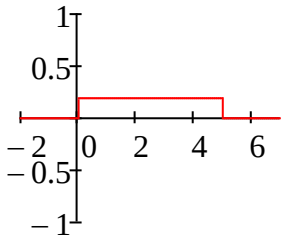
c) $M(X)=\frac{5+0}{2}=2,5$; $D(X)=\frac{(5-0)^2}{12}=2,08$;

d) it is clear, that passenger will wait for train less than 3 minutes, if he will arrive to the station at interval of time (0; 3) or in interval (2; 5) (event A), i.e. probability of this event is equal to $P(A)=P(0<X<3)=\frac{3-0}{5}$=0,6; passenger will wait for the train more than 3 minutes, if he will come to the station at interval (0;2) or in interval (3;5) (event B), i.e. probability of this event is equal to $P(B)=P(3<X<5)=\frac{5-3}{5}=0,4$ or $P(B)=1-P(A)$.

Let's construct graphs of $F(x)$ or $f(x)$ in MathCAD system:

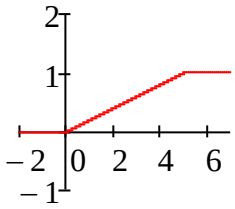
$$f(x):=\begin{cases}0 & \text{if } x\le 0\\ 0.2 & \text{if } 0<x\le 5\\ 0 & \text{if } x>5\end{cases}\qquad F(x):=\begin{cases}0 & \text{if } x\le 0\\ (0.2\cdot x) & \text{if } 0<x\le 5\\ 1 & \text{if } x>5\end{cases}$$

f(x)



x

F(x)



x

In MathCAD system uniform distribution is corresponded by special functions with root word *unif*: *dunif*(*x*,*a*,*b*) – gives values of distribution density function; *punif*(*x*,*a*,*b*) – gives values of distribution function; *runif*(*n*,*a*,*b*) – gives set of *n* values of independent random numbers, distributed uniformly at interval (*a*,*b*).

10. Time of non-failure operation of element (random variable *T*) has exponential distribution with parameter $\lambda = 0,5$, where λ - is intensity of faults in time unit. Find:

a) distribution density function *f(t);*

b) distribution function *F(t)*, indicate its probabilistic sense;

c) reliability function *R(t)*, indicate its probabilistic sense;

d) mathematical expectation, variance;

e) probability, that for time *t*=5 hours, the element will fail, and probability, that for time *t*=5 hours, the element will not fail.

Construct graphs *F(t), R(t)* and *f(t)*.

Solution: distribution law of continuous random variable *X* with distribution density function $f(x) = \begin{cases} 0, \text{ if } x < 0 \\ \lambda e^{-\lambda x}, \text{ if } x \geq 0 \end{cases}$ is called exponential distribution law. Other notions and formulas for exponential distribution: $F(x) = \begin{cases} 0, \text{ if } x < 0 \\ 1 - e^{-\lambda x}, \text{ if } x \geq 0 \end{cases}$ - distribution function; if random variable *X*=*T* – is non-failure operation time of element, then $F(t) = P(T < t) = 1 - e^{-\lambda t}$ defines element failure probability for time *t*; $R(t) = P(T > t) = e^{-\lambda t}$ – reliability function, defines non-failure operation probability for time *t*;

$$M(X) = \frac{1}{\lambda},\ D(X) = \frac{1}{\lambda^2},\ \sigma(X) = \frac{1}{\lambda};\ P(\alpha < X < \beta) = e^{-\lambda\alpha} - e^{-\lambda\beta}.$$

In our task, taking in mind that $t \geq 0$, we have:

a) $f(t) = 0,5e^{-0,5t}$;

b) $F(t) = P(T < t) = 1 - e^{-0,5t}$, defines element failure probability for time *t*;

c) $R(t) = e^{-0,5t}$, defines element non-failure operation probability for time *t*;

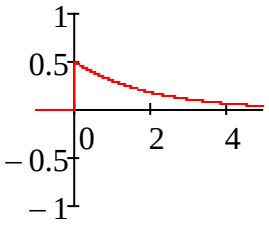
d) $M(X) = \frac{1}{0,5} = 2;\ D(X) = \frac{1}{0,5^2} = 4;$

f) since distribution function defines failure probability for time *t*, then, taking *t*=5, we get failure probability for time *t*=5 hours: $F(5) = 1 - e^{-0,5 \cdot 5} = 1 - e^{-2,5} = 0,918$; event «element will fail» and «element won't fail » - are opposite, so element non-failure operation probability for time *t*=5 is equal 1-0,918=0,082. The same result, we can obtain directly, using reliability function: $R(5) = e^{-0,5 \cdot 5} = e^{-2,5} = 0,082$.

Let's construct graphs of *F*(*t*), *R*(*t*) и *f*(*t*) and make some calculations in MathCAD system:

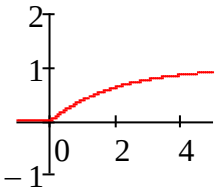
$$f(t) := \begin{cases} 0 & \text{if } t < 0 \\ 0.5 \cdot e^{-0.5 \cdot t} & \text{if } t \geq 0 \end{cases} \qquad F(t) := \begin{cases} 0 & \text{if } t < 0 \\ 1 - e^{-0.5 \cdot t} & \text{if } t \geq 0 \end{cases}$$

f(t)



t

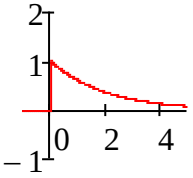
F(t)



t

$$R(t) := \begin{cases} 0 & \text{if } t < 0 \\ e^{-0.5 \cdot t} & \text{if } t \geq 0 \end{cases}$$

R(t)



t

$$e^{-2.5} = 0.082$$

$$1 - e^{-2.5} = 0.918$$

In MathCAD system exponential distribution law is corresponded by special functions with root word *exp*: *dexp*(x, λ) – gives values of distribution density; *pexp*(x, λ) – gives values of distribution function.

11. Random deviation of product size from standard size (random variable X) obeys the normal distribution law with parameters a=10 and σ=2. Find:

a) distribution density function $f(x)$;

b) distribution function $F(x)$;

c) mathematical expectation, variance;

d) probability of hitting in interval $(12;14)$;

e) probability, that deviation from standard size is not more than δ=3 by absolute value.

Construct graphs of $F(t)$ and $f(t)$.

Solution: normal distribution law of continuous random variable X is distribution law with density $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}$, where $a = M(X)$ – is mathematical expectation, $\sigma = \sigma(X)$ – is root-mean-square deviation of X. Other notions and formulas for normal distribution: distribution function– $F(x) = \int_{-\infty}^{x} f(t)dt = \dfrac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[\dfrac{-(t-a)^2}{2\sigma^2}\right] dt$ or $F(x) = \Phi\left(\dfrac{x-a}{\sigma}\right) + 0{,}5$, where $\Phi(x) = \dfrac{1}{\sqrt{2\pi}} \int_{0}^{x} e^{-\frac{t^2}{2}} dt$ – Laplace function, you can find its values in special tables or in MathCAD system;

$$P(\alpha < X < \beta) = F(\beta) - F(\alpha) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right);$$

probability, that random variable will deviate more than by δ from its mathematical expectation, is equal to $P(|X - a| \le \delta) = 2\Phi\left(\frac{\delta}{\sigma}\right)$.

In our task:

a) $f(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-10)^2}{8}}$;

b) $F(x) = \Phi\left(\frac{x-10}{2}\right) + 0{,}5$;

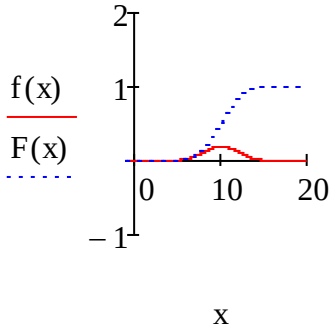
c) $M(X) = a = 10$, $\sigma(X) = \sigma = 2$, $D(X) = \sigma^2 = 4$;

d) $P(12 < X < 14) = \Phi\left(\frac{14-10}{2}\right) - \Phi\left(\frac{12-10}{2}\right) = \Phi(2) - \Phi(1)$=0,4772-0,3413 = 0,1359;

e) probability, that deviation from nominal is not more than δ=3 by absolute value, will be equal to $P(|X - 10| \le 3) = 2\Phi\left(\frac{3}{2}\right)$= $2\Phi(1{,}5)$=0,4332.

Values of Laplace function are taken from table. Also, it can be found in MathCAD system, where normal distribution law is corresponded by function with root word norm. For example, *dnorm* (*x*,*a*,σ) – gives values of distribution density *f*(*x*); *pnorm* (*x*,*a*,σ) – give values of distribution function *F*(*x*). Let's use these functions for construction of respective graphs. Copy of file from MathCAD is given below.

$f(x) := dnorm(x, 10, 2)$ $F(x) := pnorm(x, 10, 2)$



2 Calculation-graphical work №2. Mathematical statistics elements.

Aims: to study basic problems of mathematical statistics: processing and analysis of results of random mass phenomena observations.

2.1 Theoretical issues

1. Subject of mathematical statistics and its main problems. Basic notions (sample, sample volume, variants, statistical series, interval series).
2. Empirical distribution function, polygon, histogram.
3. Defining of unknown distribution parameters (sample mean, sample and corrected sample variance).
4. Point and interval estimators of distribution parameters.
5. Accuracy and reliability of estimator. Confidence interval.
6. Confidence interval for estimator of mathematical expectation of normal distributed random variable with known σ.
7. Confidence interval for estimation of root-mean-square deviation of normally distributed random variable with known σ.
8. Correlation dependence notion. Regression functions and lines.

2.2 Calculation tasks

1. For given sample make processing and systematization task, find:
a) variational series (sampling in increasing order);
b) statistical series of frequencies and relative frequencies;
c) interval statistical series of frequencies and relative frequencies (minimal and maximal variants, sample range, intervals number, intervals length);
d) discrete (grouped) statistical series of frequencies and relative frequencies.

2. For given sample, perform analysis and estimation task, find:
a) construct histogram of frequencies and relative frequencies by interval statistical series;
b) by discrete statistical series, find:
- polygon of frequencies and relative frequencies;
- empirical distribution function;
- sample mean;
- sample and corrected sample variance;
- corrected sample root-mean-square deviation;
- sample mode and median.

1	112	101	155	137	109	129	152	128	132	116
	125	125	142	140	125	118	125	135	149	145
	106	109	138	145	118	128	125	105	122	138
	120	118	133	118	129	149	124	153	132	118
	132	132	138	128	122	115	143	140	122	152
	128	118	126	132	134	123	122	159	112	110
	112	121	105	117	112	129	129	118	112	116
2	87	85	91	94	102	80	75	102	99	101
	120	122	101	88	80	97	92	91	94	82
	115	100	97	91	87	116	121	101	123	97
	88	90	101	95	93	92	88	94	98	99
	95	105	112	116	118	108	95	99	92	100
	94	106	112	122	100	92	93	82	111	102
	100	101	123	97	90	104	108	101	96	111
3	547	565	587	553	548	554	561	562	551	572
	565	555	563	568	586	549	575	537	581	553
	543	568	574	564	547	549	553	572	535	555
	552	545	554	571	569	539	549	553	562	561
	558	563	563	547	552	562	554	563	558	572
	577	554	552	566	557	551	552	571	551	552
	599	561	552	551	561	538	533	541	588	558
4	90	123	132	85	122	105	125	142	99	125
	118	105	115	92	115	142	98	123	103	144
	106	92	118	105	118	86	125	105	122	138
	102	130	112	98	115	120	118	103	118	129
	112	115	88	118	103	102	95	124	106	135
	95	124	103	102	118	112	115	92	115	119
	103	122	94	112	97	128	102	116	125	132
5	139	112	132	85	122	105	125	142	99	125
	116	105	92	115	98	123	103	144	115	142
	106	92	118	86	125	105	122	138	105	118
	102	130	112	98	115	120	118	103	118	129
	112	115	88	118	103	102	95	124	106	135
	95	124	103	102	118	112	115	103	95	122
	125	118	96	126	98	106	128	118	126	103
	134	112	101	105	117	92	129	99	118	112
6	154	143	155	113	155	171	168	153	135	168
	145	168	122	163	117	165	132	139	107	125
	146	152	142	132	152	161	148	136	138	149
	157	178	149	195	146	166	182	135	136	170
	155	152	145	198	192	143	159	116	126	155
	163	169	165	148	151	153	139	166	138	128
	168	157	143	179	165	159	149	141	102	169

7	470	801	790	764	764	950	533	402	520	780
	699	840	869	551	707	635	703	801	859	475
	797	797	789	875	698	580	821	737	910	856
	950	741	473	988	737	787	667	649	797	939
	532	885	590	590	975	910	731	869	435	889
	584	967	950	531	775	485	756	656	680	741
	950	458	511	857	536	699	474	789	889	533
8	450	434	424	432	440	443	415	446	423	472
	442	452	444	425	403	458	455	431	446	424
	438	442	482	432	416	477	431	432	412	462
	496	468	424	438	452	446	418	474	432	452
	466	488	452	489	451	422	442	492	473	402
	481	468	404	498	467	398	440	449	417	425
	444	498	466	442	483	462	492	435	449	422
9	250	244	224	232	240	224	244	226	253	232
	248	216	230	254	258	202	225	224	252	234
	242	212	231	251	204	246	232	282	242	252
	296	242	254	218	226	252	238	224	298	260
	276	254	282	242	270	254	260	232	268	242
	244	276	224	240	272	268	281	234	268	251
	271	212	234	262	204	261	254	266	278	248
10	165	143	152	167	164	199	171	171	156	149
	147	155	158	145	158	177	161	181	153	171
	175	153	174	154	163	174	152	188	162	197
	187	158	154	171	163	172	152	178	151	172
	153	186	147	169	147	166	161	171	161	186
	148	161	189	199	162	167	198	168	135	152
	154	175	163	149	162	161	161	193	172	175
	161	164	178	138	164	172	187	178	143	161
11	153	174	154	163	174	152	188	162	197	234
	188	158	154	171	163	172	152	178	151	172
	155	186	147	169	147	166	161	171	161	186
	149	161	189	199	162	167	198	168	135	152
	156	175	163	149	162	161	161	193	172	175
	162	164	178	138	164	172	187	178	143	161
	165	163	177	161	149	146	152	139	156	152

12	212	231	251	204	246	232	282	242	252	276
	297	242	254	218	226	252	238	224	298	260
	277	254	282	242	270	254	260	232	268	242
	345	276	224	240	272	268	281	234	268	232
	272	212	234	292	204	261	254	266	278	248
	253	262	256	264	272	242	244	246	253	234
	237	264	252	248	247	268	229	235	262	212
	238	242	254	263	261	266	254	264	248	251
13	165	143	52	166	164	199	171	171	156	171
	148	155	158	145	158	177	161	181	153	197
	176	153	174	154	163	174	152	188	162	172
	189	158	154	171	163	172	152	178	151	186
	157	186	147	169	147	166	161	171	161	152
	150	161	189	199	162	167	198	168	135	175
	158	175	163	149	162	161	161	193	172	
14	216	230	254	258	202	225	224	252	234	250
	243	212	231	251	204	246	232	282	242	252
	298	242	254	218	226	252	238	224	298	260
	278	254	282	242	270	254	260	232	268	242
	246	276	224	240	272	268	281	234	268	232
	273	212	234	262	201	261	254	266	278	248
	254	262	256	264	272	242	244	246	253	234
	239	264	252	248	247	268	229	235	262	212
15	165	143	152	167	165	199	171	171	156	152
	149	155	158	145	158	177	161	181	153	171
	153	174	154	163	174	152	188	162	197	178
	190	158	154	171	163	172	152	178	151	172
	159	186	147	169	147	166	161	171	161	186
	151	161	189	199	162	167	198	168	135	152
	160	175	163	149	162	161	161	193	172	175
	165	164	178	137	164	172	187	178	143	161
16	147	153	179	165	159	149	141	102	169	157
	169	154	143	155	113	155	171	168	153	135
	150	152	142	132	152	161	148	136	138	149
	157	178	149	195	146	166	182	135	136	170
	156	152	145	198	192	143	159	116	126	155
	164	169	165	148	151	153	139	166	138	128
	169	169	155	152	175	177	131	154	174	187
	180	177	162	149	146	113	151	152	134	125

17	558 561 547 566 567 544 553 577	563 552 565 555 556 568 545 552	569 551 587 563 546 574 554 566	547 561 553 568 552 564 571 557	552 538 548 586 543 547 569 551	562 533 554 549 554 549 539 552	554 547 561 575 556 553 549 546	549 552 564 564 566 578 538 584	575 557 562 553 592 557 575 572	578 543 558 555 562 561 554 535
18	577 554 558 547 555 577 601 555	568 5455 563 595 563 554 561 541	557 554 563 587 568 552 552 588	564 571 547 553 586 566 551 558	547 569 552 548 549 557 561 563	549 539 562 554 575 551 538 558	553 549 554 561 564 552 533 572	578 538 549 564 553 546 547 578	557 575 575 562 585 584 552 539	575 566 558 544 592 556 557 556
19	77 48 38 63 68 12 55 74	45 25 53 55 55 39 69 51	49 59 48 16 49 57 31 82	92 57 68 78 65 51 62 52	13 65 52 52 79 65 48 46	69 69 73 95 48 66 46 75	52 55 42 77 59 43 51 49	26 68 62 66 53 52 43 55	22 49 71 35 41 63 16 57	36 63 45 54 38 43 34 54
20	347 365 343 352 366 377 399	365 355 368 345 358 354 363	387 363 374 354 363 352 361	348 368 364 352 347 366 352	354 359 347 371 352 357 351	361 375 349 369 362 351 361	364 364 353 349 354 352 338	362 353 378 338 349 346 353	346 385 357 375 375 384 333	358 363 358 388 341 351 357
21	9 6 6 7 5 6 3	9 10 6 7 5 13 6	6 6 7 5 12 11 9	9 7 12 11 9 11 8	9 6 5 9 8 5 12	7 8 7 7 7 8 11	6 6 8 6 9 10 9	11 5 5 5 8 9 10	6 5 10 7 5 4 4	7 4 9 6 5 7 14

22	39	40	38	43	41	42	40	38	41	42
	41	40	42	39	41	41	36	43	41	42
	34	36	37	42	42	42	40	41	41	46
	47	48	52	56	68	70	68	64	56	58
	41	42	39	33	34	37	43	45	47	71
	43	42	43	41	42	47	48	49	52	53
	57	52	41	42	46	48	49	39	32	40
	39	37	42	43	54	58	59	64	66	68
23	10	15	16	17	18	19	20	15	16	11
	17	12	13	14	15	11	18	16	15	18
	20	20	21	23	26	28	23	28	27	24
	27	24	25	25	26	32	33	31	34	43
	26	32	26	27	28	29	30	21	22	23
	42	24	23	35	23	25	36	37	24	21
	58	54	49	47	32	36	43	23	24	28
24	150	144	124	132	140	124	144	153	151	148
	116	130	154	158	102	125	124	152	134	148
	142	121	112	131	151	104	146	132	182	142
	152	196	142	154	158	118	126	152	138	124
	144	176	124	140	172	168	181	134	168	132
	144	112	134	162	104	161	154	166	178	148
	162	164	164	172	142	144	146	112	171	
25	128	105	115	92	115	142	98	123	103	144
	112	115	88	118	103	102	95	124	106	135
	95	124	103	102	118	112	115	92	115	119
	92	112	132	85	122	105	125	142	99	125
	106	92	118	105	118	86	125	105	122	138
	102	130	112	98	115	120	118	103	118	129
	103	122	94	112	97	128	102	116	125	132
26	102	112	118	85	112	115	103	95	122	125
	157	178	149	195	146	166	182	135	136	170
	157	143	179	165	159	149	141	102	169	168
	151	168	122	163	117	165	132	139	107	125
	152	152	142	132	152	161	148	136	138	149
	153	154	143	155	113	155	171	168	153	135
	157	152	145	198	192	143	159	116	126	155
	165	169	165	148	151	153	139	166	138	128

27	242	254	218	226	252	238	224	298	260	287
	250	216	230	254	258	202	225	224	252	234
	244	212	231	251	204	246	232	282	242	252
	299	254	282	242	270	254	260	232	268	242
	276	224	240	272	268	281	234	268	232	300
	274	212	234	262	204	261	254	266	278	248
	255	262	256	264	272	242	244	246	253	234
	240	264	252	248	247	268	229	235	262	212
28	262	267	275	266	246	252	261	269	262	268
	259	248	266	259	252	248	252	232	269	287
	253	286	275	235	202	239	225	236	237	224
	253	268	277	249	248	263	243	266	212	255
	249	288	213	264	247	242	228	277	256	251
	267	232	258	246	278	279	257	255	243	258
	254	244	265	274	252	265	222	269	254	278
	249	252	294	232	269	263	269	271	245	235
29	558	565	587	553	548	554	561	564	562	544
	563	568	586	549	575	564	553	585	577	553
	563	564	547	552	562	554	549	575	558	592
	546	577	568	574	564	547	549	553	578	557
	557	577	568	574	564	547	549	538	575	566
	558	554	552	566	557	551	552	546	584	532
	602	561	552	551	561	538	533	547	552	557
	556	541	588	558	563	558	572	578	539	556
30	165	143	152	167	164	199	171	171	156	151
	155	155	158	145	158	177	161	181	153	171
	177	153	174	154	163	174	152	188	162	197
	191	158	154	171	163	172	152	178	151	172
	161	186	147	169	147	166	161	171	161	186
	161	189	199	162	167	198	168	135	152	146
	162	175	163	149	162	161	161	193	172	175
	153	164	178	138	164	172	187	178	1433	161

3. Find confidence interval for estimation of mathematical expectation a of normal distribution with reliability γ . Sample mean $\bar{x}_в$, sample volume n and root-mean-square deviation σ are given.

	γ	$\bar{x}_в$	n	σ		γ	$\bar{x}_в$	n	σ		γ	$\bar{x}_в$	n	σ
3.1	0.95	75.17	36	6	3.11	0.97	5.21	46	6	3.21	0.92	11.48	36	6
3.2	0.97	7.27	56	7	3.12	0.96	55.23	38	5	3.22	0.94	23.38	39	8
3.3	0.93	75.17	35	5	3.13	0.92	5.21	36	7	3.23	0.93	30.44	56	7
3.4	0.94	8.27	58	9	3.14	0.95	55.23	68	7	3.24	0.99	15.32	38	5

3.5	0.98	76.17	46	6	3.15	0.98	7.21	56	6	3.25	0.95	10.48	46	6
3.6	0.99	7.37	58	7	3.16	0.93	65.23	78	5	3.26	0.98	13.38	39	8
3.7	0.93	65.13	34	6	3.17	0.92	8.21	49	7	3.27	0.93	20.44	66	7
3.8	0.94	9.27	53	8	3.18	0.95	51.23	58	9	3.28	0.97	14.32	58	6
3.9	0.93	85.17	35	6	3.19	0.94	5.21	39	6	3.29	0.94	30.44	86	7
3.10	0.95	8.27	57	9	3.20	0.95	85.23	58	7	3.30	0.99	16.32	38	9

2.3 Solution of exemplary embodiment

1. For given sample make processing and systematization task, find:

a) variational series (sampling in increasing order);

b) statistical series of frequencies and relative frequencies;

c) interval statistical series of frequencies and relative frequencies (minimal and maximal variants, sample range, intervals number, intervals length);

d) discrete (grouped) statistical series of frequencies and relative frequencies.

2. For given sample, perform analysis and estimation task, find:

a) construct histogram of frequencies and relative frequencies by interval statistical series;

b) by discrete statistical series, find:

- polygon of frequencies and relative frequencies;
- empirical distribution function;
- sample mean;
- sample and corrected sample variance;
- corrected sample root-mean-square deviation;
- sample mode and median.

20	15	17	19	23	18	21	15	16	13
20	16	19	20	14	20	16	14	20	19
15	19	17	16	15	22	21	12	10	21
18	14	14	18	18	13	19	18	20	23
16	20	19	17	19	17	21	17	19	17
13	17	11	18	19					

Solution: note, that calculations and graphs construction are made in MathCAD system. Copy of file from MathCAD and explanations are given below.

1. a) sample volume $n = 55$. Variational series (sample in increasing order.

$Y^T =$

	0	1	2	3	4	5	6	7	8	9
0	10	11	12	13	13	14	14	14	14	...

(in MathCAD this table is looked through by pressing at direction button);

b) by variational series, let's calculate, how many time each variant is taken place, i.e. frequency (n_i) of each variant. Obtained data we put into the table – statistical series of frequencies.

x_i	10	11	12	13	14	15	16	17	18	19	20	21	22	23
n_i	1	1	1	2	4	4	5	7	6	8	7	5	1	2

Relative frequencies of variants, we find by formula $p_i^* = \frac{n_i}{n}$, where n is sample volume. Obtained results we put into the table – statistical series of relative frequencies:

x_i	10	11	12	13	14	15	16	17	18
p_i^*	0,018	0,018	0,018	0,036	0,073	0,073	0,091	0.127	0,109
x_i	19	20	21	22	23				
p_i^*	0,145	0,127	0,091	0,018	0,036				

c) in order to construct interval statistical series, let's find: the biggest and the least variants $a = x_{\min} = 10$, $b = x_{\max} = 23$; sample range $R = b - a = 13$; value of intervals we find by Sturges formula $h = \frac{x_{\max} - x_{\min}}{1 + \log_2 n}$, $h =$ 1,917, or approximately $h \approx 2$; number of intervals – is denominator in Sturges formula $m = 1 + \log_2 n$ or $m = \frac{R}{h}$=6,781, or approximately $m \approx 7$; as beginning of the first interval we recommend to take variable $x_{initial} = x_{\min} - \frac{h}{2}$, $x_{initial} = 9{,}041 \approx 9$; to find number of variants which hit in each interval (i.e. frequencies n_i) and relative frequencies (i.e. $p_i = \frac{n_i}{n}$) in MathCAD system you can use $w1^T$ and $w2^T$.

Thus, desired interval series is as follows.

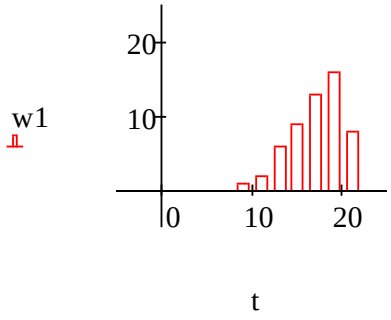
Intervals	[9,11)	[11,13)	[13,15)	[15,17)	[17,19)	[19,21)	[21,23]
n_i	1	2	6	9	13	16	8
$p_i = \frac{n_i}{n}$	0,018	0,036	0,109	0,164	0,236	0,291	0,145

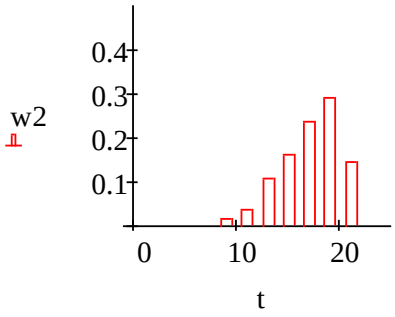
c) for construction of discrete statistical series (sometimes it is called the grouped statistical series) we find midpoints of intervals $\frac{x_i + x_{i+1}}{2}$ (see x^T in MathCAD). That points are corresponded by respective frequencies and relative frequencies from interval series.

Desired discrete statistical series.

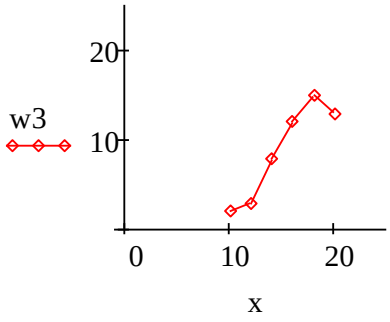
$\frac{x_i + x_{i+1}}{2}$	10	12	14	16	18	20	22
n_i	1	2	6	9	13	16	8
p_i	0,018	0,036	0,109	0,164	0,236	0,291	0,145

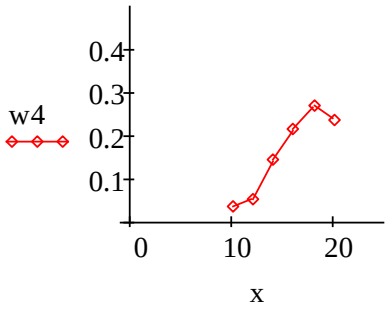
2. a) by interval statistical series, we construct histograms of frequencies and relative frequencies (in MathCAD system):





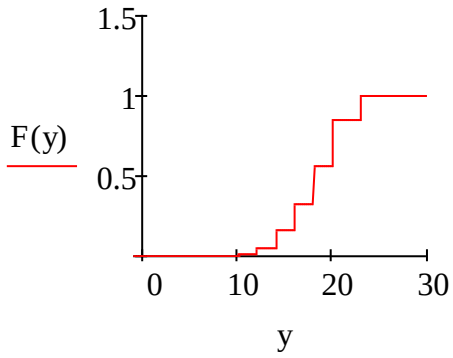
б) by discrete statistical series, we find:
- polygon of frequencies and relative frequencies:





- empirical distribution function (see F^T and $F(y)$ in MathCAD):

$$F(x)=\begin{cases} 0, if\ x\le 10 \\ 0{,}018, if\ 10<x\le 12 \\ 0{,}054, if\ 12<x\le 14 \\ 0{,}163, if\ 14<x\le 16 \\ 0{,}327, if\ 16<x\le 18 \\ 0{,}563, if\ 18<x\le 20 \\ 0{,}854, if\ 20<x\le 23 \\ 1, if\ x>23 \end{cases}$$



- sample mean (see mean(*X*) in MathCAD): $\bar{x}_в=\frac{\sum_i x_i n_i}{n}$ or $\bar{x}_в=\sum_i x_i\cdot p_i=17{,}564$;

- sample and corrected sample variance (see var(*X*) and s2 in MathCAD): $D_в=\frac{\sum n_i(x_i-\bar{x}_в)^2}{n}$ or $D_в=\frac{\sum_i n_i\cdot x_i^2}{n}-(\bar{x}_в)^2$ = 8,428 – sample variance; $s^2=\frac{n}{n-1}D_в$ = 8,584 – corrected sample variance;

5) sample root-mean-square deviation (see stdev(X) or σ in MathCAD) and corrected sample root-mean-square deviation (see s in MathCAD): $\sigma=\sqrt{D_в}=2{,}903$; $s=\sqrt{s^2}=2{,}93$

- smaple mode and median (see mode(*X*) and median(*X*) in MathCAD):

mode M_0 = 19 defines variant, which has the biggest frequency (there can be not only one mode, it can be found easily by statistical series of frequencies); median M_e =18 defines midpoint of variational series and depends on parity of sample volume: $M_e=\begin{cases} x_{k+1}, if\ n=2k+1 \\ \frac{x_k+x_{k+1}}{2}, if\ n=2k\cdot \end{cases}$

Copy of file from MathCAD:

n := 55

X := (20 15 17 19 23 18 21 15 16 13 20 16 19 20 14 20 16 14 20 19

Y := sort(X^T)

$Y^T =$

	0	1	2	3	4	5	6	7	8	9
0	10	11	12	13	13	14	14	14	14	...

$X^T =$

	0
0	20
1	15
2	17
3	19
4	23
5	18
6	21
7	15
8	16
9	13
10	20
11	16
12	19
13	20
14	14
15	...

$a := \min(X)$ $\quad b := \max(X)$

$a = 10 \quad b = 23$

$R := b - a \quad R = 13$

$h := \dfrac{b - a}{1 + \dfrac{\ln(55)}{\ln(2)}} \quad h = 1.917$

$h1 := 2$

$x0 := a - \dfrac{h}{2} \quad m := \dfrac{R}{h}$

$x0 = 9.041$

$a1 := 9 \quad m1 := 7 \quad m = 6.781$

$j := 0 .. m1 \quad i := 0 .. m1 - 1$

$t_j := a1 + h1 \cdot j \quad x_i := t_i + \dfrac{h1}{2}$

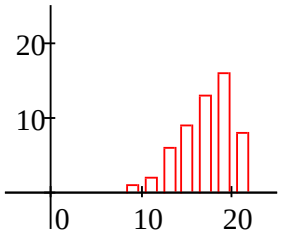
$w1 := hist(t, X) \quad w2 := \dfrac{w1}{n}$

$t^T = (9 \quad 11 \quad 13 \quad 15 \quad 17 \quad 19 \quad 21 \quad 23)$

$x^T = (10 \quad 12 \quad 14 \quad 16 \quad 18 \quad 20 \quad 22)$

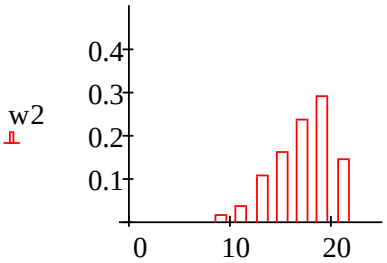
$w1^T = (1 \quad 2 \quad 6 \quad 9 \quad 13 \quad 16 \quad 8)$

w1



$w2^T = (0.018 \quad 0.036 \quad 0.109 \quad 0.164 \quad 0.236 \quad 0.291 \quad 0.145)$

$0.018 + 0.036 + 0.109 + 0.164 + 0.236 + 0.291 + 0.145 = 0.999$



mode(X) = 19

x1 := mean(X) x1 = 17.564

M := median(X) M = 18

x2 := var(X) x2 = 8.428

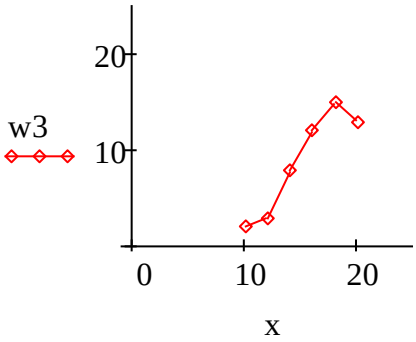
$s2 := \frac{n}{n-1} \cdot x2$

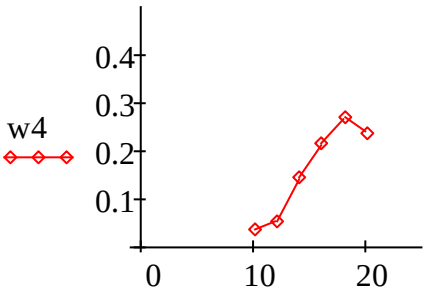
stdev(X) = 2.903

s2 = 8.584 s := $\sqrt{s2}$ s = 2.93

w3 := hist(x, X)

$w4 := \frac{w3}{n}$



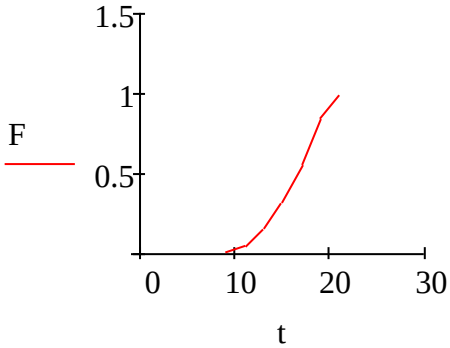


i := 0 .. 6

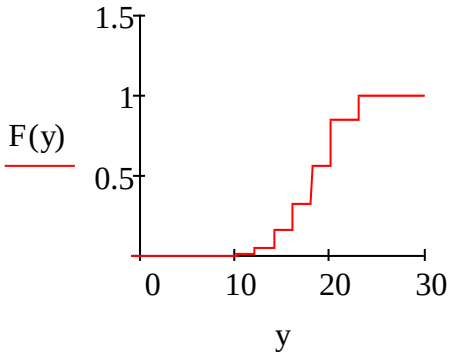
w := (0.018 0.036 0.109 0.164 0.236 0.291 0.145)^T

$F_i := \sum_{j=0}^{i} w_j$

F^T = (0.018 0.054 0.163 0.327 0.563 0.854 0.999)



$$F(y) := \begin{cases} 0 & \text{if } y \le 10 \\ 0.018 & \text{if } 10 < y \le 12 \\ 0.054 & \text{if } 12 < y \le 14 \\ 0.163 & \text{if } 14 < y \le 16 \\ 0.327 & \text{if } 16 < y \le 18 \\ 0.563 & \text{if } 18 < y \le 20 \\ 0.854 & \text{if } 20 < y \le 23 \\ 1 & \text{if } y > 23 \end{cases}$$



3. Find confidence interval for estimation of mathematical expectation *a* of normal distribution with reliability 0,95 . Sample mean (18), sample volume (25) and root-mean-square deviation (3) are given.

Solution: confidence interval for estimation of mathematical expectation a is as follows: $\bar{x}_в - t\frac{\sigma}{\sqrt{n}} < a < \bar{x}_в + t\frac{\sigma}{\sqrt{n}}$. All variables, excluding *t*, are known. Let's find *t* from formula $\Phi(t) = \frac{\gamma}{2} = \frac{0{,}95}{2} = 0{,}475$, where $\Phi(t)$ is Laplace function, its values are tabulated. By table $t = 1{,}96$. Thus, $18 - 1{,}96\frac{3}{\sqrt{25}} < a < 18 + 1{,}96\frac{3}{\sqrt{25}}$.

Answer: (16,824; 19,176).

Bibliography

1 Гмурман В.Е. Теория вероятностей и математическая статистика. Учебное пособие для вузов. – М.: Высш. школа, 2003.- 279 с.

2 Гмурман В.Е. Руководство к решению задач по теории вероятностей и математической статистике. – М.: Высш. школа, 2004.- 400 с.

3 Ивановский Р.И. Теория вероятностей и математическая статистика. Основы, прикладные аспекты с примерами изадачами в среде Mathcad. - СПб.: БХВ- Петербург, 2008. – 528 с.

4 Письменный Д. Конспект лекций по теории вероятностей и математической статистики, случайные процессы. – М.: Айрис -пресс, 2006. – 288 с.

5 Лунгу К.Н. и др. Сборник задач по высшей математике. 2 курс – М.: Айрис-пресс, 2011. – 576 с.

6 Khasseinov K. Canons of Mathematics. – Moscow: Nauka, 2007. – 592 p.

7 Астраханцева Л.Н., Байсалова М.Ж. Теория вероятностей и математическая статистика. Конспект лекций для студентов специальностей 5В070400 – Вычислительная техника и программное обеспечение, 5В070300 – Информационные системы.– Алматы: АУЭС, 2013.- 49 с.

Content

1 Calculation-graphical work №1. Elements of probability theory ……... 3
1.1 Theoretical issues……. 3
1.2 Calculation tasks ……. 3
1.3 Exemplary embodiment solution…… 12
2 Calculation-graphical work №2 Elements of mathematical statistics … 30
2.1 Theoretical issues …… 30
2.2 Calculation tasks…….. 30
2.3 Exemplary embodiment solution ……. 37
Bibliography ……..…….. 44

Summary plan 2017, pos. 133

Astrakhantseva Lyudmila Nikolaevna
Baisalova Manshuk Zhumamuratovna

PROBABILITY THEORY AND MATHEMATICAL STATISTICS

Methodological Guidelines and Tasks for carring out
the calculation- graphical works for students of specialty
5B070200 - Automation and management

Editor R.E.Kim
Specialist on standardization N.K.Moldabekova

Signed in print_____ Format 60x84 1/16
Circulation 20 copies Typographical paper №1
Volume 2,8 ed.sheets. Ordering___ Price 1375 t.

Multiple copying Office of
Non-Profit Joint Stock Company
«Almaty University of Power Engineering and Telecommunications»
050013, Almaty, 126, Baytursynov st.