Non-Profit Joint Stock Company

ALMATY UNIVERSITY OF POWER ENGINEERING AND TELECOMMUNICATIONS

The department of Mathematical modeling and software

PROBABILITY THEORY AND MATHEMATICAL STATISTICS

Methodological Guidelines for carrying out
the laboratory works for students of speciality
5B070200 - Automation and management

Almaty 2017

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Methodological Guidelines contain four laboratory works. First two works introduce students to MathCAD system. Third and fourth works contain tasks from basic parts of probability theory and statistical mathematics course. Methodological Guidelines for students of specialty 5B070200 – Automation and management, are compiled in accordance with syllabus “Probability theory and mathematical statistics”.

Tables 20, figures 10, bibl. 6.

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1 Laboratory work №1

The aim of the laboratory work is introduction to computer system MathCAD and solution of tasks of elementary mathematics, vector and linear algebra in this system.

Task 1. Calculate**.**

1.1	$\left(\left(4{,}625-\frac{13}{18}\cdot\frac{9}{26}\right):\frac{9}{4}+2{,}5:1{,}25:6{,}75\right)\cdot 7:1\frac{53}{68}$
1.2	$\left(\frac{1}{2}-0{,}375\right)\cdot 0{,}125+\left(\frac{5}{6}-\frac{7}{12}\right):(0{,}358-1{,}4796:1{,}3)$
1.3	$3{,}75:1\frac{1}{2}+\left(1{,}5:3\frac{3}{4}\right)\cdot 2\frac{1}{2}+\left(1\frac{1}{72}-\frac{23}{49}\right):\frac{22}{147}$
1.4	$8{,}4\cdot\frac{6}{7}\cdot\left(6-\frac{(2{,}3+5:6{,}25)\cdot 7}{8\cdot 0{,}0125+6{,}9}\right)-20{,}384:1{,}3$
1.5	$(3{,}4-1{,}275)\cdot\frac{16}{17}+0{,}5\cdot\left(2+12{,}5:\left(5{,}75+\frac{1}{2}\right)\right)$
1.6	$\left(520\cdot 0{,}43:0{,}26-217\cdot 2\frac{3}{7}\right)-\left(31{,}5:12\frac{3}{5}+114\cdot 2\frac{1}{3}\right)$
1.7	$\left(1\frac{1}{7}-\frac{23}{47}\right):\frac{22}{147}-\left(0{,}6:3\frac{3}{4}\right)\cdot 2\frac{1}{2}+3{,}75:1\frac{1}{2}$
1.8	$0{,}125:0{,}25+1\frac{9}{16}:2{,}5+\left(\frac{17}{20}+1{,}9\right)\cdot 0{,}5$
1.9	$\left(1\frac{7}{8}\cdot 8-\left(8{,}9-2{,}6:\frac{2}{3}\right)\right)\cdot 34\frac{2}{5}+5:2\frac{1}{2}$
1.10	$2:3\frac{1}{5}+\left(3\frac{1}{4}:13\right):\frac{2}{3}+\left(2\frac{5}{18}-\frac{17}{36}\right)\cdot 18{,}6$
1.11	$\left(0{,}3275-\left(2\frac{15}{88}+\frac{4}{33}\right):12\frac{2}{9}\right):0{,}07-22{,}5$
1.12	$\left(16\frac{1}{2}-13\frac{7}{9}\right)\cdot\frac{18}{33}+2{,}2\cdot\left(0{,}24-0{,}09\cdot 2\frac{1}{2}\right)+\frac{2}{11}$
1.13	$\left(3{,}5\cdot 1\frac{1}{3}-4{,}75\right)\cdot 0{,}6+2\frac{4}{11}\cdot 0{,}2:0{,}33-0{,}6$
1.14	$3\frac{1}{3}:10+0{,}175:0{,}35-\left(\frac{11}{18}-\frac{1}{15}\right)\cdot 1{,}4-12{,}35$
1.15	$\left(4{,}5\cdot 1\frac{2}{3}-6{,}75\right)\cdot 0{,}66+1\frac{4}{11}\cdot 0{,}22:0{,}3-0{,}96$
1.16	$\left(\left(3{,}625-\frac{12}{18}\cdot\frac{9}{24}\right):\frac{5}{4}+2{,}5:1{,}25:6{,}75\right)\cdot 9:1\frac{5}{6}$
1.17	$\left(\frac{1}{2}-0{,}175\right)\cdot 0{,}325+\left(\frac{7}{6}-\frac{5}{12}\right):(0{,}35-1{,}47:1{,}3)$

1.18	$2{,}75:1\frac{3}{4}+\left(1{,}2:3\frac{3}{4}\right)\cdot 2\frac{1}{2}+\left(1\frac{1}{7}-\frac{2}{49}\right):\frac{2}{17}$
1.19	$8{,}2\cdot\frac{1}{7}\cdot\left(5-\frac{(2{,}3+5:6{,}25)\cdot 7}{8\cdot 0{,}0125+4{,}9}\right)-2{,}38:1{,}3$
1.20	$(1{,}4-1{,}25)\cdot\frac{1}{17}+0{,}3\cdot\left(2+12{,}5:\left(1{,}75+\frac{1}{2}\right)\right)$
1.21	$\left(20\cdot 0{,}4:0{,}26-21\cdot 2\frac{3}{5}\right)-\left(30{,}5:12\frac{2}{5}+14\cdot 2\frac{1}{3}\right)$
1.22	$\left(2\frac{1}{5}-\frac{2}{47}\right):\frac{22}{47}-\left(0{,}3:4\frac{3}{4}\right)\cdot 1\frac{1}{2}+3{,}15:1\frac{1}{2}$
1.23	$0{,}25:0{,}125+1\frac{3}{16}:1{,}5+\left(\frac{7}{20}+1{,}8\right)\cdot 0{,}5$
1.24	$\left(1\frac{2}{7}\cdot 7-\left(6{,}9-1{,}6:\frac{2}{3}\right)\right)\cdot 3\frac{2}{5}+5:2\frac{1}{2}$
1.25	$4:3\frac{2}{5}+\left(5\frac{1}{4}:13\right):\frac{1}{3}+\left(2\frac{5}{8}-\frac{7}{36}\right)\cdot 1{,}6$
1.26	$\left(0{,}275-\left(2\frac{5}{8}+\frac{4}{33}\right):2\frac{2}{9}\right):0{,}08-21{,}5$
1.27	$\left(6\frac{1}{2}-13\frac{2}{9}\right)\cdot\frac{8}{33}+1{,}2\cdot\left(0{,}4-0{,}09\cdot 1\frac{1}{2}\right)+\frac{2}{13}$
1.28	$4:2\frac{1}{5}+\left(6\frac{1}{4}:13\right):\frac{1}{3}+\left(2\frac{5}{8}-\frac{17}{36}\right)\cdot 8{,}5-3{,}15$
1.29	$2\frac{1}{3}:11+0{,}125:0{,}15-\left(\frac{13}{18}-\frac{1}{15}\right)\cdot 1{,}2-12{,}25$
1.30	$\left(3{,}5\cdot 1\frac{1}{3}-4{,}75\right)\cdot 0{,}6+2\frac{4}{11}\cdot 0{,}2:0{,}33-0{,}6$

Task 2. Remove brackets and collect terms.

2.1	$(n+5k)^2(n-k)+(n-5k)^2(n+k)-2n(n+4k)(n-4k)$
2.2	$(a+3z)^2(a-z)-(a-3z)^2(a+z)-2z(3z+2a)(2a-3z)$
2.3	$2(5a+6x)(3a+2x)-(4a+7x)^2-(3a-5x)(3a+5x)$
2.4	$(3a-2b)(3a+2b)+(a-3b)^2-(5a-b)(2a-5b)$
2.5	$(4x-7y)^2-(3x+5y)^2-2(2x+3y)(4y-17x)$
2.6	$(3a-5b)^2+(2a+9b)^2-(2a+3b)(27b-16a)$
2.7	$3(5x+2y)^2+2(3x-5y)^2$
2.8	$\left(y^3-3y^2z+2yz^2-6z^3\right)(2y-z)-\left(y^3-5y^2z-6yz^2+2z^3\right)(2y+3z)$
2.9	$2(7a-3x)^2-3(a-14x)^2$

2.10	$\left(2x^3+x^2y+3xy^2-y^3\right)(x+5y)-\left(x^3-2x^2y+3xy^2+5y^3\right)(2x-y)$
2.11	$\left(a^2+4ax+x^2\right)(a-2x)-\left(a^2-5ax-2x^2\right)(a+x)$
2.12	$\left(a^2-ab+2b^2\right)(a-b)-\left(a^2+2ab-b^2\right)(a+2b)$
2.12	$\left(b^3-b^2+b-1\right)(b+1)$
2.14	$\left(2a^3-a^2+2a-1\right)(a-1)$
2.15	$3(5a-4x)^2-2(a+4x)^2$
2.16	$(n-4k)^2(n+k)+(n+4k)^2(n-k)-n(n+2k)(n-2k)$
2.17	$(a+4z)^2(a-z)-(a-4z)^2(a+z)-z(z+2a)(2a-z)$
2.18	$(4a+6x)(a+2x)-(a+6x)^2-(3a-x)(3a+x)$
2.19	$(a-2b)(a+2b)+(2a-3b)^2-(3a-b)(2a-b)$
2.20	$(4x-y)^2-(2x+5y)^2-2(2x+3y)(y-7x)$
2.21	$(a-3b)^2+(2a+7b)^2-(2a+3b)(17b-26a)$
2.22	$4(x+2y)^2+2(x-5y)^2$
2.23	$\left(y^3-2y^2z+3yz^2-4z^3\right)(y-z)-\left(2y^3-y^2z-2yz^2+z^3\right)(y+3z)$
2.24	$4(5a-3x)^2-3(a-15x)^2$
2.25	$\left(x^3+3x^2y+3xy^2-y^3\right)(x+2y)-\left(x^3-2x^2y+2xy^2+4y^3\right)(3x-y)$
2.26	$\left(a^2+4ax+x^2\right)(a-2x)-\left(a^2-5ax-2x^2\right)(a+x)$
2.27	$\left(2a^2-ab+2b^2\right)(2a-b)-\left(a^2+ab-b^2\right)(a+2b)$
2.28	$\left(2b^3-b^2+3b-1\right)(b+2)$
2.29	$\left(3a^3-a^2+2a-1\right)(a+1)$
2.30	$5(3a-x)^2-3(a+7x)^2$

Task 3.

1) Factorize the given linear polynomial $f(x)$.

2) Solve the equation $f(x)=0$.

Make conclusions comparing results obtained in 1) and 2):

	f(x)		*f(x)*
3.1	$x^5-x^4+5x^3-5x^2+9x-9$	3.2	$x^3-x^2-29x+45$
3.3	$x^7-x^6-x^5+x^4-x^3+x^2+x-1$	3.4	$x^4-7x^3+8x^2+28x-48$
3.5	$27x^3-27x^2+18x-4$	3.6	$x^3-6x^2+11x-6$
3.7	$x^3+2x^2-20x+24$	3.8	$x^3-10x^2+23x-14$
3.9	$2x^4+7x^3-2x^2-13x+6$	3.10	$x^3+9x^2+23x+15$
3.11	$2x^4-x^3-9x^2+13x-5$	3.12	x^3-6x^2-x+30

3.13	$x^4-2x^3-3x^2+4x+4$	3.14	$x^4+x^3-11x^2-5x+30$
3.15	$x^4-2x^3-11x^2+12x+36$	3.16	$x^4+2x^3-12x^2-38x-24$
3.17	$x^4+2x^3-16x^2-2x+15$	3.18	$x^4+x^3-2x^2+4x-24$
3.19	$27x^4-9x^2+14x-4$	3.20	$x^5-2x^4-4x^3+4x^2-5x+6$
3.21	$-12x^4+4x^3+9x^2-1$	3.22	$x^5+x^4-6x^3-14x^2-11x-3$
3.23	$x^5-6x^4+16x^3-32x^2+48x-32$	3.24	$x^5-5x^4+7x^3-2x^2+4x-8$
3.25	x^4-12x^2+32	3.26	x^4-20x^2+96
3.27	$6x^4-5x^2+1$	3.28	$6x^3-36x^2+72x-48$
3.29	x^4-20x^2+64	3.30	$128x^4-96x^3+24x^2-2a$

Task 4. Vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and numbers α, β, γ are given. Find:

1) $\beta\vec{a}$;
2) $\beta\vec{a}-\alpha\vec{b}+\gamma\vec{c}$;
3) inner (scalar, dot) product of vectors $\vec{a}$ and $\vec{b}$;
4) vector (cross) product of vectors $\vec{a}$ and $\vec{b}$;
5) length of vector $\vec{a}$ and vector obtained in previous item;
6) triple product of vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

	$\vec{a}$	$\vec{b}$	$\vec{c}$	α	β	γ
4.1	(2, -3, 1)	(1, 2, 5)	(6, 2, -3)	2	1	3
4.2	(4, -2, 0)	(4, -2, 0)	(1, 2, -5)	1	2	2
4.3	(5, -1, 0)	(3, 2, 4)	(3, 2, -3)	-1	-2	1
4.4	(1, 2,-3)	(1, -2,5)	(4, 1, -3)	7	3	-1
4.5	(5, 1, 2)	(2, 1, -4)	(6, 2, -3)	2	4	-3
4.6	(7, -1, 0)	(3, -6, 5)	(1, 5,- 4)	5	1	2
4.7	(2, -3, 4)	(7, 2, 4)	(6, 2,-3)	6	2	-1
4.8	(5, -1, 3)	(3, -1, 6)	(7, 2,-3)	3	-3	2
4.9	(6, 2, -5)	(2, 2, -3)	(1,-7, 5)	4	-5	1
4.10	(4, -1, 0)	(3, -3, 4)	(5, 2, -1)	-2	4	2
4.11	(7, 0, 6)	(1, 2, -5)	(3, -2,-1)	1	3	4
4.12	(1, -1, 5)	(-1, -5, 1)	(1, 3,-3)	2	4	3
4.13	(5, -1, 2)	(-3, 2, 4)	(4, 2,-5)	4	2	5
4.14	(6, -1, 4)	(1, 0, 7)	(2, -1, 0)	3	-1	1
4.15	(5, -1, 3)	(6, 2, -3)	(-5, 1 ,-3)	1	-2	5
4.16	(5, -1, 0)	(4, 3, 1)	(4, 6, -1)	2	1	3
4.17	(4, -2, 0)	(3, 1, 4)	(2, 2, -5)	1	2	2
4.18	(5, -5, 4)	(7, 2, 4)	(1, 2, -3)	-1	-2	1
4.19	(7, 2,-3)	(1, 2, 4)	(4, 1, -3)	7	3	-1
4.20	(4, 1, 2)	(3, 1, -4)	(6, 2, -3)	2	4	-3
4.21	(7, -1, 0)	(3, -6, 5)	(1, 5,- 4)	5	1	2
4.22	(2, -3, 4)	(7, 2, 4)	(6, 2,-3)	6	2	-1
4.23	(5, -1, 3)	(3, -1, 6)	(7, 2,-3)	3	-3	2

4.24	(6, 2, -5)	(2, 2, -3)	(1,-7, 5)	4	-5	1
4.25	(4, -1, 0)	(3, -3, 4)	(5, 2, -1)	-2	4	2
4.26	(7, 0, 6)	(1, 2, -5)	(3, -2,-1)	1	3	4
4.27	(1, -1, 5)	(-1, -5, 1)	(1, 3,-3)	2	4	3
4.28	(5, -1, 2)	(-3, 2, 4)	(4, 2,-5)	4	2	5
4.29	(6, -1, 4)	(1, 0, 7)	(2, -1, 0)	3	-1	1
4.30	(5, -1, 3)	(6, 2, -3)	(-5, 1 ,-3)	1	-2	5

Task 5. Matrices A, B, C are given. Find:

- determinants of matrices *A* and *C*;
- matrix B^T;
- inverse matrices to matrices *A* and *C* (if it is possible);
- ranks of matrices *A* and *C*;
- product of matrices *A* and *B*;
- matrix A^2.

	A	*B*	*C*
5.1	$\begin{pmatrix} 1 & 2 & -1 \\ 3 & -5 & 0 \\ 4 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 7 & 1 \\ 2 & 4 & 1 \\ 1 & 1 & 1 \end{pmatrix}$
5.2	$\begin{pmatrix} 3 & 0 & -1 \\ 2 & 4 & 1 \\ 9 & 7 & 5 \end{pmatrix}$	$\begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & 2 \\ 2 & 2 & 5 \\ 5 & 3 & 7 \end{pmatrix}$
5.3	$\begin{pmatrix} 1 & 4 & 1 & 2 \\ 2 & -2 & 1 & -1 \\ 4 & 1 & 1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 2 \\ -3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 & 3 & 5 \\ 5 & 1 & 4 & 3 \\ 2 & -4 & 6 & 8 \\ 3 & 0 & 1 & 9 \end{pmatrix}$
5.4	$\begin{pmatrix} 5 & 4 & -1 \\ 2 & -1 & 1 \\ 0 & 6 & 13 \end{pmatrix}$	$\begin{pmatrix} 7 \\ -1 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$
5.5	$\begin{pmatrix} 3 & 6 & -4 \\ 7 & 0 & 1 \\ 2 & 2 & 5 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 \\ 0 & 3 \\ 4 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 6 & 6 \end{pmatrix}$
5.6	$\begin{pmatrix} 11 & 6 & -1 \\ 2 & 4 & 3 \\ 5 & 0 & 2 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 2 & -1 & 3 \\ 5 & 7 & 0 \\ 4 & -2 & 6 \end{pmatrix}$
5.7	$\begin{pmatrix} 6 & -1 & 3 \\ 5 & 1 & 0 \\ 21 & 4 & 2 \end{pmatrix}$	$\begin{pmatrix} 7 \\ -1 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 3 & 1 & 7 \end{pmatrix}$

5.8	$\begin{pmatrix} 6 & -2 & 1 \\ 4 & 3 & 2 \\ 5 & 9 & -1 \end{pmatrix}$	$\begin{pmatrix} 8 \\ 5 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 4 & 7 \end{pmatrix}$
5.9	$\begin{pmatrix} 5 & -1 & 3 \\ 1 & 2 & 4 \\ 6 & 0 & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 \\ 0 & 3 \\ 4 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 5 & 9 \end{pmatrix}$
5.10	$\begin{pmatrix} 2 & -1 & 3 \\ 6 & 0 & 1 \\ 7 & 3 & 5 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 5 & 6 & 3 \end{pmatrix}$
5.11	$\begin{pmatrix} 6 & 2 & -1 \\ 1 & 3 & 21 \\ -2 & 4 & 0 \end{pmatrix}$	$\begin{pmatrix} 7 \\ 3 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 4 & 6 \\ 1 & 7 & 9 \end{pmatrix}$
5.12	$\begin{pmatrix} 3 & 1 & -2 \\ 0 & 4 & 5 \\ 7 & 3 & -1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 3 \\ 4 & 1 & 6 \\ 7 & 1 & 9 \end{pmatrix}$
5.13	$\begin{pmatrix} 4 & 2 & 1 \\ -3 & 0 & 5 \\ 9 & -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 6 & 7 \end{pmatrix}$
5.14	$\begin{pmatrix} 3 & 1 & 0 & 2 \\ 1 & 1 & 1 & -1 \\ 2 & 1 & -1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 3 \\ 1 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 4 & 2 & 1 \\ 1 & 1 & 1 \\ 7 & 3 & 1 \end{pmatrix}$
5.15	$\begin{pmatrix} 3 & 5 & 2 \\ -4 & 1 & 3 \\ 0 & -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & 2 & 9 \end{pmatrix}$
5.16	$\begin{pmatrix} 4 & 1 & 5 \\ 1 & 2 & 7 \\ 3 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$	$\begin{pmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \\ 5 & 1 & 4 \end{pmatrix}$
5.17	$\begin{pmatrix} 9 & 1 & 5 \\ 1 & 2 & 7 \\ 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix}$	$\begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & 0 \\ 1 & -1 & 1 \end{pmatrix}$
5.18	$\begin{pmatrix} 2 & 1 & 5 \\ -1 & 2 & 7 \\ 5 & 6 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 2 \\ 5 & 2 & -1 \\ 3 & -3 & 6 \end{pmatrix}$
5.19	$\begin{pmatrix} 8 & 5 & 1 \\ 0 & -1 & 2 \\ 3 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 4 & 3 & 6 \\ 7 & 1 & 2 \\ 5 & -1 & -2 \end{pmatrix}$

5.20	$\begin{pmatrix} 6 & 0 & -1 \\ -1 & 2 & 4 \\ 5 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 & 2 \\ -5 & 8 & 0 \\ 2 & 6 & 4 \end{pmatrix}$
5.21	$\begin{pmatrix} 1 & 1 & 5 \\ 3 & 2 & 7 \\ 7 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix}$	$\begin{pmatrix} 6 & 2 & 2 \\ -1 & 4 & 7 \\ 3 & 1 & 1 \end{pmatrix}$
5.22	$\begin{pmatrix} 5 & 1 & 5 \\ 4 & 2 & 7 \\ 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 5 & 1 & 2 \\ 7 & 2 & 4 \\ 3 & 1 & 2 \end{pmatrix}$
5.23	$\begin{pmatrix} 1 & 1 & 5 \\ -5 & 2 & 7 \\ 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix}$	$\begin{pmatrix} 7 & -5 & 0 \\ 2 & 3 & 1 \\ 4 & 6 & 2 \end{pmatrix}$
5.24	$\begin{pmatrix} 5 & 1 & 5 \\ 2 & 2 & 7 \\ 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$	$\begin{pmatrix} 5 & 3 & 6 \\ 7 & 2 & 4 \\ -3 & 1 & 2 \end{pmatrix}$
5.25	$\begin{pmatrix} 3 & 1 & 5 \\ 5 & 2 & 7 \\ 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 4 \\ 5 & -1 & -2 \\ 7 & 1 & 2 \end{pmatrix}$
5.26	$\begin{pmatrix} 5 & 1 & 5 \\ -4 & 2 & 7 \\ 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 2 & 7 & 0 \\ -1 & 3 & 4 \\ -2 & 6 & 8 \end{pmatrix}$
5.27	$\begin{pmatrix} 2 & 1 & 5 \\ 1 & 2 & 7 \\ 3 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & -1 \\ 1 & 5 & 4 \\ 9 & 6 & -3 \end{pmatrix}$
5.28	$\begin{pmatrix} 3 & 1 & 5 \\ 4 & 2 & 7 \\ 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 & 5 \\ 3 & 4 & 1 \\ 6 & 8 & 2 \end{pmatrix}$
5.29	$\begin{pmatrix} 2 & 1 & 5 \\ 5 & 2 & 7 \\ 7 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 4 \\ 0 & -1 & -2 \\ 5 & 4 & 8 \end{pmatrix}$
5.30	$\begin{pmatrix} 1 & 1 & 5 \\ 5 & 2 & 7 \\ 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 7 \\ 1 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 2 & 7 & 0 \\ 1 & -1 & 2 \\ 3 & -3 & 6 \end{pmatrix}$

Task 6. System of equations $AX=B$ is given. Solve the system:

- by Cramer's rule;
- by matrix method, i.e. by formula $X=A^{-1}B$;
- by means of operation *lsolve*(*A*,*B*);
- by means of operation *rref*(*A*).

	A	B		A	B
6.1	$\begin{pmatrix} 2 & 1 & 1 & 2 \\ 1 & -3 & 1 & -1 \\ 1 & 4 & -1 & 2 \\ 1 & 1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ -2 \\ 10 \\ 3 \end{pmatrix}$	6.2	$\begin{pmatrix} 1 & 4 & 1 & 2 \\ 2 & -2 & 1 & -1 \\ 4 & 1 & 1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ -2 \\ 6 \\ -1 \end{pmatrix}$
6.3	$\begin{pmatrix} 2 & 1 & 5 & 2 \\ 1 & -2 & 1 & -1 \\ 6 & 1 & -2 & 1 \\ 1 & 5 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 7 \\ 1 \\ -8 \end{pmatrix}$	6.4	$\begin{pmatrix} 1 & -1 & 1 & 4 \\ 3 & -1 & 2 & 2 \\ 1 & -1 & 2 & -1 \\ 3 & 0 & 1 & 2 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 4 \\ -1 \\ 5 \end{pmatrix}$
6.5	$\begin{pmatrix} 2 & 0 & 1 & 2 \\ 1 & -2 & 1 & -1 \\ 3 & 1 & -1 & 2 \\ 2 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 3 \\ 7 \\ 6 \end{pmatrix}$	6.6	$\begin{pmatrix} 2 & -1 & 1 & 4 \\ 3 & -1 & 2 & 2 \\ 1 & -1 & 2 & -1 \\ 3 & 0 & 1 & 2 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 4 \\ -1 \\ 6 \end{pmatrix}$
6.7	$\begin{pmatrix} 1 & 0 & 2 & 2 \\ 1 & -2 & 1 & -1 \\ 3 & 1 & -1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 6 \\ 7 \\ 5 \end{pmatrix}$	6.8	$\begin{pmatrix} 3 & -2 & 1 & 4 \\ 2 & -1 & -1 & 2 \\ 0 & 1 & 2 & -1 \\ 3 & 2 & 1 & 2 \end{pmatrix}$	$\begin{pmatrix} 3 \\ -1 \\ 4 \\ 5 \end{pmatrix}$
6.9	$\begin{pmatrix} 1 & 4 & 3 & 1 \\ 2 & -2 & 1 & 1 \\ 3 & 1 & -1 & 2 \\ 2 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 3 \\ 7 \\ 5 \end{pmatrix}$	6.10	$\begin{pmatrix} 4 & 6 & 3 & 1 \\ 1 & -2 & 1 & 1 \\ 3 & 1 & 1 & 2 \\ 2 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} -9 \\ 7 \\ 3 \\ 4 \end{pmatrix}$
6.11	$\begin{pmatrix} 5 & 1 & 4 & 1 \\ 2 & -2 & 3 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 6 \\ 5 \\ 7 \end{pmatrix}$	6.12	$\begin{pmatrix} 2 & 1 & 4 & 1 \\ 2 & -2 & 1 & 1 \\ 3 & 1 & 1 & 2 \\ 2 & -1 & 5 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 1 \\ 6 \\ 2 \end{pmatrix}$
6.13	$\begin{pmatrix} 7 & 1 & -3 & 1 \\ 5 & 2 & 1 & 9 \\ 1 & 4 & -1 & 0 \\ 8 & -1 & 3 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ -8 \\ 8 \\ 7 \end{pmatrix}$	6.14	$\begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & -2 & 1 & 1 \\ 3 & 1 & 1 & 2 \\ 2 & -1 & 5 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 2 \\ 7 \\ 7 \end{pmatrix}$
6.15	$\begin{pmatrix} 2 & 0 & -1 & 1 \\ 2 & -2 & 1 & 1 \\ 6 & 1 & 1 & 1 \\ 2 & -1 & 5 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 4 \\ 8 \\ 4 \end{pmatrix}$	6.16	$\begin{pmatrix} 1 & 2 & 1 & 4 \\ 6 & -1 & 1 & 2 \\ 1 & -1 & 2 & -1 \\ 3 & 0 & 1 & 2 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 2 \\ 7 \\ 7 \end{pmatrix}$
6.17	$\begin{pmatrix} 3 & 1 & 0 & 2 \\ 1 & 1 & 1 & -1 \\ 2 & 1 & -1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 3 \\ 1 \\ 2 \end{pmatrix}$	6.18	$\begin{pmatrix} 5 & 1 & 1 & 2 \\ 2 & 0 & 1 & -1 \\ 4 & 1 & 1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 4 \\ 6 \\ 0 \end{pmatrix}$
6.19	$\begin{pmatrix} 5 & 1 & 1 & 2 \\ -1 & -3 & 1 & -1 \\ 1 & 4 & -1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ -3 \\ 5 \\ 2 \end{pmatrix}$	6.20	$\begin{pmatrix} 8 & 2 & 1 & 3 \\ -1 & 9 & 1 & 0 \\ 3 & 7 & 2 & -1 \\ 3 & 1 & 8 & 2 \end{pmatrix}$	$\begin{pmatrix} 10 \\ -2 \\ 0 \\ -3 \end{pmatrix}$

6.21	$\begin{pmatrix} 6 & 1 & 5 & 3 \\ 1 & 2 & 1 & -1 \\ 2 & 4 & -1 & 2 \\ 7 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 1 \\ 9 \\ 5 \end{pmatrix}$	6.22	$\begin{pmatrix} 2 & -1 & 1 & 2 \\ 1 & 0 & 1 & -1 \\ 4 & 1 & 1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 1 \\ 2 \\ 3 \end{pmatrix}$
6.23	$\begin{pmatrix} 4 & -1 & 1 & 2 \\ 1 & -3 & 1 & -1 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ -1 \\ 5 \\ 2 \end{pmatrix}$	6.24	$\begin{pmatrix} 3 & 1 & 0 & 2 \\ 1 & 2 & 1 & -1 \\ 4 & 1 & -1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 3 \\ 6 \\ 3 \end{pmatrix}$
6.25	$\begin{pmatrix} 7 & 2 & 1 & 3 \\ -1 & -1 & 1 & 0 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 1 & 2 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 4 \\ 3 \\ 2 \end{pmatrix}$	6.26	$\begin{pmatrix} 7 & 0 & 1 & 2 \\ 1 & 2 & 1 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 2 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 1 \\ 2 \\ 3 \end{pmatrix}$
6.27	$\begin{pmatrix} 3 & 1 & 1 & 2 \\ 1 & -3 & 1 & -1 \\ 1 & 2 & 0 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 7 \\ -2 \\ 3 \\ 4 \end{pmatrix}$	6.28	$\begin{pmatrix} 4 & 1 & 1 & 2 \\ -1 & -1 & 1 & 0 \\ 1 & 2 & 2 & -1 \\ 3 & 1 & 1 & 2 \end{pmatrix}$	$\begin{pmatrix} 5 \\ -1 \\ -5 \\ 5 \end{pmatrix}$
6.29	$\begin{pmatrix} 5 & 1 & -1 & 2 \\ 1 & 2 & 1 & -1 \\ 2 & -1 & -1 & 2 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 8 \\ 6 \\ 0 \\ 2 \end{pmatrix}$	6.30	$\begin{pmatrix} 6 & 2 & 1 & 2 \\ 1 & 2 & 1 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 2 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 2 \\ 3 \\ 4 \end{pmatrix}$

Questions to the laboratory work №1.

1. How to call the bar “Mathematics”, where shortcuts of all basic working mathematical bars are indicated?
2. How many basic working mathematical bars do you know and what are their names?
3. What is the sense of three types of equals sign in MathCAD?
4. Function and control by blue angular cursor.
5. What you should keep in mind working with formulas (decimal recording, expression place for calculation)?
6. How to solve equation in MathCAD?
7. How to record vector in coordinate form?
8. Which ways to solve systems of linear equations there are in MathCAD?

Recommendations for performing of laboratory work №1

Task 1. Calculate $\left(4\frac{2}{5}-2{,}7:8\frac{5}{6}\right):1{,}7+3{,}5\cdot 21-0{,}62$.

Recommendation. Input the expression from the keyboard. Mixed fraction is entered as sum of integer and fractional parts. In decimal fraction instead of comma the point is entered. Multiplication sign is not removed. Highlight everything by blue angular cursor and press the button “=”.

Performing of the example

$$\frac{4\frac{2}{5}-\frac{2.7}{8\frac{5}{6}}}{1.7}+3.521-0.62=75.$$ or

$$\left(4\frac{2}{5}-2.7\div 8\frac{5}{6}\right)\div 1.7+3.521-0.62:$$

Task 2. Remove brackets and collect terms in the expression

$$(x+1)(x-1)(x^2-x+1)(x^2+x+1).$$

Recommendation. Enter the expression from the keyboard (see the 1-st example), highlight everything by blue angular cursor, click position "*Symbols*", "*Expand*" or call the bar "*Symbolic*", "*Expand*".

Performing of the example:

$(x+1)\cdot(x-1)\cdot(x^2-x+1)\cdot(x^2+x+1)$ expand, $x \rightarrow x^6-1$.

Task 3.

1) Factorize the polynomial $f(x)=x^4-4x^3+4x^2$.

2) Solve the equation $f(x)=0$.

Make conclusions comparing results obtained in 1) and 2):

Recommendation:

1) Enter the expression from the keyboard (see the 1-st example), highlight everything by angular cursor, click position "*Symbols*", "*Factor*"

Performing of the example $x^4-4x^3+4x^2$

Answer: $x^2\cdot(x-2)^2$

2) Reduce the equation to the form f(x)=0, input from the keyboard the left part of the equation, call the bar "*Symbolic*", "*Solve*", in the gap - write the variable and click the free place of the page.

Performing of the example in the working window of MathCAD program

$$x^4 - 4x^3 + 4x^2 \text{ solve, } x \rightarrow \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}.$$

Since equation $x^4 - 4x^3 + 4x^2 = 0$ has two double roots x=0 and x=2, then the left part of the equation expands into factors - $x^2(x-2)^2$, which coincides with result in the item 1.

Task 4. Vectors $\bar{a} = (1,2,3)$, $\bar{b} = (4,-2,-1)$, $\bar{c} = (-3,2,-1)$ and numbers $\alpha = 2$, $\beta = -3$, $\gamma = -2$ are given. Find:

- $\beta\vec{a}$;
- $\beta\vec{a} - \alpha\vec{b} + \gamma\vec{c}$;
- inner (scalar, dot) product of vectors $\vec{a}$ and $\vec{b}$;
- vector (cross) product of vectors $\vec{a}$ and $\vec{b}$;
- length of vector $\vec{a}$ and vector obtained in previous item;
- triple product of vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

Recommendation. Type from the keyboard expressions $\beta\vec{a}$ and $\beta\vec{a} - \alpha\vec{b} + \gamma\vec{c}$ (sing of multiplication should not be omitted), highlight everything by blue angular cursor and press sign "=": Enter, using bar *Matrix*, three vectors as matrices-columns (three rows and one column) and three numbers. Then sequentially perform all 6 tasks.

Performing of the example

$$a := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad b := \begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix} \qquad c := \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$$

$$\alpha = 2 \qquad \beta = -3 \qquad \gamma = -2$$

1) - 2) Enter from the keyboard expressions $\beta\vec{a}$ and $\beta\vec{a} - \alpha\vec{b} + \gamma\vec{c}$ (sign of multiplication should not be omitted), highlight everything by blue angular cursor and press sign "=".

3) - 4) Call the bar *Matrix*, *Dot product (*then *Cross product*), put multipliers (factors) in gaps, highlight everything by blue angular cursor and press sign "=":

$$a \cdot b = -3 \qquad a \times b = \begin{bmatrix} 4 \\ 13 \\ -10 \end{bmatrix}.$$

5) Call the bar *Matrix, Determinant,* put vector a or $a \times b$ in the gap, highlight everything by blue angular cursor and press sign "=":

$$|a| = 3.742 \qquad |a \times b| = 16.882.$$

6) Enter by means of bar *Matrix, Dot product and Cross product* the expression $(a \times b) \cdot c$, highlight everything by blue angular cursor and press sign "=":

$$(a \times b) \cdot c = 24 \text{ or } abc := (a \times b) \cdot c \to 24.$$

Task 5. Given matrices

$$A = \begin{pmatrix} 6 & 5 & 2 \\ 11 & 9 & 2 \\ 4 & 5 & 2 \end{pmatrix}, \; B = \begin{pmatrix} 2 \\ 0 \\ 9 \end{pmatrix}, \; C = \begin{pmatrix} 1 & -10 & 2 \\ -1 & 5 & 1 \\ 1 & -5 & 0 \end{pmatrix}.$$

- find determinants of matrices A and C;
- find matrix B^T;
- if there are matrices inverse to matrices A and C, find them;
- find rank of matrices A and C;
- find product of matrices A and B;
- find matrix A^2.

Recommendation. Enter, using bar *Matrix,* these three matrices and then sequentially perform all 6 tasks.

Performing of the task

$$A := \begin{pmatrix} 6 & 5 & 2 \\ 11 & 9 & 2 \\ 4 & 5 & 2 \end{pmatrix}, \; B := \begin{pmatrix} 2 \\ 0 \\ 9 \end{pmatrix}, \; C := \begin{pmatrix} 1 & -10 & 2 \\ -1 & 5 & 1 \\ 1 & -5 & 0 \end{pmatrix}.$$

- find determinants of matrices A and C as it is indicated in task 5:

$|A| = 16$, $|C| = 0$;

- the task is performed by means of bar *Matrix*, *Transpose*: $B^T = (2 \quad 0 \quad 9)$;

- the task performs by means of bar *Matrix*, *Inverse*, if the determinant of the matrix is zero, then the matrix doesn't have inverse matrix:

C^{-1} - doesn't exist because determinant of matrix C is zero.

$$A^{-1} = \begin{bmatrix} 0.5 & 0 & -0.5 \\ -0.875 & 0.25 & 0.625 \\ 1.188 & -0.625 & -0.063 \end{bmatrix};$$

- matrices ranks can be found by means of function "*rank*", which can be entered from the keyboard: $rank(A) = 3, \quad rank(C) = 2;$

- tasks are performed by means of operations of multiplication and powering from the keyboard or by using bar *Arithmetic*:

$$A \cdot B = \begin{bmatrix} 30 \\ 40 \\ 26 \end{bmatrix} \quad A^2 = \begin{bmatrix} 99 & 85 & 26 \\ 173 & 146 & 44 \\ 87 & 75 & 22 \end{bmatrix}.$$

Task 6. Given system of equations $AX = B$, where

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 2 & -3 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 30 \\ 10 \\ 3 \\ 10 \end{pmatrix}, \; X = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}.$$

- solve this system by Cramer's formulas;
- solve the system by matrix method;
- solve the system by means of function *lsolve*(A,B).
- solve the system by means of function *rref* (A).

Recommendation. Enter matrices A and B using bar *Matrix*, then sequentially perform first 3 tasks.

Performing of the tasks:

$$A := \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 2 & -3 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad B := \begin{bmatrix} 30 \\ 10 \\ 3 \\ 10 \end{bmatrix}.$$

1. To solve the system by Cramer's rule you should enter four matrices and find solution by Cramer's formulas:

$$A1 := \begin{bmatrix} 30 & 2 & 3 & 4 \\ 10 & 2 & -3 & 4 \\ 3 & 1 & -1 & 1 \\ 10 & 1 & 1 & 1 \end{bmatrix} \quad A2 := \begin{bmatrix} 1 & 30 & 3 & 4 \\ -1 & 10 & -3 & 4 \\ 0 & 3 & -1 & 1 \\ 1 & 10 & 1 & 1 \end{bmatrix} \quad A3 := \begin{bmatrix} 1 & 2 & 30 & 4 \\ -1 & 2 & 10 & 4 \\ 0 & 1 & 3 & 1 \\ 1 & 1 & 10 & 1 \end{bmatrix} \quad A4 := \begin{bmatrix} 1 & 2 & 3 & 30 \\ -1 & 2 & -3 & 10 \\ 0 & 1 & -1 & 3 \\ 1 & 1 & 1 & 10 \end{bmatrix}$$

$$x := \frac{|A1|}{|A|} \qquad y := \frac{|A2|}{|A|} \qquad z := \frac{|A3|}{|A|} \qquad t := \frac{|A4|}{|A|}$$

x = ▮ y = ▮ z = ▮ t = ▮ .

Thus, the system has one solution $X = (1, 2, 3, 4)$.

2. To solve the system by matrix method you need enter matrix A^{-1} , multiply A^{-1} to B and press the sign "=":

$A^{-1} =$ ∎ $A^{-1} \cdot B =$ ∎

.

Answer: $X = (1, 2, 3, 4)$.

3. To solve the system by means of function *lsolve* you should enter this function from the keyboard, write A and B as arguments, call sing "→" from the bar *Symbolic*, click at white space and after appeared notes press sign "=":

$$\text{lsolve}(A, B) \rightarrow \text{lsolve}\left(\begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 2 & -3 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 30 \\ 10 \\ 3 \\ 10 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

Answer: $X = (1, 2, 3, 4)$.

4. To solve the system by means of function *rref* (A), you should enter expanded matrix of the system. Operation *rref* (A) transforms matrix A to matrix A1 where rows are corresponded by equations of system resolving with respect to unknowns. Operation *rref* (A) is entered from the keyboard.

$$A := \begin{pmatrix} 1 & 2 & 3 & 4 & 30 \\ -1 & 2 & -3 & 4 & 10 \\ 0 & 1 & -1 & 1 & 3 \\ 1 & 1 & 1 & 1 & 10 \end{pmatrix}; \quad A1 := rref(A); \quad A1 := \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix}.$$

From the first row of matrix A1 we have x=1, from the second row – y=2, from the third row – z=3, from the fourth – t=4.

2 Laboratory work №2

The aim of this laboratory work is study by students the MathCAD rules and techniques for construction of functions graphs, calculation of limits, derivatives, integrals and research of functions by means of derivatives.

Task 1. Function *f(x)* and point x_0 are given:

- find limit of *f(x)* at point x_0;
- find derivatives $f'(x)$, $f'(x_0)$, $f''(x)$, $f''(x_0)$;
- construct graph of function *y=f(x)* in Cartesian coordinates. Construct tangent and normal to the function at the indicated point in the same graph.

	$f(x)$	x_0		$f(x)$	x_0		$f(x)$	x_0
1.1	$\sqrt[3]{(x-3)^2 x}$	-1	1.11	$\sqrt[3]{(x+2)^2 x}$	1	1.21	$\sqrt[3]{x^2(x+2)}$	-5
1.2	$\frac{x^2+2}{\sqrt{x^2-4}}$	-3	1.12	$\frac{x^2+8}{\sqrt{x^2-4}}$	3	1.22	$\frac{x^2-8}{\sqrt{4x^2-1}}$	-5
1.3	$\sqrt[3]{x(x+6)^2}$	-8	1.13	$\sqrt[3]{x^2(x+6)}$	8	1.23	$\sqrt[3]{x^2(x-6)}$	-3
1.4	$\frac{x(x-2)}{x+2}$	-3	1.14	$\frac{x(x-1)}{x+2}$	3	1.24	$\frac{x^2+6}{\sqrt{x^2-1}}$	-5
1.5	$2+\frac{1}{x}$	-1	1.15	$2-\frac{1}{x}$	1	1.25	$\frac{3}{x}$	-1
1.6	$\frac{4}{x}$	-2	1.16	$\frac{2}{x}$	2	1.26	$\frac{5}{x}$	5
1.7	$1-\frac{1}{x}$	-1	1.17	$1+\frac{1}{x}$	1	1.27	$\frac{x^2-3}{\sqrt{4x^2-1}}$	5
1.8	$\frac{x(x-1)}{x+1}$	-2	1.18	$\frac{x(x-4)}{x+1}$	2	1.28	$\frac{x^2+6}{\sqrt{x^2-1}}$	3
1.9	$\sin 2x$	0,5	1.19	$\sin 3x$	-0,5	1.29	$\sqrt[3]{x^2(x+1)}$	5
1.10	$\cos 2x$	0,5	1.20	$\cos 3x$	-0,5	1.30	$\sqrt[3]{x^2(x+2)}$	1

Task 2. Construct graph of function *y=f(x)*, which is given parametrically.

	$x(t)$	$y(t)$		$x(t)$	$y(t)$
2.1	$-t^2$	$2t^3$	2.16	t^2-2t	t^2+2t
2.2	$t-2\sin t$	$1-2\cos t$	2.17	$2\cos^3 t$	$2\sin^3 t$
2.3	$t-\sin t$	$1-\cos t$	2.18	t^3+3t+1	t^3-3t+1
2.4	$4\cos^3\frac{t}{4}$	$4\sin^3\frac{t}{4}$	2.19	$t+\frac{1}{t}$	$t+\frac{1}{t^2}$
2.5	$2\cos^2 t+\cos t$	$2\cos t\sin t+\sin t$	2.20	$\cos t-\cos 2t$	$\sin t-\sin 2t$
2.6	$-\frac{3t^2}{1+t^3}$	$\frac{3t}{1+t^3}$	2.21	$\frac{t^2}{t-1}$	$\frac{t}{t^2-1}$
2.7	$\frac{3t^2}{1+t^3}$	$\frac{3t}{1+t^3}$	2.22	$t+\frac{2}{t}$	$t+\frac{2}{t^2}$
2.8	t^2	$2t^3$	2.23	$2t-t^2$	$3t-t^3$
2.9	$\frac{t^2}{1+t^2}$	$\frac{t^3}{1+t^2}$	2.24	$\frac{t^2}{1+t^2}$	$\frac{t(1-t^2)}{1+t^2}$
2.10	$\cos^2 t+\cos t$	$\cos t\sin t+\sin t$	2.25	$\cos t(1+\cos t)$	$\sin t(1+\cos t)$

2.11	$\frac{t^2}{1+t^3}$	$-\frac{t}{1+t^3}$	2.26	$\frac{3t^2}{1+t^3}$	$-\frac{3t}{1+t^3}$
2.12	t^2	t^3	2.27	$t-t^2$	$2t-t^3$
2.13	$-t^2$	t^3	2.28	$1-2t$	$2-t^2$
2.14	$3\cos t$	$4\sin t$	2.29	$5\cos t$	$2\sin t$
2.15	$1-t$	$1-t^2$	2.30	te^t	te^{-t}

Task 3. Construct graph of function $\rho = \rho(\varphi)$ in Polar system of coordinates.

	$\rho(\varphi)$		$\rho(\varphi)$		$\rho(\varphi)$
3.1	3φ	3.11	$-2ctg\varphi$	3.21	$4\sqrt{\cos 2\varphi}$
3.2	$2ctg\varphi$	3.12	$2\cos\varphi+3$	3.22	$2\sqrt{\cos 2\varphi}$
3.3	$2\cos\varphi+2$	3.13	$2\sin 3\varphi$	3.23	$2\cos 2\varphi$
3.4	$\frac{3}{\varphi}+3$	3.14	$\frac{2}{\sin\varphi}+2$	3.24	$\frac{3}{\varphi^2}$
3.5	$2\cos\varphi+1$	3.15	$2\varphi+1$	3.25	2^{φ}
3.6	$5\sin\frac{\varphi}{3}$	3.16	$\frac{2}{\cos\varphi}+1$	3.26	$\frac{1}{\varphi^2}$
3.7	$\frac{1}{\varphi}+2$	3.17	$\frac{2}{\sin\varphi}+3$	3.27	$\frac{2}{\sin\varphi}+1$
3.8	$2tg\varphi$	3.18	3^{φ}	3.28	$2\sin 6\varphi$
3.9	$5\sin\frac{4\varphi}{3}$	3.19	$\frac{2}{\cos\varphi}+3$	3.29	$\frac{2}{\cos\varphi}$
3.10	$2(1-\cos\varphi)$	3.20	$1+tg\varphi$	3.30	$2\cos 6\varphi$

Task 4. Construct graph of piecewise continuous function *f(x)* (i.e. function which is given by different analytical expressions on different intervals of definition domain).

f(x)	*f(x)*
4.1 $\begin{cases} x+3, & x\le -1 \\ x^2+2, & -1<x\le 1 \\ 2x, & x>1 \end{cases}$	4.2 $\begin{cases} x+1, & x\le 0 \\ (x+1)^2, & 0<x\le 2 \\ -x+4, & x>2 \end{cases}$

4.3 $\begin{cases} x+2, & x \le -1 \\ x^2+1, & -1 < x \le 1 \\ -x+3, & x > 1 \end{cases}$	4.4 $\begin{cases} -x, & x \le 0 \\ -(x-1)^2, & 0 < x \le 2 \\ -x+4, & x > 2 \end{cases}$
4.5 $\begin{cases} -2(x+1), & x \le -1 \\ (x+1)^3, & -1 < x \le 1 \\ x, & x > 1 \end{cases}$	4.6 $\begin{cases} -x, & x \le 0 \\ x^2, & 0 < x < 2 \\ x+1, & x \ge 2 \end{cases}$
4.7 $\begin{cases} x^2+1, & x \le 1 \\ 2x, & 1 < x \le 3 \\ x+2, & x > 3 \end{cases}$	4.8 $\begin{cases} x-4, & x \le 0 \\ x+1, & 0 < x \le 5 \\ x+4, & x > 5 \end{cases}$
4.9 $\begin{cases} x-2, & x \le -1 \\ (x+1)^2, & -1 < x \le 2 \\ x+3, & x > 2 \end{cases}$	4.10 $\begin{cases} 2x^2, & x \le 0 \\ x, & 0 < x \le 1 \\ x+2, & x > 1 \end{cases}$
4.11 $\begin{cases} \sin x, & x < 0 \\ x, & 0 \le x \le \pi \\ 4, & x > \pi \end{cases}$	4.12 $\begin{cases} \cos x, & x \le 0 \\ 2, & 0 < x \le \pi \\ x, & x > \pi \end{cases}$
4.13 $\begin{cases} x+1, & x \le 0 \\ x^2-1, & 0 < x \le 2 \\ 4, & x > 2 \end{cases}$	4.14 $\begin{cases} x+1, & x \le 0 \\ x^2-1, & 0 < x < 1 \\ -x, & x \ge 1 \end{cases}$
4.15 $\begin{cases} -x, & x \le 0 \\ x^2+1, & 0 < x \le 2 \\ x+1, & x > 2 \end{cases}$	4.16 $\begin{cases} x+3, & x \le 0 \\ 1, & 0 < x \le 2 \\ x^2-3, & x > 2 \end{cases}$
4.17 $\begin{cases} x-1, & x \le 0 \\ \cos x, & 0 < x \le \pi \\ 3, & x > \pi \end{cases}$	4.18 $\begin{cases} -x+1, & x \le -1 \\ 2, & -1 < x \le 1 \\ \ln x, & x > 1 \end{cases}$
4.19 $\begin{cases} x+1, & x \le 0 \\ 2^x, & 0 < x \le 2 \\ 1, & x > 2 \end{cases}$	4.20 $\begin{cases} -x+1, & x \le 0 \\ x^2, & 0 < x \le 1 \\ 3, & x > 1 \end{cases}$
4.21 $\begin{cases} 2x+1, & x \le -2 \\ x^2, & -2 < x \le 2 \\ 4, & x > 2 \end{cases}$	4.22 $\begin{cases} x^3, & x \le -1 \\ x-1, & -1 < x \le 2 \\ -x+3, & x > 2 \end{cases}$

4.23 $\begin{cases} x-2, & x \le -3 \\ x^2-1, & -3 < x \le 2 \\ 0, & x > 2 \end{cases}$	4.24 $\begin{cases} e^x, & x \le 0 \\ x, & 0 < x \le 2 \\ 3, & x > 2 \end{cases}$
4.25 $\begin{cases} -x, & x \le -1 \\ (x+1)^2, & -1 < x < 2 \\ 5, & x \ge 2 \end{cases}$	4.26 $\begin{cases} x^2+1, & x \le 0 \\ 2x+3, & 0 < x \le 2 \\ 1, & x > 2 \end{cases}$
4.27 $\begin{cases} x+1, & x \le 0 \\ \sin x, & 0 < x \le \pi \\ 2, & x > \pi \end{cases}$	4.28 $\begin{cases} 3x+1, & x \le 0 \\ \cos x, & 0 < x \le \pi \\ 1, & x > \pi \end{cases}$
4.29 $\begin{cases} 3x-1, & x \le -1 \\ x^3, & -1 < x \le 2 \\ 2, & x > 2 \end{cases}$	4.30 $\begin{cases} 4, & x \le -1 \\ (x+1)^2, & -1 < x \le 1 \\ \ln x, & x > 1 \end{cases}$

Task 5. Function *f(x)* is given:

- decompose *f(x)* by sum of partial fractions;
- find indefinite integral $\int f(x)dx$;
- calculate definite integral $\int_a^b f(x)dx$.

	f(x)	*a*	*b*		*f(x)*	*a*	*b*
5.1	$\frac{x^2+4x+4}{x(x-1)^2}$	3	5	5.16	$\frac{1}{x(x^2+2)}$	1	2
5.2	$\frac{1}{x(x-1)}$	2	4	5.17	$\frac{x-1}{(x^2+1)^3}$	0	3
5.3	$\frac{3x^2+2x-1}{(x+2)}$	-1	3	5.18	$\frac{x^3+2}{x^3-4x}$	4	5
5.4	$\frac{2x^2-1}{x^3-5x^2+6x}$	0	2	5.19	$\frac{x}{(x-1)(x^2+1)}$	4	5
5.5	$\frac{2x-5}{(x+4)^3}$	0	3	5.20	$\frac{x^2-x+4}{(x-2)(x+1)}$	3	4
5.6	$\frac{5x-13}{(x^2-5x+6)^2}$	0	1	5.21	$\frac{x^3-2x^2-3x+4}{x^4-4x^3+4x^2}$	3	5
5.7	$\frac{x^2+9x+2}{(x+2)(x^2-1)}$	3	5	5.22	$\frac{-x^3+13x^2-27x+25}{(x-2)^3(x+3)}$	3	5

5.8	$\frac{x}{2x^2-3x-2}$	6	8	5.23	$\frac{2x^2+41x-91}{(x+3)(x-4)(x-1)}$	8	10
5.9	$\frac{x^3-6x^2+11x-5}{(x-2)^4}$	3	4	5.24	$\frac{x^3-1}{4x^3-x}$	5	7
5.10	$\frac{1}{6x^3-7x^2-3x}$	3	15	5.25	$\frac{x^3-6x^2+9x+7}{(x-2)(x-5)^2}$	-2	-1
5.11	$\frac{x^5+x^4-8}{x^3-4x}$	-3	0	5.26	$\frac{x^5}{(x-1)^2(x^2-1)^2}$	2	5
5.12	$\frac{x}{x^4-3x^2+2}$	3	9	5.27	$\frac{2x^2}{x^3-5x^2}$	7	10
5.13	$\frac{x^2-2x+3}{x-9}$	0	1	5.28	$\frac{7x^3-9}{x^4-5x^3+6x^2}$	5	6
5.14	$\frac{x^2}{x^3+5x^2+8x+4}$	0	2	5.29	$\frac{x^3+1}{x^3-x^2}$	3	9
5.15	$\frac{1}{x^4-x^2}$	2	15	5.30	$\frac{x^2}{(x+2)^2(x+4)^2}$	3	7

Task 6. Perform the full research of function *f(x)*, i.e. find:
- definition domain and break points;
- function graph asymptotes;
- cross points of graph and coordinates axes;
- evenness and oddness;
- monotony intervals and extremum points;
- concavity and convexity intervals, inflection points;
- construct the graph.

	$f(x)$		$f(x)$		$f(x)$
6.1	$\frac{x^2-2x+2}{x-1}$	6.2	$\frac{x+1}{(x-1)^2}$	6.3	$\frac{2}{x^2+2x}$
6.4	$\frac{x}{9-x}$	6.5	$\frac{4x-x^2-4}{x}$	6.6	$\frac{x^2}{4x^2-1}$
6.7	$\frac{7}{x^2-1}$	6.8	$\frac{8}{x^2-4x}$	6.9	$\frac{x^2+2x-15}{x-1}$
6.10	$\frac{x^3}{x^2-x+1}$	6.11	$\frac{x^3}{x^2-1}$	6.12	$\frac{x^2-3x-10}{x-4}$
6.13	$\frac{x^2-x-1}{x^2-2x}$	6.14	$\frac{(x-2)^2}{x+1}$	6.15	$\frac{x^2-5x+6}{x-4}$
6.16	$\frac{x^2+3}{x}$	6.17	$\frac{x^2+6}{x^2+1}$	6.18	$\frac{x^2-4x-10}{x-5}$

6.19	$\frac{7}{x^2-3x}$	6.20	$\frac{x^2-3x+2}{x+1}$	6.21	$\frac{2x-1}{(x-1)^2}$
6.22	$\frac{x^5}{x^4-1}$	6.23	$\frac{x^3+4}{x^2}$	6.24	$\frac{4}{x^2+7x}$
6.25	$\frac{x^3}{x^4-1}$	6.26	$\frac{x^3-8}{x^2}$	6.27	$x^2+\frac{1}{x^2}$
6.28	$\frac{5x^4+3}{x}$	6.29	$\frac{4-2x}{1-x^2}$	6.30	$\frac{5x}{4-x^2}$

Questions to the laboratory work №2.

1. How to construct graph of function in rectangular system of coordinates by such a way, that it would be comfortable for reading (i.e. how to change scale, represent coordinate axes, etc)?
2. How to construct several graphs (for example graphs of function, tangent and normal) in one system of coordinate?
3. How to change scale in polar system of coordinates?
4. By means of which bar we can find limits, derivatives and integrals?
5. How to find and represent vertical, horizontal and incline asymptotes of function graph?
6. How to construct graph of piecewise continuous function?
7. How to construct graph of parametrically given function?
8. What should be included in tables to define extremums and monotony intervals?

Recommendations for performing of laboratory work №2

Task 1. Function $f(x)=\cos 2x+1$ and point $x_0=\frac{\pi}{3}$ are given:

- find limit *f(x)* at point x_0;
- find derivatives $f'(x)$, $f'(x_0)$, $f''(x)$, $f''(x_0)$;
- construct graph of function *y=f(x)* in Cartesian system of coordinates; construct tangent and normal to the graph at the indicated point.

Recommendation. Input the given function from the keyboard and point x_0:

- call sign *Limits* from the bar *Calculus* , fill it in and calculate as it indicated below;
- call signs *Derivative* and *Second derivative* from the bar *Calculus,* sign it and calculate derivatives at point x_0 as it indicated below;

- input from the keyboard slope of tangent *k*, equation of tangent *yk(x)* and normal *yn(x)*. Call *X-Y plot* from the bar *Graph*, fill in labels (write three functions at *Oy* axe with comma) and click the free place out of the graph field.

The task performing:

$f(x) := \cos(2x) + 1 \qquad x0 := \frac{\pi}{3}$

1) $\lim_{x \to x0} f(x) \to \frac{1}{2}$

2) $\frac{d}{dx} f(x) \to -2\cdot\sin(2\cdot x) \text{ expand},x \to -4\cdot\cos(x)\cdot\sin(x)$;

$\frac{d}{dx0} f(x0) \to -\sqrt{3} \qquad \frac{d^2}{dx^2} f(x) \to -4\cdot\cos(2\cdot x) \qquad \frac{d^2}{dx0^2} f(x0) \to 2,$

3) $k := \frac{d}{dx0} f(x0) \qquad yk(x) := k\cdot(x - x0) + f(x0)$,

$yn(x) := \frac{-1}{k}\cdot(x - x0) + f(x0).$

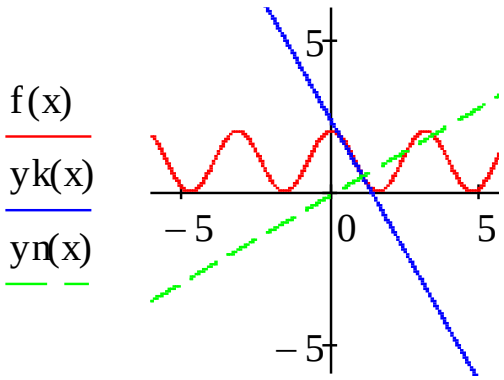


Figure 1

Task 2. Construct graph of function *y=f(x)*, given parametrically $\begin{cases} x = t - 2\sin t \\ y = 1 - 2\cos t \end{cases}$.

Recommendation. Input the given function from the keyboard, call *Graph, X-Y plot* from the bar, fill in labels, write *x(t)* by *x*-axis, and *y(t)* – by *y*-axis, click by free space out of the graph field.

Performing of the task:

$x(t) := t - 2\cdot\sin(t)$;

$y(t) := 1 - 2\cdot\cos(t)$.

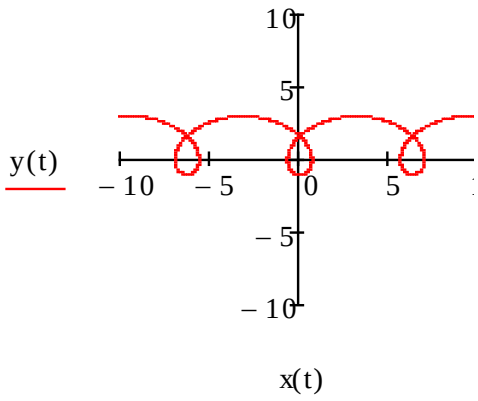


Figure 2

Task 3. Construct graph of function $\rho = tg2\varphi$ in polar system of coordinates.

Recommendation. Input given function from the keyboard, call *Graph, Polar plot* from the bar, fill in labels and click the free space out of the graph field.

Task performing

$\rho(\phi) := \tan(2\cdot\phi)$.

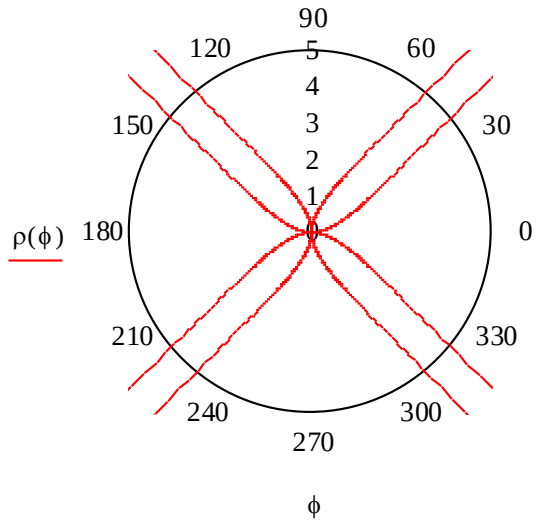


Figure 3

Task 4. Construct graph of piecewise continuous function

$$f(x)=\begin{cases} -x, \ if \ \ x\le 0 \\ \sin 2x, \ if \ \ 0<x\le 2 \\ 1, \ if \ \ x>2 \end{cases}.$$

Recommendation. Call the bar *Programming*. Call *Add line* from this bar and type the given function. Press *Add line* several times if there are not sufficient labels.

Performing of the task:

$$f(x):=\begin{cases} -x & \text{if } x\le 0 \\ \sin(2\cdot x) & \text{if } 0<x\le 2 \\ 1 & \text{if } x>2 \end{cases}$$

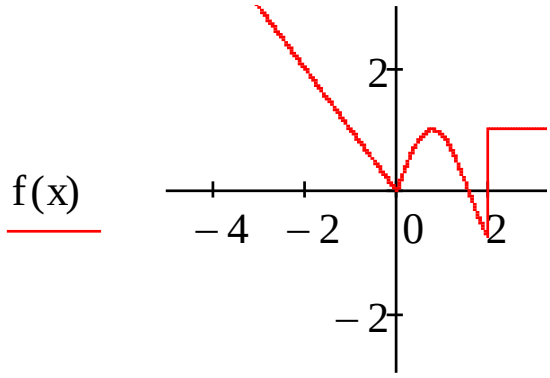


Figure 4

Task 5. Function $f(x)=\dfrac{x^2-3x+7}{(x-1)^2(x+1)}$ is given:

- decompose *f(x)* by sum of partial fractions;
- find indefinite integral $\int f(x)dx$;
- calculate definite integral $\int_2^3 f(x)dx$.

Recommendation. Enter the expression from the keyboard. Call the bar *Simbolic, Parfrac.* To calculate integrals, call the bar *Calculus.* To calculate definite integral in decimals, mark its value by blue angular cursor and press the button "=".

Performing of the task:

$$f(x) := \frac{x^2 - 3\cdot x + 7}{(x-1)^2\cdot(x+1)}$$

1) $f(x)\ \text{parfrac} \rightarrow \frac{11}{4\cdot(x+1)} - \frac{7}{4\cdot(x-1)} + \frac{5}{2\cdot(x-1)^2}.$

2) $\int f(x)\,dx \rightarrow \frac{11\cdot\ln(x+1)}{4} - \frac{7\cdot\ln(x-1)}{4} - \frac{5}{2\cdot(x-1)}.$

3) $\int_2^3 f(x)\,dx \rightarrow \frac{15\cdot\ln(2)}{4} - \frac{11\cdot\ln(3)}{4} + \frac{5}{4} = 0.828$

.

Task 6. Function $f(x)=\frac{x^3-3x}{x^2-1}$ is given. Find:

- definition domain and break points;
- function graph asymptotes;
- cross points of graph and coordinates axes;
- evenness and oddness;
- monotony intervals and extremum points;
- concavity and convexity intervals, inflection points;
- construct the graph.

Recommendation.

1) Find break points for *f(x)* (i.e. points, where the function is not defined), write domain of definition.

2) Calculate one-sided limits at break points. Write equations of vertical asymptotes ($x=a$, if a – is a break point). One-sided limits define behavior of function nearby break point. Calculate limits $k=\lim_{x\to\infty}\frac{f(x)}{x}$, $b=\lim_{x\to\infty}\left(f(x)-kx\right)$. Work out equation of slant (or horizontal, if k=0) asymptote $y=kx+b$.

3) Calculate value *f(0)* – it is a cross point with *y*-axis. Solve the equation *f(x)*=0 – its roots are points of intersection with *x*-axis.

4) If *f(-x)=f(x)*, then function is even, if *f(-x)=-f(x)*, then it is odd.

5) Find derivative of function $f'(x)$. Find critical (or extremum) points, i.e. points where $f'(x)=0$ or it doesn't exist. Break the domain of definition by critical points on sections; find derivative sign in each section; if it is "plus" then the function increases in this section, if "minus" – decreases. All data insert into the table.

6) Find second order derivative $f''(x)$. Find points, where $f''(x)=0$ or it doesn't exist. Break the domain of definition by these points onto sections and define second derivative signs in each section; if it is "plus" then graph of the function is concave in this section; if "minus" – then convex. Insert all data into the table.

7) Define function and its asymptotes, vertical asymptote *x=a* define by $f1(x)=x+a$. Inputting labels by ordinate axis in Cartesian system, enter names of function and asymptotes by comma, inputting arguments of these function by abscissa axe, enter them in the same order by comma; for vertical asymptote – the argument is *a*.

Performing of the task:

1) f(x)= $\frac{x^3-3x}{x^2-1}$, $x^2-1\neq 0$, $x=\pm 1$ – break points. D(f): $(-\infty,-1)\cup(-1,1)\cup(1,+\infty)$.

2) f(x)= $\frac{x^3-3x}{x^2-1}$, since

$\lim_{x\to -1^+} f(x) \to -\infty$ $\lim_{x\to 1^+} f(x) \to -\infty$

$\lim_{x\to -1^-} f(x) \to \infty$ $\lim_{x\to 1^-} f(x) \to \infty$

then $x=\pm 1$ are vertical asymptotes.

Let's find slant asymptote *y=kx+b:*

$$k := \lim_{x\to\infty} \frac{x^3-3x}{(x^2-1)\cdot x} \to 1$$

$$b := \lim_{x\to\infty} \left(\frac{x^3-3x}{x^2-1}-x\right) \to 0$$

, so, *y=x* – is slant asymptote.

3) Cross point of the function graph and coordinates axes:

with OX: $y=0\to\frac{x^3-3x}{x^2-1}=0\to x^3-3x=0\to x=0, x=\pm\sqrt{3}\to(0,0),(\pm\sqrt{3},0)$;

with OY: $x=0 \to y=0 \to (0,0)$.

4) Since $f(-x)=\frac{(-x)^3-3(-x)}{(-x)^2-1}=-\frac{x^3-3x}{x^2-1}=-f(x)$, then the function is odd.

5) Let's find monotony intervals and extremum points:

$$\frac{d}{dx}f(x) \to \frac{\left(3\cdot x^2 - 3\right)}{\left(x^2 - 1\right)} - 2\cdot\frac{\left(x^3 - 3\cdot x\right)}{\left(x^2 - 1\right)^2}\cdot x \text{ simplify } \to \frac{\left(x^4 + 3\right)}{\left(x^2 - 1\right)^2},$$

since $y'=\frac{df(x)}{dx}>0 \forall x$, then the function *f(x)* increases everywhere in the domain of definition. There are no extremum points.

6) Let's find concavity / convexity intervals and inflection points as well:

$$f2(x) := \frac{d^2}{dx^2} f(x);$$

$$\frac{d^2}{dx^2} f(x) \to 6\cdot\frac{x}{\left(x^2 - 1\right)} - 4\cdot\frac{\left(3\cdot x^2 - 3\right)}{\left(x^2 - 1\right)^2}\cdot x + 8\cdot\frac{\left(x^3 - 3\cdot x\right)}{\left(x^2 - 1\right)^3}\cdot x^2 - 2\cdot\frac{\left(x^3 - 3\cdot x\right)}{\left(x^2 - 1\right)^2} \text{ simplify } \to -4\cdot x\cdot\frac{\left(x^2 + 3\right)}{\left(x^2 - 1\right)^3}$$

f2(x)=0:

$$-4\cdot x\cdot\frac{\left(x^2 + 3\right)}{\left(x^2 - 1\right)^3} \text{ solve, } x \to \begin{bmatrix} 0 \\ i\cdot\sqrt{3} \\ -i\cdot\sqrt{3} \end{bmatrix},$$ so, *x=0* can be an abscissa of the inflection point.

Let's fill in the table:

x	$(-\infty,-1)$	$(-1,0)$	0	$(0,1)$	$(1,\infty)$
y''	+	-	0	+	-
y	$\cup$	$\cap$	0	$\cup$	$\cap$

Defining of signs for second derivative at indicated intervals:

$f2(-2) = 2.074$ $\quad f2\left(\frac{-1}{2}\right) = -15.407$

$f2\left(\frac{1}{2}\right) = 15.407$ $\quad f2(2) = -2.074$

Thus, (0,0) – inflection point.

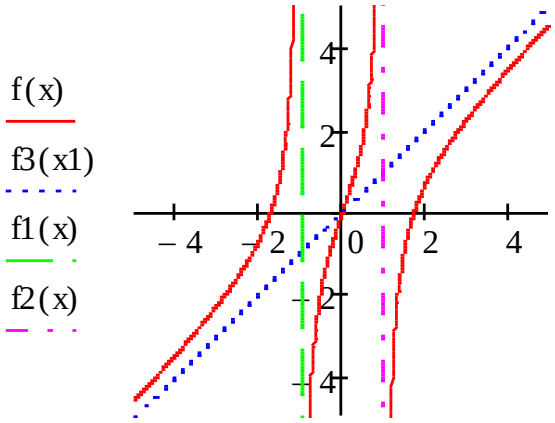
7) Let's construct the function graph and asymptotes:

$$f(x) := \frac{x^3 - 3 \cdot x}{x^2 - 1}$$

$$f1(x) := x - 1$$

$$f2(x) := x + 1$$

$$f3(x1) := x1$$



$$x, x1, -1, 1$$

Figure 5

3 Laboratory work №3

The aim of the laboratory work is to study application of MathCAD for probability theory different tasks solution – probabilities calculation; total probability formula; Bayes, Bernoulli, Poisson formulas; local and integral Laplace theorem; discrete and continuous random variables; basic distribution laws for random variables.

Task 1. There are *N* balls in the box. All balls have the same size and weight. *M* balls are white and the rest are black. All balls are mixed thoroughly. Find:

- relative frequency of white balls in the box;
- probability, that all *m* balls, taken at random from the box, will be white;
- probability, that among *m* balls, taken at random from the box, will be m_1 white balls.

	N	*M*	*m*	m_1		*N*	*M*	*m*	m_1
1.1	100	25	10	8	1.16	70	8	5	3

1.2	90	15	12	7	1.17	75	9	8	4
1.3	85	10	7	4	1.18	85	6	5	2
1.4	80	9	5	3	1.19	90	12	7	4
1.5	95	15	9	3	1.20	87	10	8	3
1.6	70	10	9	5	1.21	100	30	15	5
1.7	80	15	7	5	1.22	90	20	9	3
1.8	90	10	6	4	1.23	95	15	10	4
1.9	75	10	8	4	1.24	85	10	7	2
1.10	100	20	10	7	1.25	90	12	6	3
1.11	90	10	8	5	1.26	85	10	5	2
1.12	80	7	5	3	1.27	75	8	5	3
1.13	95	10	8	5	1.28	100	15	9	4
1.14	96	12	7	1	1.29	80	10	7	4
1.15	89	13	5	2	1.30	85	7	5	2

Task 2. At the assembling shop 1000 components from three shops are arrived: n_1 components are from shop №1, n_2 – from the second shop, and the rest - from the third shop. In the first, second and third shops, m_1, m_2 and m_3 non-standard components are produced respectively. One component is taken at random:

- find probability that it is non-standard;
- let taken component is non-standard. Find probability that it is produced in i –th shop (i=1,2,3).

	n_1	n_2	m_1	m_2	m_3	i
2.1	100	250	7	8	5	1
2.2	430	180	5	4	7	2
2.3	170	540	6	5	8	3
2.4	650	120	10	9	8	2
2.5	400	180	7	10	5	1
2.6	120	380	10	6	9	2
2.7	270	340	9	5	4	3
2.8	430	120	10	7	6	2
2.9	360	120	5	10	8	1
2.10	420	210	8	7	6	1
2.11	370	130	10	6	5	2
2.12	410	200	5	10	8	3
2.13	280	510	10	6	5	3
2.14	710	120	2	10	4	3
2.15	460	240	5	9	7	1
2.16	520	220	5	8	7	1
2.17	270	410	10	5	9	2
2.18	250	140	8	7	4	2
2.19	190	380	5	9	30	1

2.20	290	610	6	3	3	2
2.21	270	430	10	6	4	2
2.22	280	360	7	10	9	1
2.23	520	110	5	7	10	1
2.24	240	290	9	8	4	3
2.25	310	410	7	2	5	3
2.26	520	110	3	6	7	2
2.27	280	310	9	8	4	2
2.28	400	320	4	5	8	1
2.29	350	240	9	8	7	1
2.30	190	520	5	2	4	3

Task 3. n tests are produced. In each test the probability of occurrence of event A equals p. Find probability that event A occurs:

- exactly k_1 times;
- less than k_1 times;
- more than k_2 times;
- at least one time;
- from k_1 to k_2 times.

For a) – use Bernoulli formula, if possible; for b) – use local and integral Laplace theorem.

	n	k_1	k_2	p		n	k_1	k_2	p
3.1 a)	5	2	3	0,9	3.16 a)	8	3	7	0.6
b)	100	80	90		b)	100	70	95	
3.2 a)	4	2	3	0.8	3.17 a)	7	5	6	0.8
b)	100	85	95		b)	100	50	60	
3.3 a)	9	5	7	0.6	3.18 a)	8	4	7	0.8
b)	100	83	93		b)	100	70	80	
3.4 a)	10	4	8	0.4	3.19 a)	6	3	5	0.3
b)	100	65	75		b)	100	62	82	
3.5 a)	11	7	9	0.2	3.20 a)	7	4	6	0.3
b)	100	40	50		b)	100	55	75	
3.6 a)	5	3	4	0.4	3.21 a)	5	2	4	0.4
b)	100	80	95		b)	100	40	60	
3.7 a)	7	3	6	0.6	3.22 a)	4	2	3	0.8
b)	100	50	70		b)	100	50	80	
3.8 a)	9	4	7	0.3	3.23 a)	5	3	4	0.7
b)	100	45	80		b)	200	80	170	
3.9 a)	10	3	6	0.4	3.24 a)	9	4	6	0,9
b)	100	35	70		b)	100	40	65	
3.10 a)	11	5	8	0.8	3.25 a)	8	4	7	0.8

b)	100	40	65		b)	100	20	60	
3.11 a)	7	4	4	0.7	3.26 a)	6	3	5	0.6
b)	100	50	80		b)	200	85	150	
3.12 a)	8	3	6	0.8	3.27 a)	8	3	7	0.4
b)	100	40	79		b)	100	35	70	
3.13 a)	6	3	4	0.7	3.28 a)	7	5	6	0.2
b)	200	45	75		b)	100	47	80	
3.14 a)	4	2	3	0,9	3.29 a)	5	2	4	0.4
b)	100	55	75		b)	100	62	82	
3.15 a)	6	4	5	0.8	3.30 a)	4	2	3	0.6
b)	100	50	70		b)	100	90	95	

Task 4. Discrete random variable *X* is given by distribution series. Find:

- its distribution function *F(x)*, construct graph of *F(x);*
- mathematical expectation (expectation value), variance (dispersion), root-mean-square deviation, mode;
- probability, that *X* will hit into interval *(a;b).*

	X	x_1	x_2	x_3	x_4	x_5	x_6	a	b
	P	p_1	p_2	p_3	p_4	p_5	p_6		
4.1	X	0	1	2	4	6	9	-2	7
	P	0.05	0.15	0.3	0.25	0.15	0.1		
4.2	X	-3	-2	-1	0	2	4	-1	3
	P	0.15	0.3	0.02	0.14	0.18	0.31		
4.3	X	1	2	3	5	7	8	-3	6
	P	0.3	0.14	0.16	0.1	0.2	0.1		
4.4	X	-4	-3	-2	0	1	2	0	1
	P	0.2	0.08	0.23	0.27	0.12	0.1		
4.5	X	1	2	4	5	7	9	3	8
	P	0.19	0.21	0.06	0.14	0.12	0.28		
4.6	X	-1	0	2	3	5	7	-4	4
	P	0.26	0.14	0.07	0.2	0.03	0.3		
4.7	X	-2	-1	0	3	5	7	1	6
	P	0.18	0.09	0.01	0.2	0.22	0.3		
4.8	X	1	2	4	5	6	8	0	6
	P	0.3	0.17	0.13	0.1	0.2	0.1		
4.9	X	1	2	3	4	7	9	5	8
	P	0.11	0.29	0.06	0.14	0.17	0.23		
4.10	X	0	1	2	3	7	9	4	8
	P	0.06	0.14	0.3	0.25	0.15	0.1		
4.11	X	-3	-2	0	1	2	4	-1	3
	P	0.15	0.3	0.01	0.14	0.19	0.31		

4.12	X	-1	0	3	5	7	8	1	6
	P	0.25	0.14	0.16	0.1	0.2	0.15		
4.13	X	-4	-3	-2	0	2	4	-1	3
	P	0.2	0.07	0.24	0.26	0.13	0.1		
4.14	X	-3	-1	0	3	4	7	-2	6
	P	0.12	0.09	0.01	0.2	0.28	0.3		
4.15	X	-1	0	1	3	7	8	2	6
	P	0.26	0.14	0.15	0.2	0.3	0.15		
4.16	X	-2	-1	0	1	2	7	-3	5
	P	0.17	0.09	0.01	0.3	0.23	0.2		
4.17	X	1	2	3	5	6	7	0	4
	P	0.1	0.14	0.16	0.1	0.2	0.3		
4.18	X	-3	-1	0	3	5	6	-2	4
	P	0.16	0.09	0.01	0.3	0.24	0.2		
4.19	X	1	2	5	6	7	8	3	6
	P	0.2	0.15	0.15	0.1	0.3	0.1		
4.20	X	-1	0	2	4	7	8	1	5
	P	0.23	0.18	0.12	0.2	0.1	0.17		
4.21	X	1	2	4	5	6	8	0	7
	P	0.3	0.14	0.16	0.03	0.2	0.17		
4.22	X	-4	-3	-1	0	1	3	-2	2
	P	0.2	0.03	0.24	0.26	0.17	0.1		
4.23	X	1	2	3	4	7	9	0	8
	P	0.17	0.23	0.09	0.11	0.12	0.28		
4.24	X	0	1	3	5	7	8	2	6
	P	0.2	0.14	0.16	0.12	0.3	0.08		
4.25	X	-5	-3	-2	0	1	3	-4	2
	P	0.2	0.06	0.21	0.29	0.14	0.1		
4.26	X	1	2	3	5	8	9	4	7
	P	0.18	0.22	0.05	0.15	0.12	0.28		
4.27	X	1	3	4	5	7	8	2	6
	P	0.3	0.16	0.14	0.01	0.2	0.19		
4.28	X	-5	-3	-1	0	1	3	-4	2
	P	0.1	0.03	0.14	0.36	0.17	0.2		
4.29	X	0	2	3	4	6	8	1	7
	P	0.26	0.14	0.05	0.15	0.12	0.28		
4.30	X	-1	0	2	3	7	8	1	6
	P	0.21	0.16	0.14	0.1	0.2	0.19		

Task 5. Continuous random variable X is given by distribution density function $f(x)$. Find:

- its distribution function $F(x)$;

- mathematical expectation, variance, root-mean-square deviation, mode, median;

- probability, that X will hit into interval $(a;b)$.

Construct graphs of $F(x)$ and $f(x)$.

	$f(x)$	a	b		$f(x)$	a	b
5.1	$\begin{cases} 0, x \le 0, x > 4 \\ \frac{x}{8}, 0 < x \le 4 \end{cases}$	1	3	5.16	$\begin{cases} 0, x \le 0, x > 3 \\ \frac{1}{3}(1-\frac{x}{3}), 0 < x \le 3 \end{cases}$	-1	2
5.2	$\begin{cases} 0, x \le -3, x > -2 \\ \frac{6}{x^2}, -3 < x \le -2 \end{cases}$	-2,5	0	5.17	$\begin{cases} 0, x \le 0, x > \frac{\pi}{6} \\ 4\sin 2x, 0 < x \le \frac{\pi}{6} \end{cases}$	0	$\frac{\pi}{12}$
5.3	$\begin{cases} 0, x \le -\frac{\pi}{2}, x > \frac{\pi}{2} \\ 0,5\cos x, -\frac{\pi}{2} < x \le \frac{\pi}{2} \end{cases}$	0	$\frac{\pi}{4}$	5.18	$\begin{cases} 0, x \le 1, x > 2 \\ \frac{2}{x^2}, 1 < x \le 2 \end{cases}$	0	1,5
5.4	$\begin{cases} 0, x \le 0, x > 1 \\ \frac{4}{\pi(1+x^2)}, 0 < x \le 1 \end{cases}$	0	$\frac{\sqrt{3}}{3}$	5.19	$\begin{cases} 0, x \le -2, x > 3 \\ \frac{2x}{5}, -2 < x \le 3 \end{cases}$	1	2,5
5.5	$\begin{cases} 0, x \le 0, x \ge 1 \\ \frac{2}{\pi\sqrt{1-x^2}}, 0 < x < 1 \end{cases}$	0	$\frac{1}{2}$	5.20	$\begin{cases} 0, x \le 0, x > \frac{1}{\sqrt{3}} \\ \frac{6}{\pi(1+x^2)}, 0 < x \le \frac{1}{\sqrt{3}} \end{cases}$	0,1	1
5.6	$\begin{cases} 0, x \le 0, x > \pi \\ 0,5\sin x, 0 < x \le \pi \end{cases}$	0	$\frac{\pi}{2}$	5.21	$\begin{cases} 0, x \le -1, x > 2 \\ \frac{1}{3}(x+1)^2, -1 < x \le 2 \end{cases}$	0	1
5.7	$\begin{cases} 0, x \le 0, x > 2 \\ \frac{x+2}{6}, 0 < x \le 2 \end{cases}$	1	2	5.22	$\begin{cases} 0, x \le 0, x \ge \frac{1}{2} \\ \frac{6}{\pi\sqrt{1-x^2}}, 0 < x < \frac{1}{2} \end{cases}$	$\frac{1}{4}$	1
5.8	$\begin{cases} 0, x \le -4, x > 5 \\ \frac{2x}{9}, -4 < x \le 5 \end{cases}$	3	4,5	5.23	$\begin{cases} 0, x \le \frac{\pi}{2}, x > \frac{5\pi}{6} \\ -2\cos x, \frac{\pi}{2} < x \le \frac{5\pi}{6} \end{cases}$	0	$\frac{2\pi}{3}$
5.9	$\begin{cases} 0, x \le 3, x > 5 \\ \frac{7,5}{x^2}, 3 < x \le 5 \end{cases}$	2	4	5.24	$\begin{cases} 0, x \le 1, x > 2 \\ 2x-2, 1 < x \le 2 \end{cases}$	0	1,5
5.10	$\begin{cases} 0, x \le 1, x > 2 \\ 3(x-1)^2, 1 < x \le 2 \end{cases}$	1,5	2	5.25	$\begin{cases} 0, x \le 0, x > \frac{\pi}{6} \\ 6\sin 3x, 0 < x \le \frac{\pi}{6} \end{cases}$	0	$\frac{\pi}{12}$

5.11	$\begin{cases} 0, x \le 0, x > \frac{\pi}{4} \\ 2\cos 2x, 0 < x \le \frac{\pi}{4} \end{cases}$	$\frac{\pi}{8}$	$\frac{\pi}{4}$	5.26	$\begin{cases} 0, x \le -2, x > 2 \\ \frac{1}{2\pi}\sqrt{4-x^2}, -2 < x \le 2 \end{cases}$	0	1
5.12	$\begin{cases} 0, x \le 0, x > 4 \\ \frac{1}{2}(1-\frac{x}{4}), 0 < x \le 4 \end{cases}$	1	3	5.27	$\begin{cases} 0, x \le 0, x > 5 \\ \frac{2}{5}(1-\frac{x}{5}), 0 < x \le 5 \end{cases}$	1	4
5.13	$\begin{cases} 0, x \le 0, x > 2 \\ \frac{x+1}{4}, 0 < x \le 2 \end{cases}$	-1	1	5.28	$\begin{cases} 0, x \le 0, x > \frac{\pi}{6} \\ 3\cos 3x, 0 < x \le \frac{\pi}{6} \end{cases}$	$\frac{\pi}{12}$	$\frac{\pi}{9}$
5.14	$\begin{cases} 0, x \le 0, x > 1 \\ 3x^2, 0 < x \le 1 \end{cases}$	0,2	1,2	5.29	$\begin{cases} 0, x \le -3, x > 3 \\ \frac{1}{2\pi}\sqrt{9-x^2}, -3 < x \le 3 \end{cases}$	0	2
5.15	$\begin{cases} 0, x \le 0, x > \frac{\pi}{3} \\ 2\sin x, 0 < x \le \frac{\pi}{3} \end{cases}$	0	$\frac{\pi}{6}$	5.30	$\begin{cases} 0, x \le 1, x > 4 \\ \frac{2x}{15}, 1 < x \le 4 \end{cases}$	2	3

Task 6. Equipment consists of *n* elements. One element failure probability for a time *t* doesn't depend on status of other elements and equals *p*. Find:

- distribution law for number of failure elements;
- probability, that not less than *m* elements will fail.

	N	*m*	*p*		*N*	*m*	*p*
6.1	2000	4	0,001	6.16	1500	3	0,002
6.2	1000	5	0,007	6.17	2000	4	0,001
6.3	3000	7	0,004	6.18	1000	5	0,007
6.4	2000	5	0,002	6.19	3500	1	0,002
6.5	1000	6	0,005	6.20	2000	5	0,001
6.6	5000	2	0,001	6.21	1000	6	0,005
6.7	2000	4	0,001	6.22	4500	2	0,003
6.8	1500	5	0,008	6.23	2000	4	0,001
6.9	3500	7	0,004	6.24	1000	5	0,007
6.10	2000	2	0,003	6.25	3000	7	0,004
6.11	1500	6	0,005	6.26	2000	5	0,002
6.12	4000	2	0,006	6.27	1000	6	0,005
6.13	8000	2	0,001	6.28	6500	8	0,007
6.14	6500	6	0,002	6.29	7000	6	0,002
6.15	3000	2	0,005	6.30	5500	9	0,004

7. Random measurement error obeys the normal distribution law with parameters *a* and σ. Find:

- distribution density function *f(x);*
- distribution function *F(x);*
- mathematical expectation, variance;
- probability of hitting in interval (α, β);
- probability, that measurement will be made with error not more than δ by absolute value.

Construct graphs of *f(x)* and *F(x).*

	a	σ	α	β	δ		a	σ	α	β	δ
7.1	10	1	8	14	2	7.16	10	2	9	14	2
7.2	12	2	7	14	3	7.17	12	4	5	14	3
7.3	14	3	10	15	5	7.18	14	1	9	15	5
7.4	11	5	9	12	3	7.19	11	6	8	12	3
7.5	13	2	6	13	2	7.20	13	4	6	17	2
7.6	12	3	7	15	4	7.21	12	9	8	15	4
7.7	10	2	8	17	2	7.22	10	3	6	17	2
7.8	12	4	6	14	6	7.23	12	5	6	13	6
7.9	14	6	11	19	5	7.24	14	2	12	19	5
7.10	15	5	8	12	3	7.25	15	3	4	12	3
7.11	17	4	6	14	2	7.26	17	1	5	14	2
7.12	12	5	7	18	4	7.27	12	4	9	18	4
7.13	18	5	6	12	3	7.28	11	3	4	12	3
7.14	10	4	6	15	2	7.29	17	2	5	19	5
7.15	12	3	5	18	4	7.30	13	5	6	18	3

Questions to the laboratory work №3.

1. Formulas for calculation of number of combinations and arrangements from *n* elements by *m*; number of permutations from *n* elements?
2. Which built-in-functions are used in MathCAD to calculate number of combinations and arrangements?
3. How to calculate number of permutations from *n* elements, i.e. *n*-factorial, in MathCAD?
4. Which approaches to definition of random event probability there exist in probability theory?
5. Classical definition of random event probability?
6. Formulate conditions, when total probability formula and Bayes formula are used?
7. Give exact and approximate formulas to calculate event occurrence probability (exactly *m* times in *n* tests, when event occurrence probability in each test is the same).

8. Prove importance of Laplace function in probability theory, where and how it is used?

Recommendations to performing of laboratory work №3

Task 1. There are 120 balls in the box, 40 of them are white. Find:

- relative frequency of white balls in the box;
- probability, that all 20 balls, taken at random from the box, will be white;
- probability, that among 20 balls, taken at random from the box, 9 balls will be white.

Recommendation. Relative frequency of event *A* is found by formula $P^*(A) = m/n$, where *n* is a total number of tests, *m* is number of event *A* occurrence; probability of event *A* occurrence is found by formula $P(A) = m/n$, where *m* – is number of tests, favorable to event *A* occurrence, *n* – is total number or tests; in items 2) and 3) the total number of tests is the same $n = C_{120}^{20}$; in item 2) $m = C_{40}^{20}$; in item 3) $m = C_{40}^{9} \cdot C_{80}^{11}$.

To calculate number of combinations in MathCAD the function *combin* is used; *combin(Q,R)* is entered as user's function *C(Q,R)*, and it allows to obtain values of combination with arbitrary *Q* and *R*.

Performing of the task

$C(Q,R) := combin(Q,R)$

$C(120,20) = 2.946 \times 10^{22}$ $\quad$ $C(40,20) = 1.378 \times 10^{11}$

$C(40,9) = 2.734 \times 10^{8}$ $\quad$ $C(80,11) = 1.048 \times 10^{13}$

$$\frac{C(40,9) \cdot C(80,11)}{C(120,20)} = 0.097$$

So, 1) $P^*(A) = 40/120 = 1/3$;

2) $P(A) = m/n = C_{40}^{20} / C_{120}^{20} = 4{,}679 \times 10^{-12}$;

3) $P(A) = m/n = C_{40}^{9} \cdot C_{80}^{11} / C_{120}^{20} = 0{,}097$.

Task 2. At the assembling shop 1000 components from three shops are arrived: 100 components - from shop №1, 300 – from the second shop, and the rest-from the third shop. In the first, second and third shops, 5, 4 and 6 percents of non-standard components are produced respectively. One component is taken at random:

- find probability that it is non-standard;
- let taken component is non-standard. Find probability that it is produced in the 2–nd shop.

Recommendation. Let A is event that non-standard component is chosen, and B_1, B_2, B_3 - are events that the component is made in the first, second and third shop respectively (that events are called the hypotheses).

- the probability of event A is found by total probability formula:

$$P(A) = P(B_1)P(A/B_1)+P(B_2)P(A/B_2)+P(B_3)P(A/B_3),$$

where $P(A/B_i)$ – are conditional probabilities that taken at random component is from i-th shop (i=1,2,3).

By task condition, we have:

$P(B_1)$ = 100/1000 = 0,1;

$P(B_2)$ = 300/1000 = 0,3;

$P(B_3)$ = 600/1000 = 0,6;

$P(A/B_1)$=0,05;

$P(A/B_2)$=0,04;

$P(A/B_3)$=0,06.

Then $P(A) = 0.1 \cdot 0.05 + 0{,}3 \cdot 0{,}04 + 0{,}6 \cdot 0{,}06 = 0{,}053$;

- in this item it is requested to find the conditional probability $P(B_2/A)$. By Bayes formula, we have:

$$P(B_2/A) = \frac{P(B_2)P(A/B_2)}{P(B_1)P(A/B_1)+P(B_2)P(A/B_2)+P(B_3)P(A/B_3)} = \frac{0{,}3 \cdot 0{,}04}{0{,}053} = 0{,}226.$$

Task 3. n tests are produced. In each test the probability of occurrence of event A equals 0,8. Find probability that event A occurs:

- exactly k_2 times (event A);
- less than k_1 times (event B);
- more than k_2 times (event C);
- at least one time (event D);
- from k_1 to k_2 times (event E):

a) n=10, k_1=3, k_2=8;

b) n=100, k_1=70, k_2=80.

Recommendation:

a) by Bernoulli formula: $P_n(k) = C_n^k p^k q^{n-k}$, the probability of some event occurrence, exactly k times in n independent tests, is found, $q = 1 - p$. Probabilities of events B, C and E are found as sums of probabilities: $P_n(k+1) + P_n(k+2) + \ldots + P_n(n)$ - which is probability that event will happen more than k times in n independent tests, i.e. either $k+1,\ldots,$ or n times; $P_n(0) + P_n(1) + \ldots + P_n(k-1)$ - is probability that event will happen less than k times in n independent tests, i.e. either 0, or $1,\ldots,$ or $k-1$ times; $P_n(k_1) + P_n(k_1+1) + \ldots + P_n(k_2)$ - is probability that event will happen from k_1 till k_2 times inclusively. These probabilities are called cumulative. All these probabilities

can be calculated in MathCAD by means of function *combin* or by means of built-in functions *dbinom* and *pbinom*.

Performing of the task.

The first variant of calculations:

$C(Q,R) := \text{combin}(Q,R)$ $\quad 1 - C(10,0)\cdot 0.8^{0}\cdot 0.2^{10} = 1$

$C(10,9)\cdot 0.8^{9}\cdot 0.2^{1} + C(10,10)\cdot 0.8^{10}\cdot 0.2^{0} = 0.376$

$C(10,8)\cdot 0.8^{8}\cdot 0.2^{2} = 0.302$

$C(10,0)\cdot 0.8^{0}\cdot 0.2^{10} + C(10,1)\cdot 0.8^{1}\cdot 0.2^{9} + C(10,2)\cdot 0.8^{2}\cdot 0.2^{8} = 7.793 \times 10^{-5}$

Event *E* probability we don't consider here because its calculation in this variant will be too cumbersome.

The second variant of calculations:

$k1 := 3 \quad k2 := 8 \quad n := 10$

$\text{dbinom}(k2,n,0.8) = 0.302 \quad R := \text{pbinom}(k2,n,0.8)$

$R = 0.624 \quad 1 - R = 0.376 \quad \text{pbinom}(2,n,0.8) = 7.793 \times 10^{-5}$

$1 - \text{dbinom}(0,n,0.8) = 1$

$T := \text{pbinom}(k1,n,0.8) \quad T = 2.139 \times 10^{-11}$

$R - T = 0.624$

Thus,

1) $P(A) = P_{10}(8) = C_{10}^{8}\cdot 0{,}8^{8}\cdot 0{,}2^{2} = 0{,}302.$

2) $P(B) = P_{10}(0) + P_{10}(1) + P_{10}(2) = 0{,}000078.$

3) $P(C) = P_{10}(9) + P_{10}(10) = 0{,}376.$

4) $P(D) = 1 - P(\overline{D}) = 1 - P_{10}(0) \approx 1$, where $\overline{D}$ is event opposite to event *D*.

5) $P(E) = P_{10}(3) + \ldots + P_{10}(8) = 0{,}624;$

b) in case, when number of independent tests *n* is big, the probability $P_n(k)$ can be found by local Moivre-Laplace theorem:

$$P_n(k) \cong \frac{1}{\sqrt{npq}}\varphi(x),$$

where $x = \dfrac{k-np}{\sqrt{npq}}$, $0 < p < 1$, $\varphi(x) = \dfrac{1}{\sqrt{2\pi}}\exp(-x^2/2)$ (the values of the function can be taken from special tables or by means of built-in function *dnorm* in MathCAD).

Application of computer allows us to find the exact value of $P_n(k)$ by Bernoulli formula.

To define probabilities of events *B*, *C* and *E*, integral Moivre-Laplace theorem can be used: probability $P_n(k_1,k_2)$ that number of occurrence of some event will be located from k_1 till k_2 is approximately equal to $P_n(k_1,k_2) \approx \Phi(x_2) - \Phi(x_1)$, where $x_2 = \frac{k_2 - np}{\sqrt{npq}}$, $x_1 = \frac{k_1 - np}{\sqrt{npq}}$, $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp(-t^2/2)dt$ - is Laplace function, which values can be taken from special tables or can be found by means of built-in function *pnorm* in MathCAD.

Performing of the task:

$n := 100$ $k1 := 70$ $k2 := 80$ $p := 0.8$ $q := 1 - p$

$x1 := \frac{k1 - n \cdot p}{\sqrt{n \cdot p \cdot q}}$ $x2 := \frac{k2 - n \cdot p}{\sqrt{n \cdot p \cdot q}}$ $x3 := \frac{n - n \cdot p}{\sqrt{n \cdot p \cdot q}}$ $x4 := \frac{0 - n \cdot p}{\sqrt{n \cdot p \cdot q}}$

$x1 = -2.5$

$x2 = 0$ $dnorm(x2, 0, 1) = 0.399$ $\frac{dnorm(x2, 0, 1)}{\sqrt{n \cdot p \cdot q}} = 0.1$

$pnorm(x1, 0, 1) = 6.21 \times 10^{-3}$ $dbinom(k2, n, 0.8) = 0.099$

$pnorm(x2, 0, 1) = 0.5$ $pnorm(x2, 0, 1) - pnorm(x1, 0, 1) = 0.494$

Or other variant:

$\Phi(x) := pnorm(x, 0, 1) - 0.5$ $\Phi(x3) - \Phi(x2) = 0.5$

$P(k1, k2) := \Phi(x2) - \Phi(x1)$

$\Phi(x1) = -0.494$ $\Phi(x2) = 0$

$P(k1, k2) = 0.494$ $x3 = 5$ $x4 = -20$

$\Phi(x3) = 0.5$ $\Phi(x4) = -0.5$

So,

1) From table: $x = \frac{80 - 100 \cdot 0{,}8}{\sqrt{100 \cdot 0{,}8 \cdot 0{,}2}} = 0$, $\varphi(0) = 0{,}3989$,

$P(A) = P_{100}(80) \cong \frac{1}{\sqrt{100 \cdot 0{,}8 \cdot 0{,}2}} \varphi(0) = 0{,}09972$;

Via built-in functions (see file): $P(A) = \frac{0{,}399}{\sqrt{100 \cdot 0{,}8 \cdot 0{,}2}} = 0{,}1$ or by Bernoulli formula: $P(A) = 0{,}099$.

2) $P(B) = P_{100}(k < 70) = P_{100}(0{,}70) \approx \Phi(x_1) - \Phi(x_4) = 0{,}006$.

3) $P(C) = P_{100}(k > 80) = P_{100}(80,100) \approx \Phi(x_3) - \Phi(x_2) = 0,5$.

4) $P(D) = 1 - P(\overline{D}) = 1 - P_{100}(0) \approx 1$,

Since $x = \frac{0 - 100 \cdot 0,8}{\sqrt{100 \cdot 0,8 \cdot 0,2}} = -20$, then $\varphi(-20) = \varphi(20) = 0$,

$$P_{100}(0) \cong \frac{1}{\sqrt{100 \cdot 0,8 \cdot 0,2}} \varphi(-20) = 0.$$

5) $P(E) = P_{100}(70,80) \approx \Phi(x_2) - \Phi(x_1) = 0,494$.

Task 4. Discrete random variable $F(x)$ is given by distribution series

X	0	10	20	30	40	50
P	0,05	0,15	0,3	0,25	0,2	0,05

Find:

- its distribution function $F(x)$, construct graph of $F(x)$;
- mathematical expectation, variance, root-square-mean deviation, mode;
- probability of hitting of X into interval (15;45).

Recommendation. Distribution function for discrete random variable is found by formula $F(x) = P(X < x) = \sum_{x_i < x} p_i = \sum_{x_i < x} P(X = x_i)$, where summation is made by all i, for which $x_i < x$.

Numerical characteristics for discrete random variables are found as follows:

- mathematical expectation: $M(X) = \sum_i x_i p_i$;
- variance : $D(X) = \sum_i (x_i - M(X))^2 \cdot p_i$ or $D(X) = \sum_i x_i^{\,2} p_i - (\sum_i x_i p_i)^2$;
- mean-square-root deviation: $\sigma(x) = \sqrt{D(x)}$;
- mode of the discrete random variable (is designated as M_0) – is its value, taken with the most probability.

Probability of hitting of X into interval *(a;b)* is found by formula $P(a;b) = F(b) - F(a)$.

Performing of the task

ORIGIN= 1 $\quad s := (0 \;\; 10 \;\; 20 \;\; 30 \;\; 40 \;\; 50)$

$p := (0.05 \;\; 0.15 \;\; 0.3 \;\; 0.25 \;\; 0.2 \;\; 0.05)$

1) Calculation of the distribution function and its graph construction:

$i := 0..5 \quad q := (0.05 \;\; 0.15 \;\; 0.3 \;\; 0.25 \;\; 0.2 \;\; 0.05)^T$

$F^T = (0.05 \;\; 0.2 \;\; 0.5 \;\; 0.75 \;\; 0.95 \;\; 1)$

$$F_i := \sum_{j=0}^{i} (q)_j \qquad F(x) := \begin{cases} 0 & \text{if } x \le 0 \\ 0.05 & \text{if } 0 < x \le 10 \\ 0.2 & \text{if } 10 < x \le 20 \\ 0.5 & \text{if } 20 < x \le 30 \\ 0.75 & \text{if } 30 < x \le 40 \\ 0.95 & \text{if } 40 < x \le 50 \\ 1 & \text{if } x > 50 \end{cases}$$

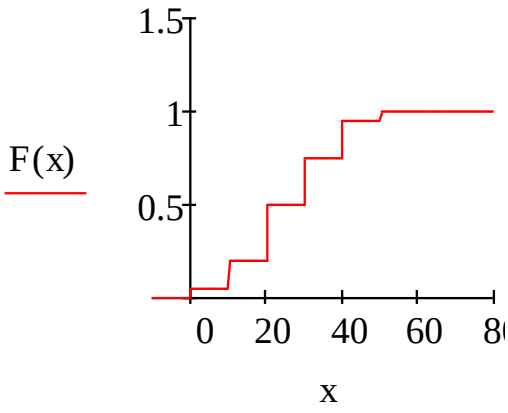


Figure 6

2) Numerical characteristics calculation:

$$M := s^T \cdot p^T \qquad s0 := s^T - M$$

$$M = 25.5$$

$$D := \left[\overrightarrow{(s0 \cdot s0)}\right]^T \cdot p^T \qquad D = 154.75$$

$$s0 = \begin{pmatrix} -25.5 \\ -15.5 \\ -5.5 \\ 4.5 \\ 14.5 \\ 24.5 \end{pmatrix}$$

$$s2 := \begin{pmatrix} 0^2 & 10^2 & 20^2 & 30^2 & 40^2 & 50^2 \end{pmatrix}$$

$$D1 := s2^T \cdot p^T - M^2 \qquad D1 = 154.75$$

$$\sigma := \sqrt{D} = 12.44$$

So, mathematical expectation $M(X)=\sum_i x_i p_i$ = 25,5; variance calculation is made by both formulas and variance equals D(x)=154,75; mode is M_0 = 20; root-mean-square deviation is $\sigma(x)=\sqrt{154,75}=12,44$.

3) Probability of hitting of X into interval (15;45): $P(15;45)=F(45)-F(15)$ = 0,95 - 0,2 = 08.

Task 5. Continuous random variable X is given by distribution density function $f(x)=\begin{cases} 0, \ if \ x\le 0 \\ \frac{2}{9}(3x-x^2), \ if \ 0<x\le 3 \\ 0, \ if \ x>3 \end{cases}$.

Find:

- its distribution function *F(x);*
- mathematical expectation, variance, standard deviation, mode, median;
- hitting probability of X into interval (1;4).

Construct graphs of $F(x)$ and $f(x)$.

Recommendation.For continuous random variables there exist the following formulas:

$F(x)=\int_{-\infty}^{x} f(x)dx$ - is distribution function;

$M(x)=\int_{-\infty(a)}^{\infty(b)} xf(x)dx$ - mathematical expectation;

$D(x)=\int_{-\infty(a)}^{\infty(b)}(x-M(x))^2 f(x)dx$ or $D(x)=\int_{-\infty(a)}^{\infty(b)} x^2 f(x)dx-[M(x)]^2$ - variance;

$\sigma(x)=\sqrt{D(x)}$ - root-square-mean deviation;

the mode of the continuous discrete variable X is its such a value M_o, that distribution density is maximal;

the median of continuous random variable X is its such a value M_e, for which it is equally probable, whether the random variable will be less or more than M_e, i.e. $P(X<M_e)=P(X>M_e)=0,5$;

$P(a<X<b)=F(b)-F(a)$ or $P(a<X<b)=\int_a^b f(x)dx$ - is hitting probability of X into interval (a,b).

Performing of the task.

1) Since $F(x)=\int_{-\infty}^{x} f(x)dx$, then if $x\le 0$, then $F(x)=\int_{-\infty}^{0} 0\,dx=0$;

if $0<x\le 3$, then $F(x)=\int_{-\infty}^{0} 0\,dx+\int_0^x \frac{2}{9}(3x-x^2)dx=-\frac{x^2(2x-9)}{27}$;

if $x>3$, then $F(x)=\int_{-\infty}^{0}0\,dx+\int_{0}^{3}\frac{2}{9}(3x-x^2)dx+\int_{3}^{x}0\,dx=1.$

So, $F(x)=\begin{cases}0, \text{ if } x\le 0\\ -\frac{x^2(2x-9)}{27}, \text{ if } 0<x\le 3\\ 1, \text{ if } x>3\end{cases}.$

2) Calculations in MathCAD:

$f(x) := \frac{2}{9}\cdot\left(3\cdot x - x^2\right)$ $\sqrt{\frac{9}{20}} = 0.671$

$\int_0^3 x\cdot f(x)\,dx \to \frac{3}{2} = 1.5$

$\int_0^3 x^2\cdot f(x)\,dx - \left(\frac{3}{2}\right)^2 \to \frac{9}{20} = 0.45$ $\frac{d}{dx}f(x) \to \frac{2}{3} - \frac{4\cdot x}{9}$

$f1(x) := \frac{d}{dx}f(x)$ $\int_0^y f(x)\,dx \to -\frac{y^2\cdot(2\cdot y - 9)}{27}$

$f1(x)\ \text{solve} \to \frac{3}{2}$ $f1(1) = 0.222$ $f1(2) = -0.222$

$-\frac{y^2\cdot(2\cdot y - 9)}{27} - 0.5\ \text{solve} \to \begin{pmatrix}4.0980762113533159403\\ -1.0980762113533159403\\ 1.5\end{pmatrix} = \begin{pmatrix}4.098\\ -1.098\\ 1.5\end{pmatrix}$

Thus, $M(X)$=1,5; $D(X)$= 0,45; $\sigma(X)$= 0,671.

To find mode, let's find maximum of function $f(x)=2/9(3x-x^2)$ using mathematical analysis tools: $f'(x)=2/3-4x/9$, $f'(x)=0$ when x=3/2 - is critical point. Since $f'(1)>0$, $f'(2)<0$, i.e. moving through the point x=3/2 the sign of derivative is changed from plus to minus, so, it is the point of maximum within the interval. Since at ends of interval the function values are $f(0)=f(3)=0$, then M_o=3/2.

Since $P(X<M_e)=0{,}5$ and $P(X<M_e)=P(-\infty<X<M_e)=P(0<X<M_e)$ $=\int_0^{M_e}2/9(3x-x^2)dx = -M_e^{\,2}(2M_e-9)/27$.

Then, solving the equation $-M_e^{\,2}(2M_e-9)/27=0{,}5$, we get three roots. One of them: $M_e = 1{,}5$ – is median.

3) Calculations in MathCAD:

$$\int_1^3 f(x)\,dx \rightarrow \frac{20}{27} = 0.741 \qquad f2(x) := -\frac{x^2\cdot(2\cdot x-9)}{27}$$

$$f2(1) = 0.259 \qquad f2(3) = 1 \qquad 1 - 0.259 = 0.741$$

So, hitting probability of X into interval (1;4) is equal to

$$P(1<X<4) = P(1<X<3) + P(3<X<\infty) = \int_1^3 2/9(3x-x^2)dx + 0 = 0{,}741$$

or

$$P(1<X<4) = P(1<X<3) + P(3<X<\infty) = F(3) - F(1) =$$
$$= 3^2(2\cdot 3-9)/27 - 1^2(2\cdot 1-9)/27 = 1 - 0{,}259 = 0{,}741.$$

Graphs of functions $F(x)$ and $f(x)$ in MathCAD:

$$f(x) := \begin{cases} 0 & \text{if } x \le 0 \\ \left[\frac{2}{9}\cdot\left(3\cdot x - x^2\right)\right] & \text{if } 0 < x \le 3 \\ 0 & \text{if } x > 3 \end{cases} \qquad F(x) := \begin{cases} 0 & \text{if } x \le 0 \\ \frac{x^2\cdot(9-2\cdot x)}{27} & \text{if } 0 < x \le 3 \\ 1 & \text{if } x > 3 \end{cases}$$

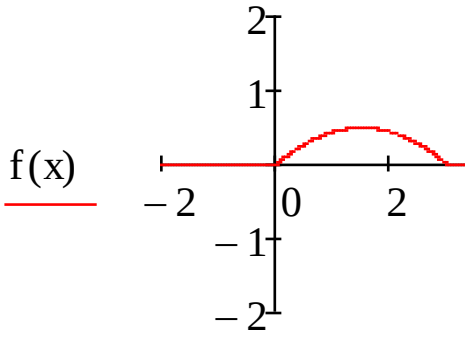


Figure 7

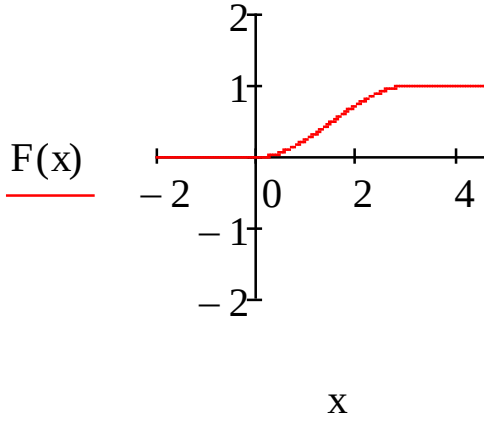


Figure 8

Task 6. Equipment consists of 1000 elements. One element failure probability for a time *t* doesn't depend on status of other elements and equals 0,001. Find:

- distribution law for number of failure elements;
- probability, that not less than 2 elements will fail.

Recommendation. Discrete random variable X – is a number of failed elements. It is distributed by Poisson law (Poisson formula $P_n(k) \approx \frac{\lambda^k}{k!} e^{-\lambda}$ defines approximate value of probability $P_n(k)$, when probability p is small, and number n is big, but $\lambda = np$ is not a big number; exact value can be found by Bernoulli formula). So: $P(X = k) = P_n(k) \approx \frac{\lambda^k}{k!} e^{-\lambda}$, where $\lambda = np = 1000 \cdot 0{,}001 = 1$, $k = 0,1,2,\ldots,1000$, $n = 1000$.

In MathCAD system Poisson distribution law is corresponded by special functions with root word *pois*.

Performing of the task

1) $p(k,\lambda) := dpois(k,\lambda)$

$p(0,\lambda) = 0.368$ $\lambda := 1$

$p(1,\lambda) = 0.368$ $p(10,\lambda) = 1.014 \times 10^{-7}$

$p(2,\lambda) = 0.184$, $p(1000,\lambda) = 0$

$p(3,\lambda) = 0.061$

Thus, $P(X = 0) = P_{1000}(0) \approx \frac{1^0}{0!} e^{-1} = 0{,}368$; $P(X = 1) = P_{1000}(1) \approx \frac{1^0}{1!} e^{-1} = 0{,}368$;

$P(X = 2) = P_{1000}(2) \approx \frac{1^2}{2!} e^{-1} = 0{,}184$; $P(X = 3) = P_{1000}(3) \approx \frac{1^3}{3!} e^{-1}$ 0,061, etc.

$P(X = 10) = P_{1000}(10) \approx \frac{1^{10}}{10!} e^{-1} = 0{,}0000001$ and e.c.t.

Desired distribution law:

X	0	1	2	3	…	10	…
p	0,368	0,368	0,184	0,061	…	0,0000001	…

2) $P(k,\lambda) := ppois(k,\lambda)$

$P(1,\lambda) = 0.736$

$1 - P(1,\lambda) = 0.264$

So, not less than two elements failure probability equals:

$P(X \geq 2) = \sum_{k=2}^{\infty} P_{1000}(k)$ or $P(X \geq 2) = 1 - \sum_{k=0}^{1} P_{1000}(k) = 1 - 0{,}736 = 0{,}264.$

Task 7. Random measurement error obeys the normal distribution law with parameters $a = 10$ and $\sigma = 2$. Find:

- distribution density function *f(x);*

- distribution function *F(x);*
- mathematical expectation, variance;
- probability of hitting in interval (12;14);
- probability, that measurement will be made with error not more than $\delta = 3$ by absolute value.

Construct graphs of *f(x)* and *F(x).*

R e c o m m e n d a t i o n. Normal distribution law is distribution law of continuous random variable X with distribution density function $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}$, where $a = M(X)$ – mathematical expectation, $\sigma = \sigma(X)$ – mean-square-root deviation of X. Formulas for normal distribution: distribution function – $F(x) = \int_{-\infty}^{x} f(t)dt = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[\frac{-(t-a)^2}{2\sigma^2}\right] dt$ or $F(x) = \Phi\left(\frac{x-a}{\sigma}\right) + 0{,}5$, where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{0}^{x} e^{-\frac{t^2}{2}} dt$ – Laplace function, its values are tabulated or it can be found in MathCAD; $P(\alpha < X < \beta) = F(\beta) - F(\alpha) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$; $P(|X - a| \le \delta) = 2\Phi\left(\frac{\delta}{\sigma}\right)$- deviation probability of X from mathematical expectation not more than δ. In MathCAD normal distribution law is corresponded by functions with root word *norm*, for example, *dnorm* (x,a,σ) – gives values of distribution density function *f(x); pnorm* (x, a,σ) – distribution functions *F(x)*; values of Laplace function is found by subtraction of 0,5 from *F(z)* which is normalized normal distribution ($a = 0, \sigma = 1$). Normalization is made by substitution $z = \frac{x-a}{\sigma}$.

Performing of the task

$a := 10$ $\sigma := 2$

$f(x) := dnorm(x, a, \sigma)$ $F(x) := pnorm(x, a, \sigma)$

$F0(z) := pnorm(z, 0, 1)$ $\Phi(z) := F0(z) - 0.5$

$2 \cdot \Phi(1.5) = 0.866$ $\Phi(2) - \Phi(1) = 0.136$

So,

1) $f(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-10)^2}{8}}$.

2) $F(x)=\Phi\left(\frac{x-10}{2}\right)+0{,}5$.

3) $M(X)=a=10$, $\sigma(X)=\sigma=2$, $D(X)=\sigma^2=4$.

4) $P(12<X<14)=\Phi\left(\frac{14-10}{2}\right)-\Phi\left(\frac{12-10}{2}\right)=\Phi(2)-\Phi(1)=0{,}136.$

5) Probability, that measurement is made with error, which is not more than $\delta=3$ by absolute value, will be equal to $P(|X-10|\le 3)=2\Phi\left(\frac{3}{2}\right)=2\Phi(1{,}5)=0{,}866.$

Graphs of *f(x)* and *F(x)*:

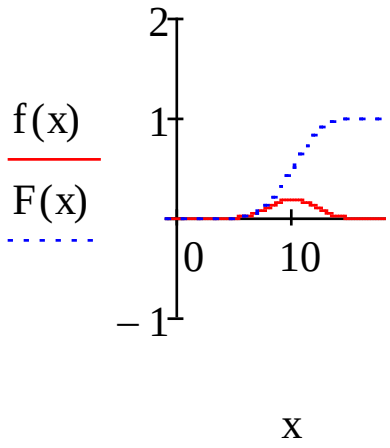


Figure 9

4 Laboratory work №4

The aim of the work: application of MathCAD for problems of mathematical statistics: obtaining of ordered statistical data (variational and statistical series); evaluation of unknown distribution laws; estimation of unknown numerical characteristics.

Task 1. Find for given sample:

- variational series (sample in growing order);
- interval statistical series (minimal and maximal variants, sample range, number of intervals, length of intervals);
- construct frequencies and relative frequencies histogram using interval statistical series;
- construct discrete statistical series;
- using discrete statistical series, find:

a) polygon of frequencies and relative frequencies;

b) empirical distribution function;
c) sample mean;
d) sample variance and corrected sample variance;
e) corrected sample standard deviation;
f) sample mode and median.

1.1	112	101	155	137	109	129	152	128	132	116
	125	125	142	140	125	118	125	135	149	145
	106	109	138	145	118	128	125	105	122	138
	120	118	133	118	129	149	124	153	132	118
	132	132	138	128	122	115	143	140	122	152
	128	118	126	132	134	123	122	159	112	110
1.2	87	85	91	94	102	80	75	102	99	101
	120	122	101	88	80	97	92	91	94	82
	115	100	97	91	87	116	121	101	123	97
	88	90	101	95	93	92	88	94	98	99
	95	105	112	116	118	108	95	99	92	100
	94	106	112	122	100	92	93	82	111	102
	100	101	123	97	90	104	108	101	96	111
1.3	547	565	587	553	548	554	561	562	551	572
	565	555	563	568	586	549	575	537	581	553
	543	568	574	564	547	549	553	572	535	555
	552	545	554	571	569	539	549	553	562	561
	558	563	563	547	552	562	554	563	558	572
	577	554	552	566	557	551	552	571	551	552
	599	561	552	551	561	538	533	541	588	558
1.4	90	123	132	85	122	105	125	142	99	125
	118	105	115	92	115	142	98	123	103	144
	106	92	118	105	118	86	125	105	122	138
	102	130	112	98	115	120	118	103	118	129
	112	115	88	118	103	102	95	124	106	135
	103	122	94	112	97	128	102	116	125	132
1.5	139	112	132	85	122	105	125	142	99	125
	116	105	92	115	98	123	103	144	115	142
	106	92	118	86	125	105	122	138	105	118
	102	130	112	98	115	120	118	103	118	129
	112	115	88	118	103	102	95	124	106	135
	95	124	103	102	118	112	115	103	95	122
	125	118	96	126	98	106	128	118	126	103
1.6	154	143	155	113	155	171	168	153	135	168
	145	168	122	163	117	165	132	139	107	125
	146	152	142	132	152	161	148	136	138	149

	157	178	149	195	146	166	182	135	136	170
	155	152	145	198	192	143	159	116	126	155
	163	169	165	148	151	153	139	166	138	128
1.7	670	801	790	606	564	1195	1033	502	1020	780
	1030	840	869	551	707	635	703	801	859	875
	779	797	789	875	698	1058	1021	1035	910	856
	1095	741	673	988	737	787	667	649	1079	939
	532	885	590	1059	975	1009	731	869	635	889
	1058	967	1095	531	775	885	756	656	680	741
	1095	758	511	857	536	699	574	789	1085	503
1.8	450	434	424	432	440	443	415	446	423	472
	442	452	444	425	403	458	455	431	446	424
	438	442	482	432	416	477	431	432	412	462
	496	468	424	438	452	446	418	474	432	452
	466	488	452	489	451	422	442	492	473	402
	481	468	404	498	467	398	440	449	417	425
	444	498	466	442	483	462	492	435	449	422
1.9	250	244	224	232	240	224	244	226	253	232
	248	216	230	254	258	202	225	224	252	234
	242	212	231	251	204	246	232	282	242	252
	296	242	254	218	226	252	238	224	298	260
	276	254	282	242	270	254	260	232	268	242
	244	276	224	240	272	268	281	234	268	251
	271	212	234	262	204	261	254	266	278	248
1.10	165	143	152	167	164	199	171	171	156	149
	147	155	158	145	158	177	161	181	153	171
	175	153	174	154	163	174	152	188	162	197
	187	158	154	171	163	172	152	178	151	172
	153	186	147	169	147	166	161	171	161	186
	148	161	189	199	162	167	198	168	135	152
1.11	153	174	154	163	174	152	188	162	197	134
	188	158	154	171	163	172	152	178	151	172
	155	186	147	169	147	166	161	171	161	186
	149	161	189	199	162	167	198	168	135	152
	156	175	163	149	162	161	161	193	172	175
	162	164	178	138	164	172	187	178	143	161
	165	163	177	161	149	146	152	139	156	152
1.12	212	231	251	204	246	232	282	242	252	276
	297	242	254	218	226	252	238	224	298	260
	277	254	282	242	270	254	260	232	268	242
	245	276	224	240	272	268	281	234	268	232
	272	212	234	292	204	261	254	266	278	248

	253	262	256	264	272	242	244	246	253	234
	237	264	252	248	247	268	229	235	262	212
1.13	165	143	152	166	164	199	171	171	156	
	148	155	158	145	158	177	161	181	153	171
	176	153	174	154	163	174	152	188	162	197
	189	158	154	171	163	172	152	178	151	172
	157	186	147	169	147	166	161	171	161	186
	150	161	189	199	162	167	198	168	135	152
1.14	216	230	254	258	202	225	224	252	234	250
	243	212	231	251	204	246	232	282	242	252
	298	242	254	218	226	252	238	224	298	260
	278	254	282	242	270	254	260	232	268	242
	246	276	224	240	272	268	281	234	268	232
	273	212	234	262	201	261	254	266	278	248
	254	262	256	264	272	242	244	246	253	234
1.15	165	143	152	167	165	199	171	171	156	152
	149	155	158	145	158	177	161	181	153	171
	153	174	154	163	174	152	188	162	197	178
	190	158	154	171	163	172	152	178	151	172
	159	186	147	169	147	166	161	171	161	186
	151	161	189	199	162	167	198	168	135	152
	160	175	163	149	162	161	161	193	172	175
	165	164	178	137	164	172	187	178	143	161
1.16	147	153	179	165	159	149	141	102	169	157
	169	154	143	155	113	155	171	168	153	135
	150	152	142	132	152	161	148	136	138	149
	157	178	149	195	146	166	182	135	136	170
	156	152	145	198	192	143	159	116	126	155
	164	169	165	148	151	153	139	166	138	128
	169	169	155	152	175	177	131	154	174	187
	180	177	162	149	146	113	151	152	134	125
1.17	558	563	569	547	552	562	554	549	575	578
	561	552	551	561	538	533	547	552	557	543
	547	565	587	553	548	554	561	564	562	558
	566	555	563	568	586	549	575	564	553	555
	567	556	546	552	543	554	556	566	592	562
	544	568	574	564	547	549	553	578	557	561
	553	545	554	571	569	539	549	538	575	554
	577	552	566	557	551	552	546	584	572	535
1.18	577	568	557	564	547	549	553	578	557	575
	554	545	554	571	569	539	549	538	575	566
	558	563	563	547	552	562	554	549	575	558

	547	595	587	553	548	554	561	564	562	544
	555	563	568	586	549	575	564	553	585	592
	577	554	552	566	557	551	552	546	584	556
	601	561	552	551	561	538	533	547	552	557
	553	562	561	572	535	555	543	556	546	538
1.19	77	45	49	92	13	69	52	26	22	36
	48	25	59	57	65	69	55	68	49	63
	38	53	48	68	52	73	42	62	71	45
	63	55	16	78	52	95	77	66	35	54
	68	55	49	65	79	48	59	53	41	38
	12	39	57	51	65	66	43	52	63	43
	55	69	31	62	48	46	51	43	16	34
	74	51	82	52	46	75	49	55	57	54
1.20	347	365	387	348	354	361	364	362	346	358
	365	355	363	368	359	375	364	353	385	363
	343	368	374	364	347	349	353	378	357	358
	352	345	354	352	371	369	349	338	375	388
	366	358	363	347	352	362	354	349	375	341
	377	354	352	366	357	351	352	346	384	351
	399	363	361	352	351	361	338	353	333	357
1.21	9	9	6	9	9	7	6	11	6	7
	6	10	6	7	6	8	6	5	5	4
	6	6	7	12	5	7	8	5	10	9
	7	7	5	11	9	7	6	5	7	6
	5	5	12	9	8	7	9	8	5	5
	6	13	11	11	5	8	10	9	4	7
	3	6	9	8	12	11	9	10	4	14
1.22	39	40	38	43	41	42	40	38	41	42
	41	40	42	39	41	41	36	43	41	42
	34	36	37	42	42	42	40	41	41	46
	47	48	52	56	68	70	68	64	56	58
	41	42	39	33	34	37	43	45	47	71
	43	42	43	41	42	47	48	49	52	53
1.23	10	15	16	17	18	19	20	15	16	11
	17	12	13	14	15	11	18	16	15	18
	20	20	21	23	26	28	23	28	27	24
	27	24	25	25	26	32	33	31	34	43
	26	32	26	27	28	29	30	21	22	23
	42	24	23	35	23	25	36	37	24	21
	58	54	49	47	32	36	43	23	24	28
1.24	150	144	124	132	140	124	144	153	151	148
	116	130	154	158	102	125	124	152	134	148

	142	121	112	131	151	104	146	132	182	142
	152	196	142	154	158	118	126	152	138	124
	144	176	124	140	172	168	181	134	168	132
	144	112	134	162	104	161	154	166	178	148
1.25	128	105	115	92	115	142	98	123	103	144
	112	115	88	118	103	102	95	124	106	135
	95	124	103	102	118	112	115	92	115	119
	92	112	132	85	122	105	125	142	99	125
	106	92	118	105	118	86	125	105	122	138
	102	130	112	98	115	120	118	103	118	129
	103	122	94	112	97	128	102	116	125	132
1.26	102	112	118	85	112	115	103	95	122	125
	157	178	149	195	146	166	182	135	136	170
	157	143	179	165	159	149	141	102	169	168
	151	168	122	163	117	165	132	139	107	125
	152	152	142	132	152	161	148	136	138	149
	153	154	143	155	113	155	171	168	153	135
	157	152	145	198	192	143	159	116	126	155
1.27	242	254	218	226	252	238	224	298	260	287
	250	216	230	254	258	202	225	224	252	234
	244	212	231	251	204	246	232	282	242	252
	299	254	282	242	270	254	260	232	268	242
	276	224	240	272	268	281	234	268	232	300
	274	212	234	262	204	261	254	266	278	248
	255	262	256	264	272	242	244	246	253	234
1.28	262	267	275	266	246	252	261	269	262	268
	259	248	266	259	252	248	252	232	269	287
	253	286	275	235	202	239	225	236	237	224
	253	268	277	249	248	263	243	266	212	255
	249	288	213	264	247	242	228	277	256	251
	267	232	258	246	278	279	257	255	243	258
	254	244	265	274	252	265	222	269	254	278
1.29	558	565	587	553	548	554	561	564	562	544
	563	568	586	549	575	564	553	585	577	553
	563	564	547	552	562	554	549	575	558	592
	546	577	568	574	564	547	549	553	578	557
	557	577	568	574	564	547	549	538	575	566
	558	554	552	566	557	551	552	546	584	532
	602	561	552	551	561	538	533	547	552	557
1.30	165	143	152	167	164	199	171	171	156	151
	155	155	158	145	158	177	161	181	153	171
	177	153	174	154	163	174	152	188	162	197

	191	158	154	171	163	172	152	178	151	172
	161	186	147	169	147	166	161	171	161	186
	161	189	199	162	167	198	168	135	152	146
	162	175	163	149	162	161	161	193	172	175

Question to the laboratory work №4.

1. Definition of basic notions of mathematical statistics: parent population and sampled population (sample), sample volume, variants.
2. Indicate methods how to order statistical data?
3. How to construct interval statistical series?
4. What is histogram of frequencies and relative frequencies?
5. How to construct discrete statistical series?
6. Definition of polygon of frequencies or relative frequencies.
7. Analogue of which theoretical function is empirical distribution function?
8. Numerical characteristics of statistical distributions.

Recommendations to laboratory work №4

Task 1.

The sample is given.

20	15	17	19	23	18	21	15	16	13
20	16	19	20	14	20	16	14	20	19
15	19	17	16	15	22	21	12	10	21
18	14	14	18	18	13	19	18	20	23
16	20	19	17	19	17	21	17	19	17
13	17	11	18	19					

Find:

- variational series (sample in growing order);
- interval statistical series (minimal and maximal variants, sample range, number of intervals, length of intervals);
- construct frequencies and relative frequencies histogram using interval statistical series;
- construct discrete statistical series;
- using discrete statistical series, find:
  a) polygon of frequencies and relative frequencies;
  b) empirical distribution function;
  c) sample mean;
  d) sample variance and corrected sample variance;
  e) corrected sample standard deviation;

f) sample mode and median.

Recommendation. Calculation and construction of graphs are made in MathCAD system.

Performing of the task

$n := 55$

$X := (20\ \ 15\ \ 17\ \ 19\ \ 23\ \ 18\ \ 21\ \ 15\ \ 16\ \ 13\ \ 20\ \ 16\ \ 19\ \ 20\ \ 14\ \ 20\ \ 16\ \ 14\ \ 20\ \ 19\ \ 15\ \ 19\ \ 17\ \ 16\ \ 15$

$a := \min(X)$ $\quad b = 23$ $\quad Y := \mathrm{sort}(X^T)$

$b := \max(X)$ $\quad a = 10$ $\quad h := \dfrac{b-a}{1+\dfrac{\ln(55)}{\ln(2)}}$

$x0 := a - \dfrac{h}{2}$

$h = 1.917$ $\quad x0 = 9.041$ $\quad m = 6.781$

$X^T =$

	0
0	20
1	15
2	17
3	19
4	23
5	18
6	21
7	15
8	16
9	13
10	20
11	16
12	19
13	20
14	14
15	...

$Y^T =$

	0	1	2	3	4	5	6	7	8	9
0	10	11	12	13	13	14	14	14	14	...

,

$R := b - a$ $\quad a1 := 9$

$R = 13$ $\quad m1 := 7$

$w1 := \mathrm{hist}(t, X)$ $\quad h1 := 2$

$m := \dfrac{R}{h}$

$x1 := \mathrm{mean}(X)$

$x1 = 17.564$

$w1^T = (1\ \ 2\ \ 6\ \ 9\ \ 13\ \ 16\ \ 8)$ $\quad j := 0..m1$

$x_i := t_i + \dfrac{h1}{2}$ $\quad t_j := a1 + h1 \cdot j$

$t^T = (\,9 \quad 11 \quad 13 \quad 15 \quad 17 \quad 19 \quad 21 \quad 23\,)$

$w2^T = (\,0.018 \quad 0.036 \quad 0.109 \quad 0.164 \quad 0.236 \quad 0.291 \quad 0.145\,)$

$w2 := \frac{w1}{n}$

$x^T = (\,10 \quad 12 \quad 14 \quad 16 \quad 18 \quad 20 \quad 22\,)$

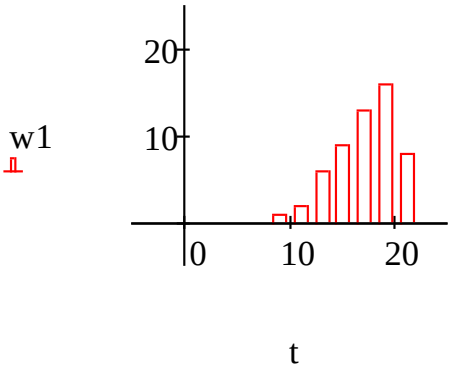


Figure 10

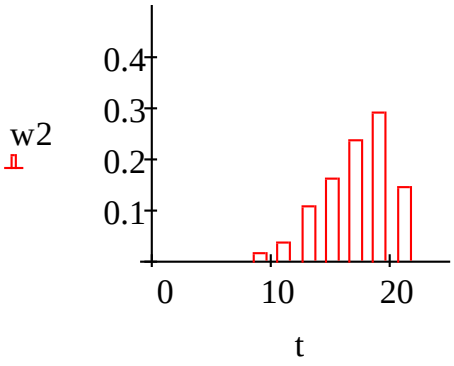


Figure 11

$0.018 + 0.036 + 0.109 + 0.164 + 0.236 + 0.291 + 0.145 = 0.999$

$x2 := \mathrm{var}(X)$ $\qquad x2 = 8.428$ $\qquad M := \mathrm{median}(X)$

$\mathrm{mode}(X) = 19$ $\qquad s := \frac{n}{n-1} \cdot x2$ $\qquad M = 18$

$s = 8.584$ $\qquad \mathrm{stdev}(X) = 2.903$

$\sigma := \sqrt{s}$ $\qquad \sigma = 2.93$

$w3 := \mathrm{hist}(x, X)$ $\qquad w4 := \frac{w3}{n}$

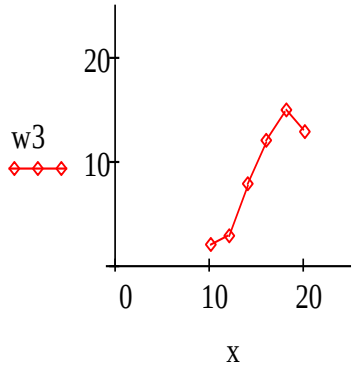


Figure 12

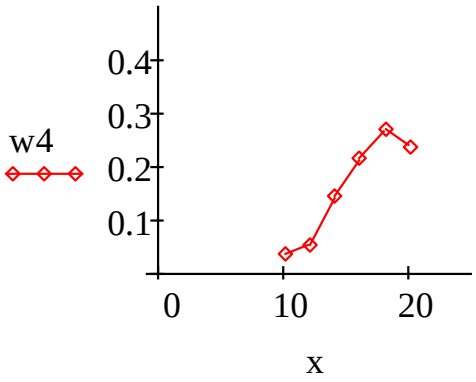


Figure 13

$i := 0..6$ $\quad w := (0.018 \quad 0.036 \quad 0.109 \quad 0.164 \quad 0.236 \quad 0.291 \quad 0.145)^T$

$$F_i := \sum_{j=0}^{i} w_j \qquad F^T = (0.018 \quad 0.054 \quad 0.163 \quad 0.327 \quad 0.563 \quad 0.854 \quad 0.999)$$

$$F(y) := \begin{cases} 0 & \text{if } y \le 10 \\ 0.018 & \text{if } 10 < y \le 12 \\ 0.054 & \text{if } 12 < y \le 14 \\ 0.163 & \text{if } 14 < y \le 16 \\ 0.327 & \text{if } 16 < y \le 18 \\ 0.563 & \text{if } 18 < y \le 20 \\ 0.854 & \text{if } 20 < y \le 23 \\ 1 & \text{if } y > 23 \end{cases}$$

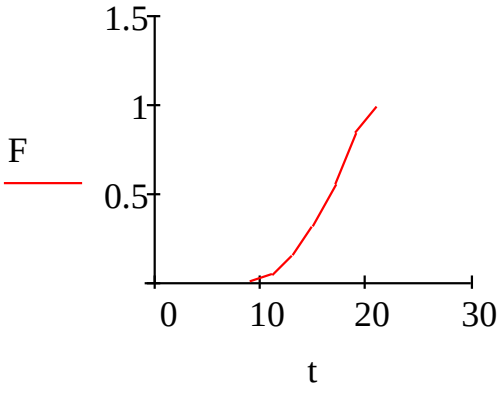


Figure 14

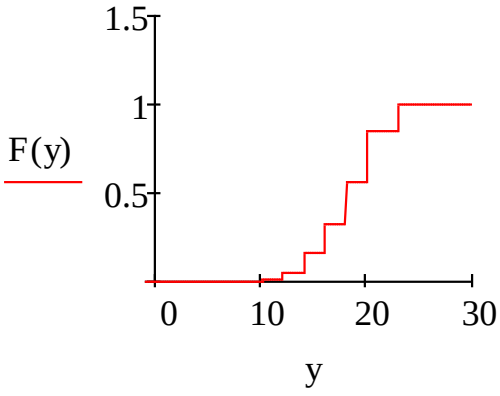


Figure 15

Comments to calculations in MathCAD:

- sample volume n=55. Variational series (sample in growing order);

$Y^T =$

	0	1	2	3	4	5	6	7	8	9
0	10	11	12	13	13	14	14	14	14	...

(in MathCAD system to see this table, you should press advance direction sign);

- in order to construct interval statistical series, we found: the biggest and the least variants $a = x_{\min} = 10$, $b = x_{\max} = 23$; sample range $R = b - a = 13$; value of interval was found by Sturges formula $h = \frac{x_{\max} - x_{\min}}{1 + \log_2 n}$, $h =$ 1,917, or approximately $h \approx 2$; number of intervals – is denominator in Sturges formula $m = 1 + \log_2 n$ or $m = \frac{R}{h}$=6,781, or approximately $m \approx 7$; as beginning of the first interval we recommend take variable $x_{initial} = x_{\min} - \frac{h}{2}$, $x_{initial} = 9{,}041 \approx 9$; number of variants which hit in each interval (i.e. frequencies n_i) and relative frequencies (i.e. $p_i = \frac{n_i}{n}$) in MathCAD system you can use $w1^T$ and $w2^T$.

Thus, interval series is as follows:

Intervals	[9,11)	[11,13)	[13,15)	[15,17)	[17,19)	[19,21)	[21,23]
n_i	1	2	6	9	13	16	8
$p_i = \frac{n_i}{n}$	0,018	0,036	0,109	0,164	0,236	0,291	0,145

- by interval statistical series, we construct histograms of frequencies and relative frequencies.

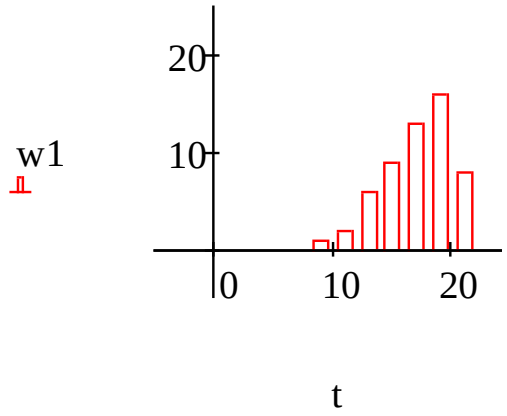


Figure 16

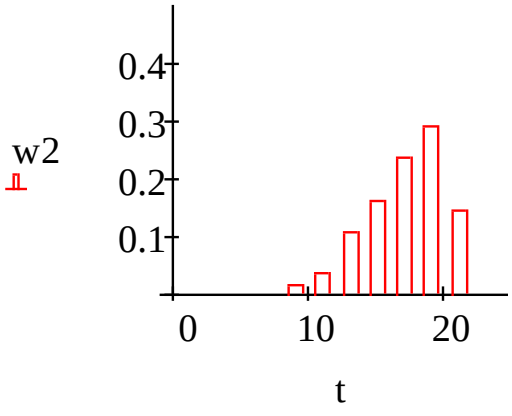


Figure 17

- for construction of discrete statistical series we found midpoints of intervals $\frac{x_i + x_{i+1}}{2}$ (in MathCAD x^T). That points are corresponded by respective frequencies and relative frequencies from interval series.

Discrete statistical series:

$\frac{x_i + x_{i+1}}{2}$	10	12	14	16	18	20	22
n_i	1	2	6	9	13	16	8
p_i	0,018	0,036	0,109	0,164	0,236	0,291	0,145

- by discrete statistical series, we construct:

a) polygon of frequencies and relative frequencies.

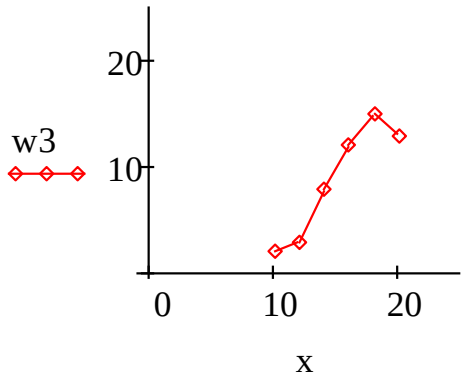


Figure 18

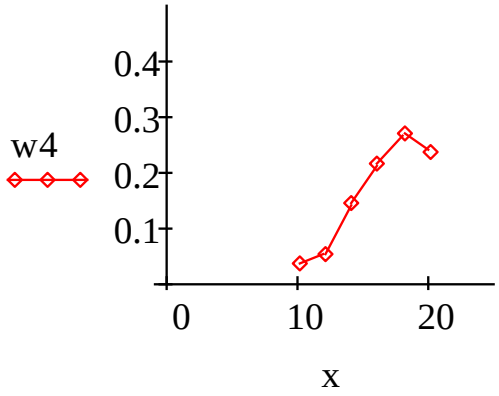


Figure 19

b) empirical distribution function (see F^T and $F(y)$ in MathCAD):

$$F(x) = \begin{cases} 0, \text{ if } x \le 10 \\ 0{,}018, \text{ if } 10 < x \le 12 \\ 0{,}054, \text{ if } 12 < x \le 14 \\ 0{,}163, \text{ if } 14 < x \le 16 \\ 0{,}327, \text{ if } 16 < x \le 18 \\ 0{,}563, \text{ if } 18 < x \le 20 \\ 0{,}854, \text{ if } 20 < x \le 23 \\ 1, \text{ if } x > 23 \end{cases};$$

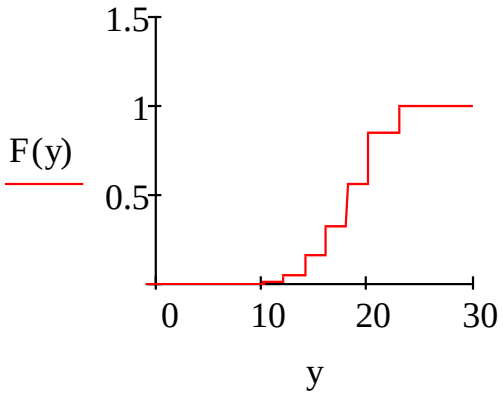


Figure 20

c) sample mean (see *mean*(*X*) in MathCAD): $\bar{x}_в = \frac{\sum_i x_i n_i}{n}$ or $\bar{x}_в = \sum_i x_i \cdot p_i$ =17,564;

d) $D_в = \frac{\sum n_i (x_i - \bar{x}_в)^2}{n}$ or $D_в = \frac{\sum_i n_i \cdot x_i^2}{n} - (\bar{x}_в)^2$ = 8,428 – sample variance;

$s^2 = \frac{n}{n-1} D_в$ = 8,584 – corrected sample variance (see *var*(*X*) and *s* in MathCAD);

e) corrected mean-root-square deviation (see σ or *stdev*(*X*) in MathCAD):

$$\sigma = \sqrt{s} = 2{,}93;$$

f) sample mode and median (see *mode*(*X*) and *median*(*X*) in MathCAD): mode M_0 = 19 defines variant, which have the biggest frequency; median M_e =18 defines midpoint of variational series and depends on parity of the sample volume:

$$M_e = \begin{cases} x_{k+1}, \text{ when } n = 2k+1 \\ \frac{x_k + x_{k+1}}{2}, \text{ when } n = 2k \cdot \end{cases}$$

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PROBABILITY THEORY AND MATHEMATICAL STATISTICS

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the laboratory works for students of speciality
5B070200 - Automation and management

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