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ALMATY UNIVERSITY OF POWER ENGINEERING AND TELECOMMUNICATIONS

The department of mathematical modeling and software

MATHEMATICS 1

Methodological Guidelines and Tasks
for carrying out the calculation-graphical works for students of specialty
5B070200 – Automation and management

Almaty 2017

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Methodological Guidelines and Tasks for carrying out the calculation-graphical works contain sections of the first semester program of course of mathematics for students of AUPET: elements of linear algebra, analytic geometry and complex numbers; introduction to analysis, differential calculus of one-variable functions; integral calculus of one variable function.

The basic theoretical questions of the program and the solution of an exemplary embodiments are given.

Tables 45, figures 9, bibl. 5

Reviewer: candidate of chemical sciences, associate professor G.S. Ospanova

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Кафедра «Математическое моделирование и программное обеспечение»

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MATHEMATICS 1

Methodological Guidelines and Tasks for carrying out the calculation-graphical works for students of specialty
5B070200 – Automation and management

МАТЕМАТИКА 1

Методические указания и задания
по выполнению расчетно-графических работ
для студентов специальности
5В070200 – Автоматизация и управление

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Introduction

Mathematics plays an important role in engineering studies. It is not only a quantitative calculation device, but also mathematics is the means of accurate research and extremely precise formulation of concepts and problems. Mathematical methods have become an integral part of any technical discipline. These facts lead to the need to strengthen the applied orientation of course of mathematics and improve basic mathematical training. CGW is performed in a separate thin notebook. In the number of each task the second digit indicates the variant.

1 Calculation-graphical work №1. Elements of linear algebra, analytic geometry and complex numbers

Purpose: to master the fundamental concepts and methods of the theory of algebra and geometry. Get the skills in calculation of determinants and operations on matrices and vectors, which are used for solving systems of equations and used in analytic geometry. By the analytical curve equations determine their geometric properties and relative location on the plane or in space.

Perform all arithmetic operations with complex numbers.

1.1 Theoretical questions

1. Determinants and their properties, the calculation.
2. Matrices, operations on them, the inverse matrix.
3. Vectors, their length, linear operations on vectors. Collinearity.
4. Scalar, vector, mixed products of vectors and their applications.
5. Different types of equation of the straight line on the plane and in space.
6. Equations of a plane.
7. Angle between the lines, planes, straight line and plane.
8. The distance from a point to the straight line and to the plane.
9. Ellipse, hyperbola, parabola. Their canonical equations.
10. Second-order surfaces.
11. Bringing of the general equations of the second-order curves and surfaces to the canonical form.
12. Various methods for solving systems of linear equations:

a) Cramer's rule;

b) by using the inverse matrix.

13. Complex numbers, the module and the main part of the complex numbers, operations over the complex numbers, De Moivre formula.

1.2 Calculated tasks

Task1. The third order determinant is given.

It is required to calculate:

a) the minor M_{23} and the algebraic complement A_{23} of the element a_{23};

b) the determinant by the triangle rule (rule of Sarrus);

c) the determinant by the method of expansion along the *i*-th row and the *j*-th column.

1.1 $\begin{vmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{vmatrix}, i=2, j=1$	1.2 $\begin{vmatrix} 2 & 3 & 2 \\ 1 & 3 & -1 \\ 4 & 1 & 3 \end{vmatrix}, i=2, j=3$	1.3 $\begin{vmatrix} 6 & 7 & 3 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{vmatrix}, i=1, j=2$
1.4 $\begin{vmatrix} 1 & 7 & 3 \\ -4 & 8 & 0 \\ 4 & 3 & 2 \end{vmatrix}, i=3, j=1$	1.5 $\begin{vmatrix} 2 & 6 & 4 \\ 1 & 3 & -2 \\ 0 & 1 & 1 \end{vmatrix}, i=1, j=2$	1.6 $\begin{vmatrix} 6 & 9 & 4 \\ -1 & -1 & 1 \\ 10 & 1 & 7 \end{vmatrix}, i=3, j=2$
1.7 $\begin{vmatrix} 1 & 0 & 3 \\ 3 & 1 & 7 \\ -2 & 1 & 8 \end{vmatrix}, i=2, j=1$	1.8 $\begin{vmatrix} 5 & 1 & -2 \\ 1 & 3 & -1 \\ 8 & 4 & -1 \end{vmatrix}, i=3, j=2$	1.9 $\begin{vmatrix} 2 & 2 & 5 \\ 3 & -3 & 6 \\ 4 & 3 & 1 \end{vmatrix}, i=3, j=1$
1.10 $\begin{vmatrix} 3 & 5 & 5 \\ -3 & 0 & 1 \\ 5 & 6 & -4 \end{vmatrix}, i=3, j=2$	1.11 $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 1 & -2 & -1 \end{vmatrix}, i=2, j=3$	1.12 $\begin{vmatrix} 4 & -3 & 2 \\ -4 & 0 & 5 \\ 3 & 2 & 3 \end{vmatrix}, i=3, j=1$
1.13 $\begin{vmatrix} 2 & -1 & -3 \\ 8 & -7 & -6 \\ 3 & 4 & 2 \end{vmatrix}, i=2, j=3$	1.14 $\begin{vmatrix} 2 & -1 & -2 \\ 3 & -5 & 4 \\ 1 & 2 & 1 \end{vmatrix}, i=1, j=2$	1.15 $\begin{vmatrix} 3 & 5 & -6 \\ 2 & 4 & 3 \\ -3 & 1 & 1 \end{vmatrix}, i=2, j=1$
1.16 $\begin{vmatrix} 2 & 8 & -5 \\ -3 & 1 & 0 \\ 4 & 5 & -3 \end{vmatrix}, i=3, j=1$	1.17 $\begin{vmatrix} 3 & 6 & 0 \\ 2 & 4 & -6 \\ 1 & -2 & 3 \end{vmatrix}, i=2, j=3$	1.18 $\begin{vmatrix} 3 & 0 & 1 \\ 0 & 2 & 7 \\ 1 & -3 & 2 \end{vmatrix}, i=3, j=3$
1.19 $\begin{vmatrix} -6 & 1 & 11 \\ 9 & 2 & 5 \\ 0 & 3 & 7 \end{vmatrix}, i=3, j=2$	1.20 $\begin{vmatrix} 1 & -2 & 5 \\ 3 & 0 & 6 \\ 4 & 3 & 4 \end{vmatrix}, i=1, j=3$	1.21 $\begin{vmatrix} -1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{vmatrix}, i=3, j=2$
1.22 $\begin{vmatrix} 5 & 4 & 2 \\ 1 & 2 & 4 \\ 3 & 0 & -5 \end{vmatrix}, i=2, j=2$	1.23 $\begin{vmatrix} 5 & 4 & -5 \\ 3 & -7 & 1 \\ 1 & 2 & 2 \end{vmatrix}, i=3, j=1$	1.24 $\begin{vmatrix} 3 & 1 & 0 \\ 4 & 3 & 2 \\ 2 & 2 & -7 \end{vmatrix}, i=2, j=1$
1.25 $\begin{vmatrix} 2 & 7 & 0 \\ 5 & 3 & 1 \\ 1 & -6 & 1 \end{vmatrix}, i=2, j=3$	1.26 $\begin{vmatrix} 8 & -1 & -1 \\ 5 & -5 & -1 \\ 10 & 3 & 2 \end{vmatrix}, i=1, j=3$	1.27 $\begin{vmatrix} 3 & 2 & 5 \\ 3 & 2 & 1 \\ -1 & 0 & 2 \end{vmatrix}, i=1, j=1$
1.28 $\begin{vmatrix} 4 & -7 & -6 \\ 3 & 2 & -1 \\ 0 & 1 & 2 \end{vmatrix}, i=2, j=3$	1.29 $\begin{vmatrix} 7 & 5 & 1 \\ 5 & 3 & -1 \\ 1 & 2 & 3 \end{vmatrix}, i=1, j=3$	1.30 $\begin{vmatrix} 1 & 2 & 1 \\ 1 & -2 & 4 \\ 3 & -5 & 7 \end{vmatrix}, i=3, j=2$

Task 2. Given matrices A, B, C.

Find:

a) the product of matrices AB or BC, or CB (if possible). Explain why if it is impossible;

b) the matrix A^{-1}, the inverse matrix for A.

2.1	$A=\begin{pmatrix}3 & -7 & 2\\ 1 & -8 & 3\\ 4 & -2 & 3\end{pmatrix}$, $B=\begin{pmatrix}2\\ -7\\ 1\end{pmatrix}$, $C=\begin{pmatrix}1 & 1 & -4\\ 0 & 1 & 3\end{pmatrix}$
2.2	$A=\begin{pmatrix}0 & 5 & -3\\ 2 & 4 & 1\\ 2 & 1 & 5\end{pmatrix}$, $B=\begin{pmatrix}3 & 1 & -1\\ -2 & 4 & 0\end{pmatrix}$, $C=\begin{pmatrix}5\\ -2\\ 4\end{pmatrix}$
2.3	$A=\begin{pmatrix}2 & -1 & -4\\ 4 & -9 & 3\\ 2 & -7 & -1\end{pmatrix}$, $B=\begin{pmatrix}-4 & 2 & 1\end{pmatrix}$, $C=\begin{pmatrix}7 & 1\\ 2 & 1\\ 3 & -3\end{pmatrix}$
2.4	$A=\begin{pmatrix}1 & 1 & -1\\ 2 & -4 & 1\\ 4 & -3 & 6\end{pmatrix}$, $B=\begin{pmatrix}-2 & 4\\ 1 & 7\\ -4 & 3\end{pmatrix}$, $C=\begin{pmatrix}4 & 3 & -2\end{pmatrix}$
2.5	$A=\begin{pmatrix}1 & 0 & -4\\ 2 & 5 & -3\\ 4 & -3 & 2\end{pmatrix}$, $B=\begin{pmatrix}3 & 1\\ 0 & -2\end{pmatrix}$, $C=\begin{pmatrix}-3 & 1 & 4 & 2\\ 7 & 1 & 5 & 6\end{pmatrix}$
2.6	$A=\begin{pmatrix}1 & 2 & 1\\ 1 & -2 & 4\\ 3 & -5 & 3\end{pmatrix}$, $B=\begin{pmatrix}-4 & 1\\ 0 & 2\\ 1 & -1\\ 6 & 5\end{pmatrix}$, $C=\begin{pmatrix}7 & 1\\ 0 & 2\end{pmatrix}$
2.7	$A=\begin{pmatrix}5 & 7 & 1\\ 5 & 3 & -1\\ 1 & 2 & 3\end{pmatrix}$, $B=\begin{pmatrix}3\\ -6\\ 2\end{pmatrix}$, $C=\begin{pmatrix}2 & 3 & -3\\ 1 & 0 & 3\end{pmatrix}$
2.8	$A=\begin{pmatrix}-3 & 4 & 0\\ 4 & 5 & 1\\ -2 & 3 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & 2 & -2\\ 3 & 5 & 1\end{pmatrix}$, $C=\begin{pmatrix}6\\ -1\\ 3\end{pmatrix}$
2.9	$A=\begin{pmatrix}1 & 7 & -1\\ 0 & 2 & 6\\ 2 & -1 & 1\end{pmatrix}$, $B=\begin{pmatrix}-5 & 3 & 1\end{pmatrix}$, $C=\begin{pmatrix}8 & -1\\ 3 & 0\\ 2 & -2\end{pmatrix}$
2.10	$A=\begin{pmatrix}-1 & 0 & 2\\ 2 & 3 & 2\\ 3 & 7 & 1\end{pmatrix}$, $B=\begin{pmatrix}-3 & 1\\ 4 & 7\\ -5 & 0\end{pmatrix}$, $C=\begin{pmatrix}5 & 4 & -2\end{pmatrix}$
2.11	$A=\begin{pmatrix}3 & 0 & 1\\ -3 & 1 & 7\\ 1 & 3 & 2\end{pmatrix}$, $B=\begin{pmatrix}4 & 2\\ 1 & -3\end{pmatrix}$, $C=\begin{pmatrix}-4 & 2 & 5 & 3\\ 1 & 5 & 6 & 1\end{pmatrix}$

2.12	$A=\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}$, $B=\begin{pmatrix}-5 & 2\\ 1 & 3\\ 2 & 0\\ 7 & 4\end{pmatrix}$, $C=\begin{pmatrix}8 & 2\\ 0 & 1\end{pmatrix}$
2.13	$A=\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 1\\ 1 & 2 & -2\end{pmatrix}$, $B=\begin{pmatrix}4\\ -3\\ 1\end{pmatrix}$, $C=\begin{pmatrix}0 & 2 & -4\\ 1 & 3 & 7\end{pmatrix}$
2.14	$A=\begin{pmatrix}6 & 9 & 4\\ -1 & -1 & 1\\ 2 & 1 & 1\end{pmatrix}$, $B=\begin{pmatrix}1 & -2 & 3\\ 2 & 4 & 7\end{pmatrix}$, $C=\begin{pmatrix}5\\ -8\\ 1\end{pmatrix}$
2.15	$A=\begin{pmatrix}1 & 0 & 3\\ 2 & -1 & 8\\ 3 & 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}6 & -2 & 4\end{pmatrix}$, $C=\begin{pmatrix}3 & -6\\ 7 & 1\\ 2 & -2\end{pmatrix}$
2.16	$A=\begin{pmatrix}4 & 5 & 1\\ -1 & 0 & 3\\ 4 & 2 & 1\end{pmatrix}$, $B=\begin{pmatrix}3 & -2\\ 4 & -2\\ 7 & 5\end{pmatrix}$, $C=\begin{pmatrix}-2 & 3 & 4\end{pmatrix}$
2.17	$A=\begin{pmatrix}3 & 3 & 6\\ 4 & 3 & -4\\ 2 & 1 & -2\end{pmatrix}$, $B=\begin{pmatrix}4 & -5\\ 1 & 2\end{pmatrix}$, $C=\begin{pmatrix}2 & -3 & 1 & 8\\ 1 & -9 & 2 & 2\end{pmatrix}$
2.18	$A=\begin{pmatrix}2 & 5 & -3\\ 4 & -3 & 2\\ 1 & 0 & 3\end{pmatrix}$, $B=\begin{pmatrix}3 & -1\\ 4 & 9\\ 1 & 2\\ -2 & 3\end{pmatrix}$, $C=\begin{pmatrix}5 & -6\\ 2 & 1\end{pmatrix}$
2.19	$A=\begin{pmatrix}4 & -1 & -2\\ 4 & 3 & 7\\ 2 & 0 & 6\end{pmatrix}$, $B=\begin{pmatrix}-8\\ 1\\ 9\end{pmatrix}$, $C=\begin{pmatrix}6 & -1 & 5\\ 2 & 4 & 3\end{pmatrix}$
2.20	$A=\begin{pmatrix}3 & 0 & 6\\ 1 & 7 & 2\\ -2 & 1 & 4\end{pmatrix}$, $B=\begin{pmatrix}3 & 8 & -4\\ 4 & 1 & 1\end{pmatrix}$, $C=\begin{pmatrix}2\\ -6\\ 8\end{pmatrix}$
2.21	$A=\begin{pmatrix}3 & 1 & 7\\ 1 & 0 & 3\\ -4 & 2 & 5\end{pmatrix}$, $B=\begin{pmatrix}7 & -4 & 0\end{pmatrix}$, $C=\begin{pmatrix}2 & -3\\ 1 & 9\\ 3 & 4\end{pmatrix}$
2.22	$A=\begin{pmatrix}-3 & 4 & 1\\ -2 & 3 & 5\\ 4 & 6 & 2\end{pmatrix}$, $B=\begin{pmatrix}-6 & 1\\ -2 & 2\\ 5 & 3\end{pmatrix}$, $C=\begin{pmatrix}0 & 6 & 5\end{pmatrix}$
2.23	$A=\begin{pmatrix}1 & -5 & 3\\ -3 & 4 & 0\\ 2 & 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}3 & 9\\ -2 & 1\end{pmatrix}$, $C=\begin{pmatrix}2 & 3 & -1 & 4\\ 1 & -3 & 9 & 2\end{pmatrix}$

2.24	$A=\begin{pmatrix}3 & -6 & 1\\ 2 & 1 & -7\\ 4 & 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}7 & 2\\ -5 & 1\\ 0 & 2\\ 3 & 5\end{pmatrix}$, $C=\begin{pmatrix}6 & 2\\ -2 & 4\end{pmatrix}$
2.25	$A=\begin{pmatrix}2 & 3 & -3\\ 4 & 1 & 5\\ 2 & 7 & -1\end{pmatrix}$, $B=\begin{pmatrix}3\\ -5\\ 4\end{pmatrix}$, $C=\begin{pmatrix}2 & -4 & 1\\ 0 & 6 & -9\end{pmatrix}$
2.26	$A=\begin{pmatrix}5 & 6 & -4\\ 3 & 1 & 8\\ 2 & -2 & 3\end{pmatrix}$, $B=\begin{pmatrix}6 & -8 & 1\\ 0 & -1 & -2\end{pmatrix}$, $C=\begin{pmatrix}7\\ 6\\ 3\end{pmatrix}$
2.27	$A=\begin{pmatrix}-2 & 3 & 5\\ 1 & 7 & 2\\ -2 & 5 & 4\end{pmatrix}$, $B=\begin{pmatrix}5 & 1 & -4\end{pmatrix}$, $C=\begin{pmatrix}-9 & 1\\ 2 & 7\\ 3 & 1\end{pmatrix}$
2.28	$A=\begin{pmatrix}-5 & 1 & 3\\ 2 & 4 & 0\\ 4 & 6 & 1\end{pmatrix}$, $B=\begin{pmatrix}5 & 8\\ -4 & 1\\ 0 & 3\end{pmatrix}$, $C=\begin{pmatrix}5 & 2 & -8\end{pmatrix}$
2.29	$A=\begin{pmatrix}10 & 0 & -2\\ 3 & 1 & -1\\ 5 & 2 & 3\end{pmatrix}$, $B=\begin{pmatrix}7 & 3\\ -4 & 2\end{pmatrix}$, $C=\begin{pmatrix}2 & 3 & -7 & 1\\ 4 & 1 & 3 & 6\end{pmatrix}$
2.30	$A=\begin{pmatrix}2 & -3 & 1\\ 4 & 1 & 9\\ 3 & 2 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & -3\\ 1 & 6\\ -3 & 7\\ 2 & 3\end{pmatrix}$, $C=\begin{pmatrix}5 & 1\\ -4 & 2\end{pmatrix}$

Task 3. Given points A and B; and vectors $\bar{b}$ and $\vec{c}$.

Find:

a) the module (length) of the vector $\left|\overrightarrow{a}\right|=\left|\overrightarrow{AB}\right|$ and the midpoint of the segment *AB*;

b) the projection of the vector $\vec{a}$ on $\vec{c}$;

c) the area of the parallelogram, obtained from the vectors $\vec{b}$ and $\vec{c}$;

d) volume of the pyramid constructed from the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

3.1	A(5, -4, 3), B(1, 2, -8), $\bar{e}$=(0, 1, 4), $\bar{c}$=(5, 2, -3)	3.2	A(-3, 1, 0), B(7, 1, -5), $\bar{e}$=(7, 1, 4), $\bar{c}$=(5, 8, -3)
3.3	A(0, 4, 5), B(3, -2, 1), $\bar{e}$=(6, 4, 4), $\bar{c}$=(0, 2, -2)	3.4	A(3, -2, 5), B(4, 5, 7), $\bar{e}$=(5, 1, 4), $\bar{c}$=(5, -3, -3)
3.5	A(2, -3, 7), B(3, 2, 8), $\bar{e}$=(6, 4, 6), $\bar{c}$=(0, 6, -2)	3.6	A(2, -1, 7), B(6, 3, 4) , $\bar{e}$=(4, 5, 4), $\bar{c}$=(7, 8, 5)

3.7	A(3, 1, 7), B(2, -3, 9), $\bar{в}$=(9, 1, 4), $\bar{c}$=(8, 2, -3)	3.8	A(2, 1, -6), B(1, 4, 9) , $\bar{в}$=(1, 1, 8), $\bar{c}$=(5, -3, 9)
3.9	A(2, -4, 8), B(5, 4, 7), $\bar{в}$=(6, 3, 4), $\bar{c}$=(7, 2, -2)	3.10	A(3, 2, 5), B(4, 0, -3) , $\bar{в}$=(7, 2, 4), $\bar{c}$=(5, 8, 4)
3.11	A(2, 3, -1),B(-6, 4, 2), $\bar{в}$=(2, 9, 6), $\bar{c}$=(0, 6, 4)	3.12	A(-4, 2, 3), B(8, 7, -2), $\bar{в}$=(5, 3, 4), $\bar{c}$=(5, 3, 4)
3.13	A(5, 3, 6), B(-2, 3, 5), $\bar{в}$=(6, 2, 4), $\bar{c}$=(7, 2, -7)	3.14	A(0, 6, 0), B(5, 3, -4), $\bar{в}$=(8, 5, 3), $\bar{c}$=(7, 8, 7)
3.15	A(4, 2, 0), B(1, -7, 8), $\bar{в}$=(2, 8, 6), $\bar{c}$=(0, 6, 3)	3.16	A(4, 2, 5), B(-1, 0, 6), $\bar{в}$=(2, 1, 8), $\bar{c}$=(5, 9, 9)
3.17	A(3,-5, 8), B(6, 3, 9), $\bar{в}$=(7, 3, 4), $\bar{c}$=(3, 2, -2)	3.18	A(7, 2, 2), B(-5, 7, -7) , $\bar{в}$=(6, 1, 3), $\bar{c}$=(1, 8, 7)
3.19	A(5, -3, 1), B(2, 3, 7), $\bar{в}$=(2, 7, 6), $\bar{c}$=(7, 6, 4)	3.20	A(8, -6, 4), B(10, 5, 1), $\bar{в}$=(9, 1, 8), $\bar{c}$=(4, 9, 1)
3.21	A(5, 6,-8), B(8, 10,7), $\bar{в}$=(1, 3, 4), $\bar{c}$=(3, 6, -2)	3.22	A(1, -1, 3), B(6, 5, 8), $\bar{в}$=(7, 8, 3), $\bar{c}$=(1, 8, 9)
3.23	A(3, 5,-7), B(8, 4, 1), $\bar{в}$=(2, 5, 6), $\bar{c}$=(7, 6, 8)	3.24	A(6, -6, 5), B(4, 9, 5), $\bar{в}$=(6, 1, 4), $\bar{c}$=(0, 9, 1)
3.25	A(4,6,11), B(9,3,-4), $\bar{в}$=(3, 7, 6), $\bar{c}$=(8, 6, 4)	3.26	A(5, 7, 4),B(4,-10, 9), $\bar{в}$=(8, 1, 7), $\bar{c}$=(4, 5, 1)
3.27	A(-9, 8, 9), B(7, 1,-2), $\bar{в}$=(7, 2, 4), $\bar{c}$=(5, 2, -1)	3.28	A(5, 2, 6), B(1, 8, -2), $\bar{в}$=(4, 2, 3), $\bar{c}$=(2, 3, 7)
3.29	A(2, 8, -9), B(7, 5,-5), $\bar{в}$=(8, 1, 6), $\bar{c}$=(6, 1, 4)	3.30	A(-2, 7, 0), B(6, 3, 5), $\bar{в}$=(2, -3, 8), $\bar{c}$=(3, 9, 2)

Task 4. Given points A_1, A_2 on a plane and the equation of the line L_1. Write the equation of a straight line:

a) $L_2 = (A_1A_2)$ – passing through these points;

b) L_2 – as a general equation of straight line;

c) L_2 – in the form of equation of straight line with slope;

d) L_2 – in the form of equation of straight line in segments;

e) L_3 , passing through the point A_2 and perpendicular to L_1.

4.1	$A_1(3,1),\ A_2(2,-3),\ L_1: x-5y+4=0$	4.2	$A_1(-4,1),\ A_2(6,2),\ L_1: 2x-3y-1=0$
4.3	$A_1(-4,7),\ A_2(2,-13),\ L_1: 3x-5y+2=0$	4.4	$A_1(5,1),\ A_2(-4,2),\ L_1: x-3y-1=0$
4.5	$A_1(2,1),\ A_2(6,-3),\ L_1: 4x-5y+1=0$	4.6	$A_1(8,3),\ A_2(-1,2),\ L_1: 2x+2y-9=0$
4.7	$A_1(-1,7),\ A_2(3,-1),\ L_1: 3x-4y+2=0$	4.8	$A_1(-2,1),\ A_2(-7,3),\ L_1: 8x-3y-1=0$
4.9	$A_1(6,1),\ A_2(5,-3),\ L_1: x-7y+4=0$	4.10	$A_1(-4,2),\ A_2(6,1),\ L_1: 2x-5y+4=0$
4.11	$A_1(-4,3),\ A_2(2,-2),\ L_1: 3x-y+2=0$	4.12	$A_1(7,1),\ A_2(-4,2),\ L_1: x-y+4=0$

4.13	$A_1(-9,1),\ A_2(1,-3),\ L_1: x-5y+1=0$	4.14	$A_1(8,1),\ A_2(-1,3),\ L_1: 2x+5y+8=0$
4.15	$A_1(5,7),\ A_2(3,-1),\ L_1: 3x-5y+2=0$	4.16	$A_1(-2,7),\ A_2(-7,-3),\ L_1: 7x-6y-1=0$
4.17	$A_1(5,1),\ A_2(7,-3),\ L_1: x-7y+5=0$	4.18	$A_1(-4,8),\ A_2(2,1),\ L_1: 2x-4y-6=0$
4.19	$A_1(-4,4),\ A_2(9,-2),\ L_1: x-y+2=0$	4.20	$A_1(7,6),\ A_2(-4,1),\ L_1: 2x-y-1=0$
4.21	$A_1(-8,1),\ A_2(1,-7),\ L_1: 2x-5y+1=0$	4.22	$A_1(3,1),\ A_2(-1,-3),\ L_1: 2x-4y+9=0$
4.23	$A_1(5,1),\ A_2(6,-1),\ L_1: x-5y+2=0$	4.24	$A_1(-2,8),\ A_2(-1,4),\ L_1: 7x-y+4=0$
4.25	$A_1(-4,2),\ A_2(9,-2),\ L_1: 9x-y+2=0$	4.26	$A_1(2,6),\ A_2(-2,1),\ L_1: 5x-5y+4=0$
4.27	$A_1(-6,1),\ A_2(1,-9),\ L_1: 2x-5y+4=0$	4.28	$A_1(10,1),\ A_2(-1,5),\ L_1: x+4y-9=0$
4.29	$A_1(3,1),\ A_2(1,-1),\ L_1: x+5y+8=0$	4.30	$A_1(-1,8),\ A_2(-1,5),\ L_1: 7x-2y-1=0$

Task 5. Given points A_1, A_2, A_3. It is required to:

a) write the equation of the plane $P_1 = (A_1A_2A_3)$;

b) write P_1 as a general equation of a plane;

c) write P_1 as equation of a plane in segments;

d) write P_1 in a general equation of a plane, passing through the point A_1;

e) make a canonical equation of the straight line $L_1 = (A_2A_3)$;

f) write a parametric equation of the straight line L_1;

g) write the equation of the straight line $L_2 = (A_1N)$ perpendicular to the plane P_1.

5.1	$A_1(3,1,5),\ A_2(2,-3,1),\ A_3(4,-6,2)$	5.2	$A_1(7,4,1),\ A_2(1,-4,2),\ A_3(1,2,7)$
5.3	$A_1(3,4,5),\ A_2(3,-9,1),\ A_3(4,5,7)$	5.4	$A_1(-3,4,-2),\ A_2(9,5,1),\ A_3(4,-3,-1)$
5.5	$A_1(0,8,5),\ A_2(3,-2,4),\ A_3(2,5,6)$	5.6	$A_1(2,5,6),\ A_2(3,-7,8),\ A_3(5,4,-2)$
5.7	$A_1(-1,4,2),\ A_2(1,-9,1),\ A_3(4,4,7)$	5.8	$A_1(1,9,-1),\ A_2(2,6,1),\ A_3(2,8,-6)$
5.9	$A_1(-1,4,5),\ A_2(3,-5,1),\ A_3(4,5,2)$	5.10	$A_1(9,5,1),\ A_2(4,-7,8),\ A_3(5,2,-2)$
5.11	$A_1(3,7,5),\ A_2(3,-9,2),\ A_3(1,5,7)$	5.12	$A_1(1,6,-1),\ A_2(2,7,1),\ A_3(2,3,-6)$
5.13	$A_1(0,9,5),\ A_2(4,-2,4),\ A_3(2,7,6)$	5.14	$A_1(7,5,-1),\ A_2(2,-4,1),\ A_3(3,-2,7)$
5.15	$A_1(3,4,-1),\ A_2(1,-7,1),\ A_3(4,3,7)$	5.16	$A_1(-4,4,-2),\ A_2(8,5,1),\ A_3(4,-3,-1)$
5.17	$A_1(1,4,1),\ A_2(1,8,1),\ A_3(4,3,-6)$	5.18	$A_1(2,-5,6),\ A_2(3,-2,-7),\ A_3(-5,4,-2)$
5.19	$A_1(2,4,1),\ A_2(3,-7,1),\ A_3(5,3,7)$	5.20	$A_1(3,9.-2),\ A_2(-2,6,1),\ A_3(2,8,5)$
5.21	$A_1(1,8,-3),\ A_2(2,8,1),\ A_3(6,3,-6)$	5.22	$A_1(1,6,-5),\ A_2(4,7,1),\ A_3(1,3,-6)$
5.23	$A_1(8,4,-1),\ A_2(1,-9,1),\ A_3(4,2,7)$	5.24	$A_1(8,5,-1),\ A_2(2,-3,2),\ A_3(-3,1,7)$
5.25	$A_1(1,4,-2),\ A_2(9,8,1),\ A_3(4,-3,-6)$	5.26	$A_1(-4,3,2),\ A_2(8,5,9),\ A_3(4,-3,-5)$
5.27	$A_1(2,5,-1),\ A_2(3,-7,8),\ A_3(5,3,-2)$	5.28	$A_1(9,-5,6),\ A_2(3,2,-7),\ A_3(-5,1,-2)$
5.29	$A_1(1,8,-1),\ A_2(2,6,1),\ A_3(2,3,-6)$	5.30	$A_1(3,9,-1),\ A_2(-4,6,1),\ A_3(2,7,5)$

Task 6. Solve the system
a) by Cramer's method;
b) by matrix method (using the inverse matrix).

6.1	$\begin{cases} 2x+y=7 \\ x+3y=1 \end{cases}$	6.2	$\begin{cases} 4x+y=-3 \\ x+2y=1 \end{cases}$	6.3	$\begin{cases} 5x+y=2 \\ -x+2y=3 \end{cases}$
6.4	$\begin{cases} x+y=3 \\ 2x+3y=1 \end{cases}$	6.5	$\begin{cases} 8x+y=-3 \\ x-2y=1 \end{cases}$	6.6	$\begin{cases} 6x+y=2 \\ -x+2y=4 \end{cases}$
6.7	$\begin{cases} x+y=13 \\ 2x-3y=1 \end{cases}$	6.8	$\begin{cases} 4x+y=-3 \\ 6x+2y=11 \end{cases}$	6.9	$\begin{cases} 9x+y=2 \\ -x+3y=3 \end{cases}$
6.10	$\begin{cases} 2x+5y=7 \\ x+3y=1 \end{cases}$	6.11	$\begin{cases} 4x+y=-3 \\ x+7y=1 \end{cases}$	6.12	$\begin{cases} 7x+y=2 \\ -x+3y=4 \end{cases}$
6.13	$\begin{cases} 2x+y=7 \\ x+3y=1 \end{cases}$	6.14	$\begin{cases} 2x+y=7 \\ 6x+2y=1 \end{cases}$	6.15	$\begin{cases} 5x+6y=2 \\ -x+3y=3 \end{cases}$
6.16	$\begin{cases} x+8y=3 \\ 2x+3y=1 \end{cases}$	6.17	$\begin{cases} 8x+y=7 \\ x-2y=1 \end{cases}$	6.18	$\begin{cases} 6x-y=6 \\ -x+2y=4 \end{cases}$
6.19	$\begin{cases} 4x+3y=7 \\ x+3y=-1 \end{cases}$	6.20	$\begin{cases} -3x+y=7 \\ 6x+3y=11 \end{cases}$	6.21	$\begin{cases} 7x+y=2 \\ -x+2y=3 \end{cases}$
6.21	$\begin{cases} x-2y=13 \\ x+3y=1 \end{cases}$	6.23	$\begin{cases} 3x-y=12 \\ x+2y=6 \end{cases}$	6.24	$\begin{cases} 7x+y=2 \\ -x+2y=4 \end{cases}$
6.23	$\begin{cases} 2x+y=7 \\ x+3y=1 \end{cases}$	6.26	$\begin{cases} 7x-y=2 \\ x+2y=6 \end{cases}$	6.27	$\begin{cases} x-4y=7 \\ x+y=6 \end{cases}$
6.28	$\begin{cases} 2x+y=-3 \\ 6x+2y=1 \end{cases}$	6.29	$\begin{cases} x-y=12 \\ x+2y=6 \end{cases}$	6.30	$\begin{cases} x-y=2 \\ x+2y=5 \end{cases}$

Task 7. Solve the system of equations.
a) by Cramer's method;
b) by matrix method (using the inverse matrix).

7.1	7.2	7.3
$\begin{cases} 2x_1+x_2+3x_3=7 \\ 2x_1+3x_2+x_3=9 \\ 3x_1-2x_2+3x_3=2 \end{cases}$	$\begin{cases} 2x_1-x_2+2x_3=10 \\ x_1+x_2+2x_3=6 \\ 3x_1+x_2+4x_3=16 \end{cases}$	$\begin{cases} 3x_1-x_2+x_3=-12 \\ x_1+2x_2+4x_3=-13 \\ 5x_1-x_2+2x_3=-17 \end{cases}$

7.4 $\begin{cases} 2x_1 - x_2 + 3x_3 = 3 \\ x_1 + 3x_2 + x_3 = 9 \\ x_1 - 2x_2 - 2x_3 = 2 \end{cases}$	7.5 $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 3 \\ 3x_1 + 4x_2 - 2x_3 = -3 \\ 2x_1 - x_2 - x_3 = -15 \end{cases}$	7.6 $\begin{cases} 2x_1 + 3x_2 - 6x_3 = 13 \\ x_1 + 3x_2 - x_3 = -9 \\ 4x_1 + x_2 + 3x_3 = -9 \end{cases}$
7.7 $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 9 \\ x_1 - 7x_2 - 5x_3 = 16 \\ 2x_1 + 4x_2 + 3x_3 = 4 \end{cases}$	7.8 $\begin{cases} 4x_1 + x_2 - 3x_3 = 11 \\ x_1 + x_2 - x_3 = 4 \\ 2x_1 - 3x_2 - 6x_3 = 7 \end{cases}$	7.9 $\begin{cases} 2x_1 + 3x_2 + 4x_3 = -2 \\ 7x_1 - 5x_2 + x_3 = -4 \\ 4x_1 + x_2 + x_3 = 11 \end{cases}$
7.10 $\begin{cases} x_1 - 4x_2 - x_3 = -8 \\ 2x_1 + 5x_2 + 4x_3 = 6 \\ 3x_1 + 2x_2 + 5x_3 = -8 \end{cases}$	7.11 $\begin{cases} 3x_1 - 2x_2 + 4x_3 = -3 \\ 3x_1 - 4x_2 - 2x_3 = -1 \\ 2x_1 - x_2 + x_3 = -5 \end{cases}$	7.12 $\begin{cases} x_1 - 2x_2 - 5x_3 = -7 \\ 2x_1 + 3x_2 + 4x_3 = -7 \\ x_1 + 2x_2 + 3x_3 = -3 \end{cases}$
7.13 $\begin{cases} 4x_1 + x_2 - 4x_3 = 15 \\ 2x_1 - x_2 - 2x_3 = 9 \\ x_1 + 3x_2 + 2x_3 = 10 \end{cases}$	7.14 $\begin{cases} 2x_1 - x_2 + 3x_3 = -3 \\ 4x_1 + 3x_2 + 4x_3 = -6 \\ x_1 + x_2 + 2x_3 = -7 \end{cases}$	7.15 $\begin{cases} 2x_1 - x_2 - 2x_3 = 6 \\ x_1 + 3x_2 + 2x_3 = -5 \\ 4x_1 + x_2 - 3x_3 = 2 \end{cases}$
7.16 $\begin{cases} 2x_1 - x_2 - 3x_3 = 4 \\ 5x_1 + x_2 + x_3 = 9 \\ 3x_1 + x_2 + 2x_3 = 4 \end{cases}$	7.17 $\begin{cases} 2x_1 - x_2 + 3x_3 = -2 \\ 3x_1 + 4x_2 - 2x_3 = 6 \\ 5x_1 + x_2 + 3x_3 = 0 \end{cases}$	7.18 $\begin{cases} 3x_1 + 5x_2 + 4x_3 = 7 \\ 3x_1 + x_2 - 2x_3 = 5 \\ x_1 + 4x_2 + 2x_3 = -5 \end{cases}$
7.19 $\begin{cases} 3x_1 + x_2 + x_3 = -5 \\ 3x_1 + 5x_2 + x_3 = 8 \\ x_1 + 4x_2 - 2x_3 = -2 \end{cases}$	7.20 $\begin{cases} 3x_1 + x_2 + x_3 = 6 \\ 5x_1 + x_2 - 2x_3 = 1 \\ x_1 + 2x_2 + 4x_3 = 8 \end{cases}$	7.21 $\begin{cases} 3x_1 - x_2 + x_3 = 9 \\ 5x_1 - 2x_2 + 2x_3 = 13 \\ x_1 + 2x_2 + 4x_3 = -1 \end{cases}$
7.22 $\begin{cases} 2x_1 + 3x_2 + x_3 = 3 \\ 2x_1 + 3x_2 + 3x_3 = 7 \\ 3x_1 + 2x_2 + x_3 = 11 \end{cases}$	7.23 $\begin{cases} 4x_1 - x_2 + 5x_3 = 8 \\ 3x_1 + x_2 + 4x_3 = 7 \\ 3x_1 - x_2 + 2x_3 = 11 \end{cases}$	7.24 $\begin{cases} x_1 + 2x_2 + 5x_3 = -8 \\ 2x_1 + 3x_2 + 4x_3 = -14 \\ 3x_1 + 2x_2 + 5x_3 = -2 \end{cases}$
7.25 $\begin{cases} x_1 - 2x_2 - 3x_3 = -5 \\ 2x_1 + 3x_2 + 4x_3 = -1 \\ 3x_1 + x_2 + x_3 = -6 \end{cases}$	7.26 $\begin{cases} x_1 + 2x_2 + 6x_3 = 1 \\ 3x_1 + x_2 - 4x_3 = -5 \\ 2x_1 + 3x_2 + x_3 = 8 \end{cases}$	7.27 $\begin{cases} 2x_1 + 8x_2 - 7x_3 = -5 \\ 2x_1 + 5x_2 - 6x_3 = -1 \\ x_1 + 3x_2 - x_3 = -1 \end{cases}$

7.28	7.29	7.30
$\begin{cases} 5x_1 + 2x_2 + 4x_3 = -4 \\ 2x_1 + x_2 - 3x_3 = 4 \\ -2x_1 - 3x_2 + x_3 = 3 \end{cases}$	$\begin{cases} 2x_1 + 3x_2 + 2x_3 = -9 \\ 3x_1 + 4x_2 - 7x_3 = 6 \\ 5x_1 + x_2 - 5x_3 = -9 \end{cases}$	$\begin{cases} 7x_1 - 4x_2 - 3x_3 = -3 \\ 3x_1 + 2x_2 - 3x_3 = 3 \\ 2x_1 - 3x_2 + x_3 = 1 \end{cases}$

Task 8. Given point *A*; radius of the circle *R*; *a*, *b* – semiaxes of curves; equation of the straight line *D*. It is required to:

а) write the equation of a circle with center *A* and radius *R*;

б) write the canonical equation of an ellipse with semiaxes *a* and *b*. Find the coordinates of it's foci and the eccentricity;

c) write the canonical equation of the hyperbola with the real semiaxis *a* and imaginary semiaxis *b*. Find the coordinates of its foci, eccentricity, equations of the asymptotes;

d) write the canonical equation of the parabola with vertex at the origin and if *D* is its directrix. Find the coordinates of its focus, eccentricity;

e) make the drawing of ellipse, hyperbola, parabola.

8.1	A(2,-4), R=4, a=1, b=3, D: x=-5	8.2	A(-8,2), R=1, a=6, b=5, D: x=-5
8.3	A(1,-4), R=5, a=8, b=3, D: y=-6	8.4	A(5,-4), R=2, a=6, b=4, D: y=-2
8.5	A(2,-5), R=7, a=3, b=2, D: x=4	8.6	A(1,8), R=5 , a=3, b=2, D: x=-3
8.7	A(3,-4), R=9, a=7, b=6, D: y=-2	8.8	A(10,1), R=8, a=1, b=6, D: y=-4
8.9	A(5,-4), R=1, a=6, b=4, D: x=-5	8.10	A(6,3), R=8 , a=2, b=3, D: x=-5
8.11	A(1,-3), R=5 , a=8, b=2, D: y=6	8.12	A(5,5), R=2, a=1, b=3, D: y=-7
8.13	A(2,-6), R=7, a=3, b=4, D: x=5	8.14	A(12,6), R=7, a=6, b=2, D: x=-5
8.15	A(3,4), R=9 , a=2, b=6, D: y=-8	8.16	A(0,5), R=4, a=6, b=4, D: y=8
8.17	A(2,-9), R=7, a=5, b=2, D: x=6	8.18	A(-5,0), R=7, a=4, b=5, D: x=1
8.19	A(8,4), R=6 , a=8, b=5, D: y=2	8.20	A(5,1), R=2, a=9, b=1, D: x=-1
8.21	A(5,-4), R=4, a=6, b=4, D: x=1	8.22	A(-3,2), R=4, a=8, b=4, D: y=1
8.23	A(1,8), R=5, a=9, b=4, D: y=-6	8.24	A(9,1), R=6, a=4, b=7, D: x=-3
8.25	A(2,-5), R=7, a=7, b=4, D: x=9	8.26	A(-9,2), R=7, a=1, b=8, D: y=7
8.27	A(7,4), R=5, a=1, b=7, D: y=8	8.28	A(11,-4), R=2, a=2, b=4, D: x=8
8.29	A(-2,5), R=5, a=7, b=1, D: x=8	8.30	A(12,4), R=7, a=3, b=5, D: y=-9

Task 9. Determine the type (name) of the second order surface and make a schematic drawing.

9.1	$a)\frac{x^2}{3}+\frac{y^2}{4}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}-\frac{z^2}{6}=3y$	9.2	$a)\frac{x^2}{16}-\frac{y^2}{16}-\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}+\frac{z^2}{3}=1$
9.3	$a)\frac{x^2}{3}+\frac{y^2}{16}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{3}+\frac{z^2}{4}=y$	9.4	$a)\frac{x^2}{10}+\frac{y^2}{10}+\frac{z^2}{10}=1;\quad b)\frac{x^2}{3}-\frac{z^2}{6}=1$
9.5	$a)-\frac{x^2}{3}+\frac{y^2}{4}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{1}+\frac{y^2}{9}=5z$	9.6	$a)\frac{x^2}{1}+\frac{y^2}{1}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{3}-\frac{y^2}{5}=z$

9.7	$a)\frac{x^2}{25}-\frac{y^2}{9}+\frac{z^2}{4}=1;\quad b)\frac{y^2}{3}+\frac{z^2}{4}=1$	9.8	$a)-\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{4}=-1;\quad b)\frac{y^2}{16}-\frac{z^2}{9}=6z$
9.9	$a)\frac{x^2}{12}+\frac{y^2}{9}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{3}-\frac{z^2}{6}=1$	9.10	$a)\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{1}=1;\quad b)\frac{x^2}{9}+\frac{z^2}{4}=-2y$
9.11	$a)\quad\frac{x^2}{9}+\frac{y^2}{16}-\frac{z^2}{16}=1;\quad b)\frac{x^2}{9}+\frac{z^2}{4}=4y$	9.12	$a)-\frac{x^2}{12}+\frac{y^2}{9}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}+\frac{y^2}{6}=-4z$
9.13	$a)-\frac{x^2}{3}-\frac{y^2}{8}+\frac{z^2}{4}=1;\quad b)\frac{y^2}{10}-\frac{z^2}{6}=2x$	9.14	$a)\frac{x^2}{2}+\frac{y^2}{16}+\frac{z^2}{1}=1;\quad b)\frac{x^2}{4}-\frac{z^2}{4}=0$
9.15	$a)-\frac{x^2}{4}+\frac{y^2}{16}-\frac{z^2}{1}=1;\quad b)-\frac{x^2}{3}+\frac{z^2}{4}=2y$	9.16	$a)-\frac{x^2}{3}-\frac{y^2}{9}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}-\frac{y^2}{6}=0$
9.17	$a)\frac{x^2}{3}+\frac{y^2}{4}-\frac{z^2}{5}=-1;\quad b)\frac{x^2}{3}=3y$	9.18	$a)\frac{x^2}{4}+\frac{y^2}{16}-\frac{z^2}{1}=-1;\quad b)-\frac{x^2}{8}+\frac{z^2}{4}=0$
9.19	$a)-\frac{x^2}{4}+\frac{y^2}{1}-\frac{z^2}{16}=-1;\quad b)\frac{x^2}{9}-\frac{z^2}{4}=y$	9.20	$a)\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{4}=-1;\quad b)\frac{x^2}{3}-\frac{z^2}{6}=1$
9.21	$a)\frac{x^2}{3}-\frac{y^2}{16}+\frac{z^2}{9}=1;\quad b)\frac{x^2}{3}+\frac{y^2}{5}=2z$	9.22	$a)\frac{x^2}{9}-\frac{y^2}{16}+\frac{z^2}{64}=1;\quad b)\frac{x^2}{3}=-8y$
9.23	$a)\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{12}-\frac{z^2}{4}=2y$	9.24	$a)\frac{x^2}{12}+\frac{y^2}{4}-\frac{z^2}{4}=1;\quad b)-\frac{z^2}{6}=4y$
9.25	$a)-\frac{x^2}{9}-\frac{y^2}{25}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}-\frac{z^2}{9}=1$	9.26	$a)\frac{x^2}{16}+\frac{y^2}{16}+\frac{z^2}{16}=1;\quad b)\frac{x^2}{16}+\frac{z^2}{16}=-2y$
9.27	$a)\frac{x^2}{25}-\frac{y^2}{16}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{4}+\frac{z^2}{4}=1$	9.28	$a)\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{4}-\frac{z^2}{4}=2y$
9.29	$a)-\frac{x^2}{9}+\frac{y^2}{1}+\frac{z^2}{25}=1;\quad b)\frac{y^2}{3}-\frac{x^2}{6}=4z$	9.30	$a)-\frac{x^2}{16}+\frac{y^2}{16}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{4}+\frac{z^2}{4}=1$

Task 10. Lead to the canonical form the equation of the second order, construct this curve.

10.1	$x^2+8x+2y+20=0$	10.2	$9x^2+4y^2-54x-32y+109=0$
10.3	$x^2+8x-2y-16=0$	10.4	$2x^2-2y^2+2x=0$
10.5	$x^2+2y^2-2x+8y+7=0$	10.6	$-5y^2+x+20y-6=0$
10.7	$4x^2+8x-6y+28=0$	10.8	$x^2+6x-2y+5=0$
10.9	$x^2+9y^2-40x+36y+100=0$	10.10	$16x^2-9y^2-64x-18y+199=0$
10.11	$4y^2-2x+8y+28=0$	10.12	$9x^2+10y^2+40y-50=0$
10.13	$3x^2-4y^2+18x+20=0$	10.14	$x^2+6x-2y+30=0$
10.15	$y^2+4x+8y+16=0$	10.16	$4y^2+8x+12y+10=0$

10.17	$4x^2+8x+2y+30=0$	10.18	$x^2-4y^2-2x-24y-64=0$
10.19	$y^2-8x+8y+32=0$	10.20	$x^2+y^2-2x+8y+9=0$
10.21	$5x^2+9y^2+30x+18y+9=0$	10.22	$4y^2+8x+16y+30=0$
10.23	$-y^2+8x-2y-9=0$	10.24	$9x^2-y^2-18x-6y-10=0$
10.25	$9x^2-16y^2-36x-64y-127=0$	10.26	$-3y^2+2x-12y+30=0$
10.27	$-y^2+12x-2y-25=0$	10.28	$x^2-y^2-4y-4=0$
10.29	$x^2+y^2+8x-12y+20=0$	10.30	$-16y^2-4x-64y-64=0$

Task 11. Given complex numbers z_1 and z_2. It is required to find:

a) the module of the complex numbers z_1;

b) the argument of the complex number z_1;

c) the representation of the complex number z_1 in the trigonometric and exponential forms;

d) the sum of complex numbers z_1 and z_2 analytically and graphically;

e) $(z_2)^5$;

f) multiplication of $z_1 \cdot z_2$ in trigonometric form;

g) all complex roots $\sqrt[3]{z_2}$ (for versions with even numbers) and $\sqrt[4]{z_2}$ (for versions with odd numbers). Show the solution in the drawing.

11.1	$z_1=8i-8;$ $z_2=8\sqrt{2}(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})$	11.2	$z_1=5i+5\sqrt{3};$ $z_2=5(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$
11.3	$z_1=-7i-7;$ $z_2=7\sqrt{2}(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2})$	11.4	$z_1=-10;$ $z_2=16(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$
11.5	$z_1=-9+9\sqrt{3}i;$ $z_2=18(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4})$	11.6	$z_1=3-3i;$ $z_2=3\sqrt{2}(\cos\frac{6\pi}{4}+i\sin\frac{6\pi}{4})$
11.7	$z_1=2i-2\sqrt{3};$ $z_2=4(\cos\pi+i\sin\pi)$	11.8	$z_1=27i-27;$ $z_2=27(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6})$
11.9	$z_1=-4+4i;$ $z_2=4\sqrt{2}(\cos(-\frac{3\pi}{4})+i\sin(-\frac{3\pi}{4}))$	11.10	$z_1=6+6\sqrt{3}i;$ $z_2=12(\cos\frac{2\pi}{6}+i\sin\frac{2\pi}{6})$

11.11	$z_1 = -9\sqrt{3} - 9i;$ $z_2 = 18(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$	11.12	$z_1 = 4 - 4\sqrt{3}i;$ $z_2 = 8(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$
11.13	$z_1 = -4\sqrt{3} - 4i;$ $z_2 = 8(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3}))$	11.14	$z_1 = -5 - 5i;$ $z_2 = 5\sqrt{2}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$
11.15	$z_1 = -9i;$ $z_2 = 9(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$	11.16	$z_1 = 3i;$ $z_2 = 3(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$
11.17	$z_1 = \sqrt{3} - 3i;$ $z_2 = 2\sqrt{3}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$	11.18	$z_1 = \sqrt{2} - \sqrt{2}i;$ $z_2 = 2(\cos\frac{6\pi}{3} + i\sin\frac{6\pi}{3})$
11.19	$z_1 = 3 - \sqrt{3}i;$ $z_2 = 2\sqrt{3}(\cos\frac{9\pi}{6} + i\sin\frac{9\pi}{6})$	11.20	$z_1 = 8 - 8i;$ $z_2 = 8\sqrt{2}(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3}))$
11.21	$z_1 = \sqrt{3}i;$ $z_2 = \sqrt{3}(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$	11.22	$z_1 = -64;$ $z_2 = 64(\cos\pi + i\sin\pi)$
11.23	$z_1 = 64i;$ $z_2 = 64(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$	11.24	$z_1 = 3\sqrt{3} - 3i;$ $z_2 = 6(\cos\frac{6\pi}{8} + i\sin\frac{6\pi}{8})$
11.25	$z_1 = 6 + 6i;$ $z_2 = 6\sqrt{2}(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})$	11.26	$z_1 = -5i;$ $z_2 = 5(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$
11.27	$z_1 = \sqrt{3} - \sqrt{3}i;$ $z_2 = 2\sqrt{3}(\cos\frac{8\pi}{6} + i\sin\frac{8\pi}{6})$	11.28	$z_1 = 8 - 8i;$ $z_2 = 8\sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4}))$
11.29	$z_1 = \sqrt{3} - i;$ $z_2 = 2(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4})$	11.30	$z_1 = \sqrt{3}i - 1;$ $z_2 = \sqrt{3}(\cos\frac{10\pi}{6} + i\sin\frac{10\pi}{6})$

1.3 Solution of an exemplary embodiment

Task1. The third order determinant is given $\begin{vmatrix} 3 & 4 & -1 \\ 0 & 2 & 8 \\ 1 & 5 & -2 \end{vmatrix}$, $i=2, j=3$.

It is required to calculate:

a) the minor M_{23} and the algebraic complement A_{23} of the element a_{23};

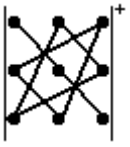
b) the determinant by the triangle rule (rule of Sarrus);

c) the determinant by the method of expansion along the 2-nd row and the 3-rd column.

Solution:

a) the minor M_{ij} of the element a_{ij} equals to the determinant, obtained from given by deleting the *i*-th row and *j*-th column. Thus cross out second row and the third column in our determinant, we obtain $M_{23} = \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} = 3 \cdot 5 - 1 \cdot 4 = 11$. The algebraic complement of the element is calculated by formula $A_{ij} = (-1)^{i+j} M_{ij}$.

So, $A_{23} = (-1)^{2+3} 11 = -11$;

b) the triangle rule: the third order determinant is the sum of six terms; terms with a plus sign are obtained by multiplication of three elements of the determinant taken by the scheme  , terms with a negative sign – by the scheme .

So, $\begin{vmatrix} 3 & 4 & -1 \\ 0 & 2 & 8 \\ 1 & 5 & -2 \end{vmatrix} = 3 \cdot 2 \cdot (-2) + 4 \cdot 8 \cdot 1 + 0 \cdot 5 \cdot (-1) - 1 \cdot 2 \cdot (-1) -$

$$- 3 \cdot 8 \cdot 5 - 4 \cdot 0 \cdot (-2) = -12 + 32 + 2 - 120 = -98;$$

c) expansion formula along the third column has the form:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33},$$

$$\begin{vmatrix} 3 & 4 & -1 \\ 0 & 2 & 8 \\ 1 & 5 & -2 \end{vmatrix} = (-1) \cdot (-1)^{1+3} \cdot \begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} + 8 \cdot (-1)^{2+3} \cdot \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} + (-2) \cdot (-1)^{3+3} \cdot \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} =$$

$$= 2 - 88 - 12 = -98;$$

similarly calculate the determinant expanding along the second row:

$$\begin{vmatrix} 3 & 4 & -1 \\ 0 & 2 & 8 \\ 1 & 5 & -2 \end{vmatrix} = 0 \cdot (-1)^{2+1} \cdot \begin{vmatrix} 4 & -1 \\ 5 & -2 \end{vmatrix} + 2 \cdot (-1)^{2+2} \cdot \begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} + 8 \cdot (-1)^{2+3} \cdot \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} =$$

$$= 0 - 10 - 88 = -98.$$

Task 2. Given matrices $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 \end{pmatrix}$, $C = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$.

Find:

a) the product of matrices *AB* or *BC*, or *CB* (if possible). Explain why if it is impossible;

b) the matrix A^{-1}, the inverse matrix for *A*.

Solution:

a) the product of matrices *AB* is possible if the number of columns of matrix *A* equals the number of rows of the matrix *B*. Matrix sizes: $A_{3\times3}$, $B_{2\times4}$, $C_{4\times1}$. So, $A_{3\times3} \cdot B_{2\times4} = |3 \neq 2|$ – multiplication is impossible. $B_{2\times4} \cdot C_{4\times1} = |4 = 4|$ – multiplication is possible. $C_{4\times1} \cdot B_{2\times4} = |1 \neq 2|$ – multiplication is impossible. Matrix multiplication *BC* is a matrix *E*, the number of rows is equal to the number of rows of the matrix *B*, the number of columns equals the number of columns of the matrix *C*: $B_{m\times k} \cdot C_{k\times n} = E_{m\times n}$. Element e_{ij} of the matrix *E* is equal to the sum of products of *i*-th row of the matrix *B* for the *j*-th column of the matrix *C*.

So, $BC = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1\cdot1+1\cdot2+1\cdot3+1\cdot4 \\ 3\cdot1+3\cdot2+3\cdot3+3\cdot4 \end{pmatrix} = \begin{pmatrix} 10 \\ 30 \end{pmatrix}$,

where $(B_{2\times4} \cdot C_{4\times1} = E_{2\times1})$.

And $B \cdot C = E = \begin{pmatrix} 10 \\ 30 \end{pmatrix}$;

b) an inverse matrix exists for a square matrix *A*, if the determinant of the matrix is not equal to zero; and an inverse matrix does not exist if $|A| = 0$.

The inverse matrix A^{-1} for matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is obtained by the formula $A^{-1} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$,

where $|A|$ – the determinant of the matrix A;

A_{ij} – the algebraic complement of the elements a_{ij}.

Let us find the determinant of the matrix A:

$$|A| = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & -1 \end{vmatrix} = -30 \neq 0, \text{ so, } A^{-1} \text{ exists.}$$

Let us find the algebraic complements of all elements of the matrix A:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & -4 \\ -2 & -1 \end{vmatrix} = -11; \quad A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -4 \\ 3 & -1 \end{vmatrix} = -10;$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -13; \quad A_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 3 \\ -2 & -1 \end{vmatrix} = -8;$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix} = -10; \quad A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -2 \\ 3 & -2 \end{vmatrix} = -4;$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 3 \\ 3 & -4 \end{vmatrix} = -1; \quad A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} = 10;$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = 7.$$

Let us form A^{-1} by the above formula

$$A^{-1} = \frac{1}{-30}\begin{pmatrix} -11 & -8 & -1 \\ -10 & -10 & 10 \\ -13 & -4 & 7 \end{pmatrix};$$

and then it is need to check:

$$AA^{-1} = A^{-1}A \equiv E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Task 3. Given points $A(7, -9, 3)$ and $B(1, 0, -5)$; and vectors $\bar{b} = (1,0,5), \bar{c} = (6,1,-2)$.

Find:

a) the module (length) of the vector $\vec{a} = |\overrightarrow{AB}|$ and the midpoint of the segment *AB*;

b) the projection of the vector $\vec{a}$ on $\vec{c}$;

c) the area of the parallelogram, obtained from the vectors $\vec{b}$ and $\vec{c}$;

d) volume of the pyramid constructed from the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

Solution:

a) for points *A* (x_1, y_1, z_1) and *B*(x_2, y_2, z_2) coordinates of the vector $\bar{a} = \overline{AB}$ are obtained by the formula $\bar{a} = \overline{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$.

So, $\bar{a}$ = (1–7, 0– (–9), –5–3) = (–6, 9, –8);

The module (length) of the vector $\bar{a}$: $|\bar{a}| = \sqrt{x_1^2 + y_1^2 + z_1^2}$.

So, $|\overline{AB}| = \sqrt{(-6)^2 + 9^2 + (-8)^2} = \sqrt{181}$;

the midpoint of the segment AB has coordinates

$$C\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = \mathrm{C}\left(\frac{7+1}{2}, \frac{0-9}{2}, \frac{-5+3}{2}\right) = \mathrm{C}(4, -9/2, -1);$$

b) the projection of the vector $\vec{a}$ on $\vec{c}$ is $\mathrm{Pr}_c\, \vec{a} = \dfrac{\vec{a} \cdot \vec{c}}{|\bar{c}|}$;

where $\bar{a} \cdot \bar{c} = x_1 \cdot x_3 + y_1 \cdot y_3 + z_1 \cdot z_3$ – scalar product of vectors $\bar{a} = (x_1, y_1, z_1)$, $\bar{c} = (x_3, y_3, z_3)$.

If vectors $\vec{a}$ and $\vec{c}$ are orthogonal, then $\bar{a} \cdot \bar{c} = 0$.

In our case $\bar{a} \cdot \bar{c} = (-6) \cdot 6 + 9 \cdot 1 + 8 \cdot (-2) = -43$, as $\bar{a} \cdot \bar{c} \neq 0$, so the vectors $\bar{a}$ and $\bar{c}$ are not orthogonal; calculate more

$$\mathrm{Pr}_c\, \vec{a} = \frac{\vec{a} \cdot \vec{c}}{|\bar{c}|} = \frac{-43}{\sqrt{6^2 + 1^2 + (-2)^2}} = \frac{-43}{41} \approx -1{,}049;$$

c) the area of the parallelogram, obtained from the vectors $\bar{b} = (x_2, y_2, z_2)$, $\bar{c} = (x_3, y_3, z_3)$, is the module of the vector, received from their vector product: $|\vec{d}| = |\vec{b} \times \vec{c}|$; at first find the coordinates:

$$\bar{b} \times \bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = \begin{vmatrix} y_2 & z_2 \\ y_3 & z_3 \end{vmatrix} \cdot \bar{i} - \begin{vmatrix} x_2 & z_2 \\ x_3 & z_3 \end{vmatrix} \cdot \bar{j} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \cdot \bar{k} = \left(\begin{vmatrix} y_2 & z_2 \\ y_3 & z_3 \end{vmatrix}, -\begin{vmatrix} x_2 & z_2 \\ x_3 & z_3 \end{vmatrix}, \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \right);$$

$$\bar{b}\times\bar{c}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 0 & 5\\ 6 & 1 & -2\end{vmatrix}=\begin{vmatrix}0 & 5\\ 1 & -2\end{vmatrix}\cdot\bar{i}-\begin{vmatrix}1 & 5\\ 6 & -2\end{vmatrix}\cdot\bar{j}+\begin{vmatrix}1 & 0\\ 6 & 1\end{vmatrix}\cdot\bar{k}=(-5,32,1);$$

further find module of the vector:
$\left|\vec{d}\right|=\left|\vec{b}\times\vec{c}\right|=|(-5,32,1)|=\sqrt{(-5)^2+32^2+1^2}=\sqrt{1050}$;

d) the volume of pyramid constructed from vectors $\bar{a}$, $\bar{b}$, $\bar{c}$, can be found from a module of their mixed product

$$V=\frac{1}{6}\left|\bar{a}\bar{b}\bar{c}\right|=\frac{1}{6}\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}=$$

$$=\frac{1}{6}\begin{vmatrix}-6 & 9 & 8\\ 1 & 0 & 5\\ 6 & 1 & -2\end{vmatrix}=\frac{1}{6}(0+270+8-0+30+18)=\frac{1}{6}\cdot 326=54\frac{1}{3}.$$

As $\bar{a}\bar{b}\bar{c}\neq 0$, these vectors are not coplanar.

Task 4. Given points $A_1(4, -2)$, $A_2(8, 1)$ on a plane and the equation of the line L_1: $-x + 4y + 5 = 0$.

Write the equation of a straight line:

a) $L_2=(A_1A_2)$ – passing through these points;

b) L_2 – as a general equation of straight line;

c) L_2 – in the form of equation of straight line with slope;

d) L_2 – in the form of equation of straight line in segments;

e) L_3, passing through the point A_2 and perpendicular to L_1.

Solution:

a) the equation of the straight line passing through two points $M_1(x_1, y_1)$, $M_2(x_2, y_2)$, is obtained by the formula $\dfrac{x-x_1}{x_2-x_1}=\dfrac{y-y_1}{y_2-y_1}$.

So, equation of the straight line L_2 is $\dfrac{x-4}{8-4}=\dfrac{y-(-2)}{1-(-2)}$ or $\dfrac{x-4}{4}=\dfrac{y+2}{3}$;

b) let us write the equation of the straight line L in the general form $Ax + By + C = 0$:

$$\frac{x-4}{4}=\frac{y+2}{3}\rightarrow 3x-12=4y+8\rightarrow 3x-4y-20=0,\ (A=3,\ B=-4,\ C=-20).$$

The geometric meaning of the coefficients: A, B – are coordinates of the normal (perpendicular) vector of the straight line L, ie $\overline{N} = (A,B) = (3,-4) \perp L_2$;

c) L_2 in the form of equation of straight line with slope $y = kx + m$:

$$\frac{x-4}{4} = \frac{y+2}{3} \rightarrow \frac{3}{4}(x-4) = y+2 \rightarrow y = \frac{3}{4}x - 5 \text{ , } (k = \frac{3}{4}, m = -5);$$

d) L_2 in the form of equation of straight line in segments $\frac{x}{a} + \frac{y}{b} = 1$:

$$3x - 4y = 20 \rightarrow \frac{x}{20/3} + \frac{y}{-5} = 1, \text{ (} a = 20/3, b = -5 \text{);}$$

e) the equation of the straight line passing through the point (x_1, y_1) and parallel to the vector $\bar{b} = (m,n)$, has the form: $\frac{x-x_1}{m} = \frac{y-y_1}{n}$.

If vector $\overline{N}_1 = (A;B) = (-1;4)$ is perpendicular to the straight line L_1 and the straight line L_3 is also perpendicular to L_1, then we find in our case that the directional vector for the straight line L_3 is $\bar{b} = (m,n) = \overline{N}_1 = (-1;4)$ and the equation has the form $L_3 \perp L_1$: $\frac{x-8}{-1} = \frac{y-1}{4}$.

Task 5. Given points $A_1(1, 2, -1)$, $A_2(3, 3, 2)$, $A_3(2, -3, 7)$. It is required to:

a) write the equation of the plane $P_1 = (A_1A_2A_3)$;

b) write P_1 as a general equation of a plane;

c) write P_1 as equation of a plane in segments;

d) write P_1 in a general equation of a plane, passing through the point A_1;

e) make a canonical equation of the straight line $L_1 = (A_2A_3)$;

f) write a parametric equation of the straight line L_1;

g) write the equation of the straight line $L_2 = (A_1N)$ perpendicular to the plane P_1.

Solution:

a) the equation of the plane passing through the points $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$, $M_3(x_3, y_3, z_3)$ has the form: $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$.

Here P_1: $\begin{vmatrix} x-1 & y-2 & z+1 \\ 3-1 & 3-2 & 2+1 \\ 2-1 & -3-2 & 7+1 \end{vmatrix} = 0 \rightarrow \begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & 1 & 3 \\ 1 & -5 & 8 \end{vmatrix} = 0$ (*);

b) expanding the determinant in the right-hand side of (*), we obtain the equation of the plane P_1 in the general form $Ax + By + Cz + D = 0$:

$$23x - 13y - 11z - 8 = 0, (A = 23, B = -13, C = -11, D = -8).$$

The geometrical sense of coefficients: A, B, C – are coordinates of a normal (perpendicular) vector of the plane, ie vector $\overline{N} = (23,-13,-11) \perp P_1$;

c) let us move free term –8 in the general equation of plane on the right side and divide both sides by 8. Obtain the equation of the plane in segments $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$:

$$\frac{x}{8/23} + \frac{y}{-8/13} + \frac{z}{-8/11} = 1,$$

where $a = 8/23$, $b = -8/13$, $c = -8/11$ – are sizes of the segments cut off by the plane on coordinate axes, considering from the origin of coordinates;

d) let us expand the determinant on the left side of (*) along the first row

$$(x-1)\cdot\begin{vmatrix} 1 & 3 \\ -5 & 8 \end{vmatrix} - (y-2)\cdot\begin{vmatrix} 2 & 3 \\ 1 & 8 \end{vmatrix} + (z+1)\cdot\begin{vmatrix} 2 & 1 \\ 1 & -5 \end{vmatrix} = 0 \rightarrow$$

$$\rightarrow 23\cdot(x-1) - 13\cdot(y-2) - 11\cdot(z+1) = 0.$$

We obtain the equation of plane in the form $A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$ $(x_0 = 1, y_0 = 2, z_0 = -1)$;

e) the canonical equation of the straight line passing through two points $M_1(x_1, y_1, z_1)$ and $M_2(x_2, y_2, z_2)$, has the form $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$. So, the equation of the straight line L_1 has the form $\frac{x-3}{2-3} = \frac{y-3}{-3-3} = \frac{z-2}{7-2}$ or $\frac{x-3}{-1} = \frac{y-3}{-6} = \frac{z-2}{5}$;

f) the equation of the straight line in the form $\frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p}$ is called canonical, where $a = (m, n, p)$ – is the direction vector. In the previous section we have already found the canonical equations of straight lines. The equations of the straight line in the form $\begin{cases} x = mt + x_0 \\ y = nt + y_0 \\ z = pt + z_0 \end{cases}$ are called parametric. To get

the parametric equations of the straight line, we equate the canonical equations to the parameter *t*, and from these equalities we find *x*, *y*, *z*.

$$L_1:\ \frac{x-3}{-1}=\frac{y-3}{-6}=\frac{z-2}{5}=t\rightarrow\begin{cases}\dfrac{x-3}{-1}=t\rightarrow x=-t+3\\ \dfrac{y-3}{-6}=t\rightarrow y=-6t+3\\ \dfrac{z-2}{5}=t\rightarrow z=5t+2\end{cases}.$$

So, the parametric equations of $L_1:\begin{cases}x=-t+3\\ y=-6t+32\\ z=5t+2\end{cases}$;

g) as $(A_1N)\perp P_1$, so vector $\overline{N}=(23,-13,-11)\perp P_1$ is the direction vector of the straight line $L_2=(A_1N)$. Then the canonical equation of the straight line $L_2=(A_1N)$ has the form: $\frac{x-1}{23}=\frac{y-2}{-13}=\frac{z+1}{-11}$.

Task 6. Solve the system $\begin{cases}x+2y=1\\ -3x+4y=3\end{cases}$:

a) by Cramer's method;

b) by matrix method (using the inverse matrix).

Solution:

a) the solution of the system $\begin{cases}a_{11}x+a_{12}y=b_1\\ a_{21}x+a_{22}y=b_2\end{cases}$ by Cramer's rule has the form $x=\frac{\Delta_1}{\Delta}$, $y=\frac{\Delta_2}{\Delta}$, where $\Delta=\begin{vmatrix}a_{11}&a_{12}\\ a_{21}&a_{22}\end{vmatrix}$ – determinant of the system, $\Delta_1=\begin{vmatrix}b_1&a_{12}\\ b_2&a_{22}\end{vmatrix}$, $\Delta_2=\begin{vmatrix}a_{11}&b_1\\ a_{21}&b_2\end{vmatrix}$ – auxiliary determinants obtained from the determinant of the system by replacing the first and second columns by column of free terms.

In this case $\Delta=\begin{vmatrix}1&2\\ -3&4\end{vmatrix}=10$, $\Delta_1=\begin{vmatrix}1&2\\ 3&4\end{vmatrix}=-2$, $\Delta_2=\begin{vmatrix}1&1\\ -3&3\end{vmatrix}=6$.

So, $x=-1/5$, $y=3/5$.

The answer can be written as a vector: $\overline{X}=\begin{pmatrix}-1/5\\ 3/5\end{pmatrix}$;

b) in matrix form solution of the system $\begin{cases}a_{11}x+a_{12}y=b_1\\ a_{21}x+a_{22}y=b_2\end{cases}$ is written as:

$X = A^{-1} \cdot B$,

where $X = \begin{pmatrix} x \\ y \end{pmatrix}$ – column matrix of unknowns;

$B = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ – column matrix of free terms;

A^{-1} – inverse matrix for the matrix of the system $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$.

Let us find the inverse matrix (look example 2b) for the matrix of the system $A = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}$. As the determinant of the system $|A| = \Delta = 10 \neq 0$, then the inverse matrix exists and is equal to $A^{-1} = \frac{1}{10}\begin{pmatrix} 4 & -2 \\ 3 & 1 \end{pmatrix}$.

Then $\begin{pmatrix} x \\ y \end{pmatrix} = X = A^{-1} \cdot B = \frac{1}{10}\begin{pmatrix} 4 & -2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -1/5 \\ 3/5 \end{pmatrix}$. Or the answer in the usual form: $x = -1/5$, $y = 3/5$.

Task 7. Solve the system of equations. $\begin{cases} x_1 - x_2 + 5x_3 = 3 \\ 2x_1 - 3x_2 + 4x_3 = 2 : \\ 3x_1 - 3x_2 - x_3 = -7 \end{cases}$

a) by Cramer's method;
b) by matrix method (using the inverse matrix).

Solution:

a) the coefficient matrix of the system is $A = \begin{pmatrix} 1 & -1 & 5 \\ 2 & -3 & 4 \\ 3 & -3 & -1 \end{pmatrix}$, column of free terms – $B = \begin{pmatrix} 3 \\ 2 \\ -7 \end{pmatrix}$.

Then the solution of the system by Cramer's rule has the form:

$$x_1 = \frac{\Delta_1}{\Delta}, x_2 = \frac{\Delta_2}{\Delta}, x_3 = \frac{\Delta_3}{\Delta},$$

where $\Delta = \begin{vmatrix} 1 & -1 & 5 \\ 2 & -3 & 4 \\ 3 & -3 & -1 \end{vmatrix} = 3 - 12 - 30 + 45 + 12 - 2 = 16$ is the determinant of the system,

$$\Delta_1 = \begin{vmatrix} 3 & -1 & 5 \\ 2 & -3 & 4 \\ -7 & -3 & -1 \end{vmatrix} = -64, \quad \Delta_2 = \begin{vmatrix} 1 & -3 & 5 \\ 2 & 2 & 4 \\ 3 & -7 & -1 \end{vmatrix} = -32, \quad \Delta_3 = \begin{vmatrix} 1 & -1 & -3 \\ 2 & -3 & 2 \\ 3 & -3 & -7 \end{vmatrix} = 16$$ –

auxiliary determinants obtained from the determinant of the system by replacing the first, second and third column by column of free terms. Finally, we obtain the solution by Cramer's method:

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{-64}{16} = -4, \quad x_2 = \frac{\Delta_2}{\Delta} = \frac{-32}{16} = -2, \quad x_3 = \frac{\Delta_3}{\Delta} = \frac{16}{16} = 1 \quad \text{or}$$

$$\overline{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix};$$

b) in matrix form the solution of the system is written:

$$\overline{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 5 \\ 2 & -3 & 4 \\ 3 & -3 & -1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 3 \\ 2 \\ -7 \end{pmatrix}.$$

Let us find the inverse matrix (see example 2b) , as $|A| = \Delta = 16 \neq 0$, the inverse matrix exists and has the form

$$A^{-1} = \begin{pmatrix} 1 & -1 & 5 \\ 2 & -3 & 4 \\ 3 & -3 & -1 \end{pmatrix}^{-1} = \frac{1}{16} \begin{pmatrix} 15 & -16 & 11 \\ 14 & -16 & 6 \\ 3 & 0 & -1 \end{pmatrix}.$$

So,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^{-1} \cdot B = \frac{1}{16} \begin{pmatrix} 15 & -16 & 11 \\ 14 & -16 & 6 \\ 3 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -7 \end{pmatrix} = \frac{1}{16} \begin{pmatrix} 45-32-77 \\ 42-32-42 \\ 9+7 \end{pmatrix} = \frac{1}{16} \begin{pmatrix} -64 \\ -32 \\ 16 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}.$$

Task 8. Given point $A(3, -7)$; radius of the circle $R = 6$; $a = 2$; $b = 3$ – semi-axes of curves; D: $y = -3$ – equation of the straight line. It is required to:

a) write the equation of a circle with center A and radius R;

b) write the canonical equation of an ellipse with semiaxes $a = 2$ and $b = 3$. Find the coordinates of it's foci and the eccentricity;

c) write the canonical equation of the hyperbola with the real semiaxis $a = 2$ and imaginary semiaxis $b = 3$. Find the coordinates of it's foci, eccentricity, equations of the asymptotes;

d) write the canonical equation of the parabola with vertex at the origin and if D: $y = -3$ is its directrix. Find the coordinates of its focus, eccentricity;

e) make the drawing of ellipse, hyperbola, parabola.

Solution:

a) the equation of a circle centered at (x_0, y_0) with the radius *R* has the form $(x-x_0)^2+(y-y_0)^2=R^2$, So, in our version: $(x-3)^2+(y+7)^2=36$;

b) the canonical equation of an ellipse with semiaxes *a* and *b* has the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; So, in our version: $\frac{x^2}{4}+\frac{y^2}{9}=1$.

If $a>b$, then $c=\sqrt{a^2-b^2}$ and points $F_1(-c,0),\ F_2(c,0)$ are foci of the ellipse, and the eccentricity ε of the ellipse is $\varepsilon=\frac{c}{a}$.

If $b<a$, then $c=\sqrt{b^2-a^2}$ and $F_1(0,-c),\ F_2(0,c)$, $\varepsilon=\frac{c}{b}$.

Since $b<a$ in our version then $c=\sqrt{b^2-a^2}=\sqrt{9-4}=\sqrt{5}$ and the eccentricity equals $\varepsilon=\frac{c}{b}=\frac{\sqrt{5}}{3}$, $(\varepsilon<1)$. Foci lie on the *y*-axis: $F_1(0,-\sqrt{5}),\ F_2(0,\sqrt{5})$.

Let us draw the ellipse on the coordinate plane (figure 1.1), where A_1, A_2, B_1, B_2 – vertices of the ellipse.

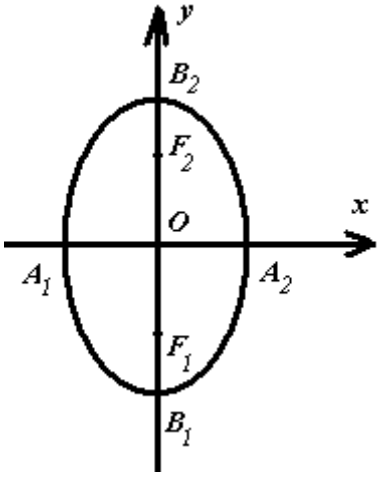


Figure 1.1

c) the canonical equation of hyperbola with real semiaxis *a* , imaginary semiaxis *b* has the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$; with the real semiaxis *b* and the imaginary semiaxis *a*: $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$. For hyperbola with real semiaxis *a*, eccentricity equals $\varepsilon=\frac{c}{a}$, where $c=\sqrt{a^2+b^2}$; asymptotes of the hyperbola

equation have the form $y = \pm\frac{b}{a}x$; foci are points $F_1(-c,0),\ F_2(c,0)$, located on the real axis.

By hypothesis *a* =2, *b* =3, so the canonical equation of the hyperbola with the real semiaxis *a* is $\frac{x^2}{4}-\frac{y^2}{9}=1$. For it half-foci distance $c=\sqrt{4+9}=\sqrt{13}$; eccentricity equals $\varepsilon=\frac{c}{a}=\frac{\sqrt{13}}{2},(\varepsilon>1)$; foci: $F_1(-\sqrt{13},0),\ F_2(\sqrt{13},0)$; equations of the asymptotes: $y=\pm\frac{3}{2}x$.

It is easy to construct a hyperbola as follows: construct a rectangle with sides $x=\pm a,\ y=\pm b$ (in our case $x=\pm 2,\ y=\pm 3$). The diagonals of a rectangle are the asymptotes of the hyperbola, the point of intersection of the rectangle with the real axis of the hyperbola – vertices of the hyperbola (figure 1.2):

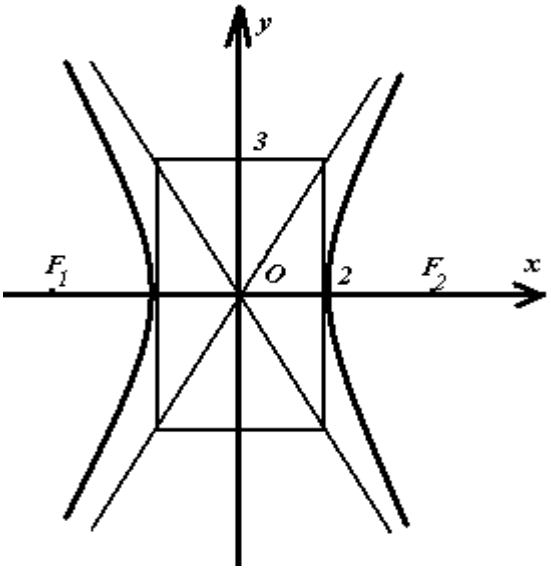


Figure 1.2

d) by hypothesis, if the directrix of the parabola *y* = –*p*/2, then the axis of symmetry of the parabola with vertex at the origin is the *y*-axis. Hence, its canonical equation is $x^2=2py$; if the directrix of the parabola *x* = –*p*/2, then the axis of symmetry of the parabola with vertex at the origin is the *x*-axis. Hence, its canonical equation has the form $y^2=2px$. As the directrix of the parabola has the equation y = –3, then $-\frac{p}{2}=-3\rightarrow p=6$ and the equation of the parabola is: $x^2=2\cdot 6\cdot y\rightarrow x^2=12y$. The focus of the parabola is the point $F(0,\frac{p}{2})$, lying on the axis of symmetry. In our case, the focus is $F(0,3)$. Let us construct the parabola (figure 1.3).

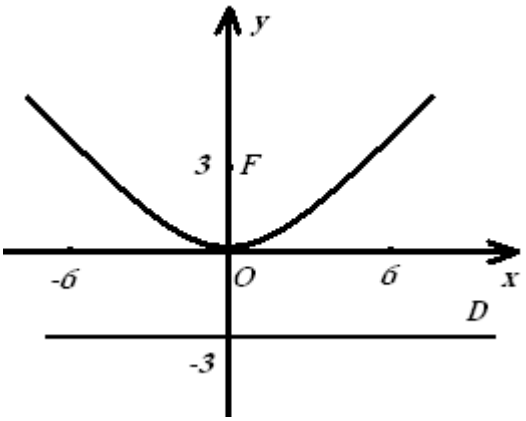


Figure 1.3

Task 9. Determine the type (name) of the second order surface and make a schematic drawing:

a) $-\frac{x^2}{12}+\frac{y^2}{2}+\frac{z^2}{4}=1$; b) $\frac{x^2}{3}+\frac{y^2}{2}-\frac{z^2}{4}=0$.

Solution:

a) $-\frac{x^2}{12}+\frac{y^2}{2}+\frac{z^2}{4}=1$ is the canonical equation of one-sheeted hyperboloid with the symmetry axis *Ox*. Here its schematic drawing (figure 1.4);

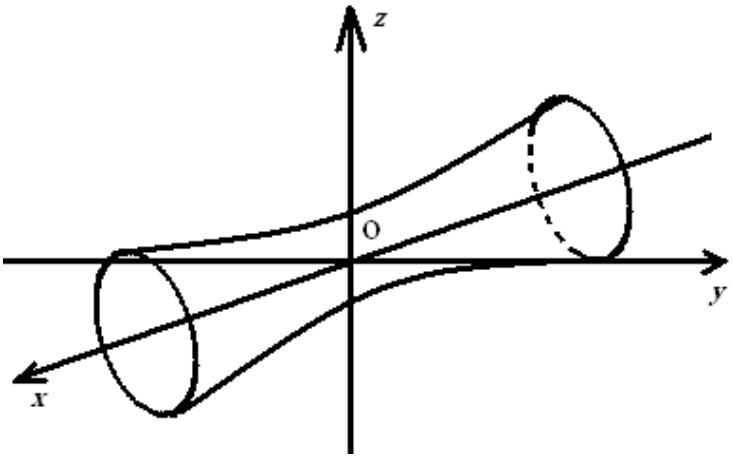


Figure 1.4

b) $\frac{x^2}{3}+\frac{y^2}{2}-\frac{z^2}{4}=0$ is the canonical equation of the cone of the second order with the symmetry axis *Oz* and vertex at the origin. Here its schematic drawing (figure 1.5);

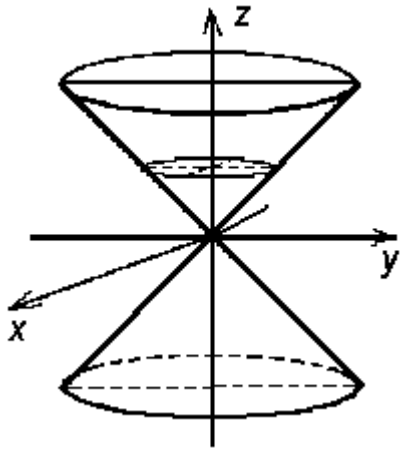


Figure 1.5

Task 10. Lead to the canonical form the equation of the second order $9x^2 - 16y^2 + 18x - 64y - 19 = 0$ and construct this curve.

Solution: the equation of the form

$$\frac{(x-x_0)^2}{a^2} \pm \frac{(y-y_0)^2}{b^2} = 1$$

defines, respectively, the ellipse or the hyperbola.

The equation of the form:

$$(x-x_0)^2 = 2p(y-y_0), \quad (y-y_0)^2 = 2p(x-x_0)^2$$

defines the parabola. These curves have symmetry center (for the ellipse and hyperbola) or vertex of the parabola at the point $C(x_0, y_0)$. To determine the type of the curve, we apply the method of allocating of perfect squares:

$$(9x^2 + 18x) + (-16y^2 - 64y) - 19 = 0 \rightarrow 9(x^2 + 2x) - 16(y^2 + 4y) - 19 = 0.$$

Supplement the terms containing x, and the terms containing y, to complete the squares:

$$9(x^2 + 2x + 1 - 1) - 16(y^2 + 4y + 4 - 4) - 19 = 0;$$

$$9(x^2 + 2x + 1) - 16(y^2 + 4y + 4) - 19 - 9 \cdot 1 - 16 \cdot (-4) = 0;$$

$$9(x+1)^2 - 16(y+2)^2 = -36;$$

$$\frac{(x+1)^2}{-36/9} + \frac{(y+2)^2}{-36/(-16)} = 1; \quad -\frac{(x+1)^2}{4} + \frac{(y+2)^2}{9/4} = 1,$$

so we have the hyperbola with center $C(-1; -2)$, real semi-axis $b = 3/2$ (along the axis Oy) and the imaginary semi-axis $a = 2$. Let us construct its schematic drawing (figure 1.6)

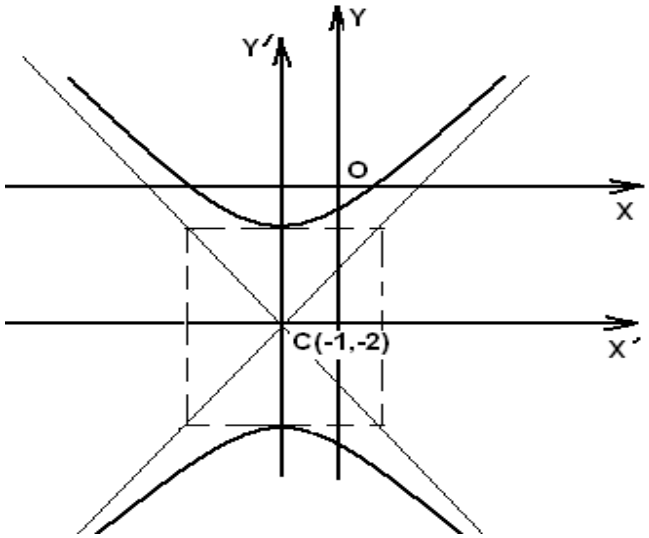


Figure 1.6

Task 11. Given complex numbers:

$$z_1 = 9 - 9i; \quad z_2 = 9(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}).$$

It is required to find:

a) the module of the complex numbers z_1;

b) the argument of the complex number z_1;

c) the representation of the complex number z_1 in the trigonometric and exponential forms;

d) the sum of complex numbers z_1 and z_2 analytically and graphically;

e) $(z_2)^5$;

f) multiplication of $z_1 \cdot z_2$ in trigonometric form;

g) all complex roots $\sqrt[3]{z_2}$ using De Moivre formula.

Solution:

a) if the complex number in algebraic form is:

$$z = \alpha + i\beta = (\alpha, \beta) \Rightarrow z_1 = 9 - 9i = (9, -9), \text{ where } \alpha = 9; \beta = -9,$$

then the module of the complex number z_1:

$$|z| = \sqrt{\alpha^2 + \beta^2} \quad or \quad |z_1| = \sqrt{9^2 + (-9)^2} = 9\sqrt{2};$$

b) the argument of the complex number z_1:

$$\varphi = \arg(z) = arctg\frac{\beta}{\alpha} \Rightarrow \varphi_1 = \arg(z_1) = arctg\frac{-9}{9} = -arctg1 = -\frac{\pi}{4} \ (\alpha = 9 > 0; \beta = -9 < 0);$$

c) in the trigonometric and exponential form complex number has the form:

$$z = \alpha + i\beta = |z|(\cos\varphi + \sin\varphi) = |z|e^{i\varphi};$$

$$z_1 = 9 - 9i = 9\sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})) = 9\sqrt{2}e^{-i\frac{\pi}{4}};$$

d) addition of complex numbers z_1 and z_2:

$$z_2 = 9(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = 9(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2});$$

$$z_1 + z_2 = 9 - 9i + 9(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) = (9 + 9\frac{\sqrt{2}}{2}) + i(9\frac{\sqrt{2}}{2} - 9) \approx 15{,}36 - 2{,}63i;$$

e) for calculating $(z_2)^5$ we use De Moivre formula:

$$z^n = |z|^n(\cos n\varphi + \sin n\varphi) = |z|^n e^{i\varphi n};$$

$$z_2^5 = \left(9\sqrt{2}\right)^5(\cos(\frac{5\pi}{4}) + i\sin(\frac{5\pi}{4})) = \left(9\sqrt{2}\right)^5(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2});$$

f) the multiplication of $z_1 \cdot z_2$ in algebraic form is:

$$z_1 \cdot z_2 = (9 - 9i)\cdot 9(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) = 81\frac{\sqrt{2}}{2} + i81\frac{\sqrt{2}}{2} - i81\frac{\sqrt{2}}{2} - i^2 81\frac{\sqrt{2}}{2} =$$

$$= 81\frac{\sqrt{2}}{2} - (\sqrt{-1})^2 81\frac{\sqrt{2}}{2} = 81\sqrt{2};$$

but multiplication $z_1 \cdot z_2$ in trigonometric form is easily calculated:

$$z_1 \cdot z_2 = |z_1| \cdot |z_2|(\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2));$$

$$z_1 \cdot z_2 = 9\sqrt{2} \cdot 9(\cos(-\frac{\pi}{4} + \frac{\pi}{4}) + i\sin(-\frac{\pi}{4} + \frac{\pi}{4})) = 81\sqrt{2}(\cos 0 + i\sin 0) = 81\sqrt{2};$$

g) all complex roots of the *n*-th degree are calculated by De Moivre formula:

$$\sqrt[n]{z} = \sqrt[n]{|z|}\left(\cos\frac{\varphi + 2\pi k}{n} + i\sin\frac{\varphi + 2\pi k}{n}\right), \quad k = 0,1,2,3,\ldots,n-1;$$

$$\sqrt[3]{z_2} = \sqrt[3]{9\sqrt{2}}\left(\cos\frac{\frac{\pi}{4} + 2\pi k}{3} + i\sin\frac{\frac{\pi}{4} + 2\pi k}{3}\right), k = 0,1,2.$$

Write all the roots obtained for all *k*:

$$k = 0: \quad z_1 = \sqrt[3]{9\sqrt{2}}(\cos\frac{\frac{\pi}{4}}{3} + i\sin\frac{\frac{\pi}{4}}{3}) = \sqrt[3]{9\sqrt{2}}(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12});$$

$$k = 1: \quad z_1 = \sqrt[3]{9\sqrt{2}}(\cos\frac{\frac{\pi}{4} + 2\pi}{3} + i\sin\frac{\frac{\pi}{4} + 2\pi}{3}) = \sqrt[3]{9\sqrt{2}}(\cos\frac{9\pi}{12} + i\sin\frac{9\pi}{12});$$

$$k = 2: \quad z_1 = \sqrt[3]{9\sqrt{2}}(\cos\frac{\frac{\pi}{4} + 4\pi}{3} + i\sin\frac{\frac{\pi}{4} + 4\pi}{3}) = \sqrt[3]{9\sqrt{2}}(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}).$$

On the graph all the cube roots are located at the vertices of an equilateral triangle.

2 Calculation-graphical work №2. Introduction to analysis. Differential calculus of one-variable function

Purpose: teach students to possess classical mathematical methods of investigation of functions. Give students the skills of finding limits, derivatives and differentials of the first and higher orders for composite and parametrically defined functions, and be able to use the gained knowledge in the study of further mathematics course and sections of other special disciplines.

2.1 Theoretical questions

1. Function of one variable. Properties.
2. Composite function, parametrically defined function.
3. Numerical sequences and their limits.
4. Limit of function. Remarkable limits.
5. Continuity of the function. Points of discontinuity and their classification.
6. Derivative of the function. Differentiation of parametrically defined function.
7. Disclosure of the uncertainties. L'Hopital rule.
8. Signs of monotonicity of the function. Extremum of function.
9. Convexity, concavity of the function graph. Inflection point.
10. Asymptotes of graph of the function.
11. Investigation of function using the derivative. Construction of the graph of function.

2.2 Calculated tasks

Task 1. Find the limits.

1.1 $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x^2-12x+20}$	1.2 $\lim\limits_{x\to 0}\dfrac{x^3-x^2+2x}{x^2+x}$	1.3 $\lim\limits_{x\to 3}\dfrac{6+x-x^2}{x^3-27}$
1.4 $\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{3x^2-x-2}$	1.5 $\lim\limits_{x\to 2}\dfrac{2x^2-7x+6}{x^2-5x+6}$	1.6 $\lim\limits_{x\to 3}\dfrac{12-x-x^2}{x^3-27}$
1.7 $\lim\limits_{x\to 1/3}\dfrac{3x^2+2x-1}{27x^3-1}$	1.8 $\lim\limits_{x\to -1}\dfrac{x^2-4x-5}{x^2-2x+6}$	1.9 $\lim\limits_{x\to -1}\dfrac{3x^2+2x-1}{-x^2+x+2}$
1.10 $\lim\limits_{x\to 3}\dfrac{3x^2-11x+6}{2x^2-5x-3}$	1.11 $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2+x-6}$	1.12 $\lim\limits_{x\to -1}\dfrac{x^2-x-2}{x^3+1}$
1.13 $\lim\limits_{x\to 4}\dfrac{x^2-16}{x^2+x-20}$	1.14 $\lim\limits_{x\to -3}\dfrac{4x^2+11x-3}{x^2+2x-3}$	1.15 $\lim\limits_{x\to 3}\dfrac{3x^2-7x-6}{2x^2-7x+3}$
1.16 $\lim\limits_{x\to -2}\dfrac{4x^2+7x-2}{3x^2+8x+4}$	1.17 $\lim\limits_{x\to -1}\dfrac{5x^2+4x-1}{3x^2+x-2}$	1.18 $\lim\limits_{x\to -1}\dfrac{x^2-4x-5}{3x^2+x-2}$

1.19 $\lim\limits_{x\to -1}\dfrac{7x^2+4x-3}{2x^2+3x+1}$	1.20 $\lim\limits_{x\to 4}\dfrac{3x^2-x-64}{x^2-x-12}$	1.21 $\lim\limits_{x\to 2}\dfrac{2x^2-9x+10}{x^2+3x-10}$
1.22 $\lim\limits_{x\to 1}\dfrac{4x^2+x-5}{x^2-2x+1}$	1.23 $\lim\limits_{x\to 2}\dfrac{-5x^2+11x-2}{3x^2-x-10}$	1.24 $\lim\limits_{x\to 7}\dfrac{x^2-5x-14}{2x^2-9x-35}$
1.25 $\lim\limits_{x\to 5}\dfrac{3x^2-6x-45}{2x^2-3x-35}$	1.26 $\lim\limits_{x\to -3}\dfrac{x^2+8x+15}{x^2-6x-27}$	1.27 $\lim\limits_{x\to -5}\dfrac{x^2-2x-35}{2x^2+11x+5}$
1.28 $\lim\limits_{x\to -8}\dfrac{2x^2+15x-8}{3x^2+25x+8}$	1.29 $\lim\limits_{x\to 4}\dfrac{3x^2-2x-40}{x^2-3x-4}$	1.30 $\lim\limits_{x\to -3}\dfrac{2x^2+5x-3}{3x^2+10x+3}$

Task 2. Find the limits.

2.1 $\lim\limits_{x\to\infty}\dfrac{3x^3-5x^2+2}{2x^3+5x^2-x}$	2.2 $\lim\limits_{x\to\infty}\dfrac{x^5-2x+4}{2x^4+3x^2+1}$	2.3 $\lim\limits_{x\to\infty}\dfrac{2x^2+3x-5}{7x^3-2x^2+1}$
2.4 $\lim\limits_{x\to\infty}\dfrac{4x^3+7x}{2x^3-4x^2+5}$	2.5 $\lim\limits_{x\to\infty}\dfrac{3x^4+x-5}{2x^2+x+7}$	2.6 $\lim\limits_{x\to-\infty}\dfrac{3x^3-7x+2}{x^4+2x-4}$
2.7 $\lim\limits_{x\to\infty}\dfrac{5x^4-3x^2+7}{x^4+2x^3+1}$	2.8 $\lim\limits_{x\to-\infty}\dfrac{3x^2+7x-4}{x^5+2x-1}$	2.9 $\lim\limits_{x\to\infty}\dfrac{7x^4-3x+4}{3x^2-2x+1}$
2.10 $\lim\limits_{x\to\infty}\dfrac{7x^3-2x^2+4x}{2x^3+5}$	2.11 $\lim\limits_{x\to\infty}\dfrac{3x-x^6}{x^2-2x+5}$	2.12 $\lim\limits_{x\to-\infty}\dfrac{2x^2-x+7}{3x^4-5x^2+10}$
2.11 $\lim\limits_{x\to\infty}\dfrac{x^3-4x^2+28x}{5x^3+3x^2-1}$	2.14 $\lim\limits_{x\to\infty}\dfrac{2x^3+7x-1}{3x^4+2x+5}$	2.15 $\lim\limits_{x\to-\infty}\dfrac{4x^3-2x^2+x}{3x^2-x}$
2.16 $\lim\limits_{x\to\infty}\dfrac{3x^2+10x+3}{2x^2+5x-3}$	2.17 $\lim\limits_{x\to-\infty}\dfrac{2x^3-7x^2+4}{x^4+5x-1}$	2.18 $\lim\limits_{x\to\infty}\dfrac{3x^4-2x+1}{3x^2+2x-5}$
2.19 $\lim\limits_{x\to\infty}\dfrac{-3x^4+x^2+x}{x^4+3x-2}$	2.20 $\lim\limits_{x\to-\infty}\dfrac{3x^6-5x^2+2}{2x^3+4x-5}$	2.21 $\lim\limits_{x\to\infty}\dfrac{2x^2-5x+2}{x^4+3x^2-9}$
2.22 $\lim\limits_{x\to\infty}\dfrac{2x^2+7x+3}{5x^2-3x+4}$	2.23 $\lim\limits_{x\to\infty}\dfrac{x^7+5x^2-4x}{3x^2+11x-7}$	2.24 $\lim\limits_{x\to-\infty}\dfrac{5x^2-4x+2}{4x^3+2x-5}$
2.25 $\lim\limits_{x\to\infty}\dfrac{-x^2+3x+1}{3x^2+x-5}$	2.26 $\lim\limits_{x\to-\infty}\dfrac{7x^2+6x+9}{1+4x-x^3}$	2.27 $\lim\limits_{x\to\infty}\dfrac{2x^3-3x^2+2x}{x^2+7x+1}$
2.28 $\lim\limits_{x\to\infty}\dfrac{x^3-3x^2+10}{7x^3+2x+1}$	2.29 $\lim\limits_{x\to\infty}\dfrac{3x^4+x^2-6}{2x^2+3x+1}$	2.30 $\lim\limits_{x\to-\infty}\dfrac{3x^2-7x+5}{4x^5-3x^3-1}$

Task 3. Find the limits.

3.1 $\lim\limits_{x\to 3}\dfrac{x^2+x-12}{\sqrt{x-2}-\sqrt{4-x}}$	3.2 $\lim\limits_{x\to -4}\dfrac{\sqrt{x+12}-\sqrt{4-x}}{x^2+2x-8}$	3.3 $\lim\limits_{x\to -3}\dfrac{\sqrt{x+10}-\sqrt{4-x}}{2x^2-x-21}$

3.4 $\lim\limits_{x\to -2}\dfrac{\sqrt{2-x}-\sqrt{x+6}}{x^2-x-6}$	3.5 $\lim\limits_{x\to 1}\dfrac{\sqrt{3+2x}-\sqrt{4+x}}{3x^2-4x+1}$	3.6 $\lim\limits_{x\to 2}\dfrac{x^2-3x+2}{\sqrt{5-x}-\sqrt{x+1}}$
3.7 $\lim\limits_{x\to -1}\dfrac{3x^2+4x+1}{\sqrt{x+3}-\sqrt{5+3x}}$	3.8 $\lim\limits_{x\to 4}\dfrac{2x^2-9x+4}{\sqrt{5-x}-\sqrt{x-3}}$	3.9 $\lim\limits_{x\to 5}\dfrac{\sqrt{2x+1}-\sqrt{x+6}}{2x^2-7x-15}$
3.10 $\lim\limits_{x\to -5}\dfrac{\sqrt{3x+17}-\sqrt{2x+12}}{x^2+8x+15}$	3.11 $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+2}-\sqrt{2}}{\sqrt{x^2+1}-1}$	3.12 $\lim\limits_{x\to 0}\dfrac{\sqrt{7-x}-\sqrt{7+x}}{\sqrt{7}x}$
3.13 $\lim\limits_{x\to 0}\dfrac{3x}{\sqrt{x+1}-\sqrt{1-x}}$	3.14 $\lim\limits_{x\to 4}\dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$	3.15 $\lim\limits_{x\to -1}\dfrac{\sqrt{5+x}-2}{\sqrt{8-x}-3}$
3.16 $\lim\limits_{x\to 5}\dfrac{\sqrt{x+4}-3}{\sqrt{x-1}-2}$	3.17 $\lim\limits_{x\to 7}\dfrac{\sqrt{x-3}-2}{\sqrt{x+2}-3}$	3.18 $\lim\limits_{x\to 3}\dfrac{\sqrt{4x-3}-3}{x^2-9}$
3.19 $\lim\limits_{x\to 3}\dfrac{\sqrt{5x+1}-4}{x^2+2x-15}$	3.20 $\lim\limits_{x\to 0}\dfrac{2-\sqrt{x^2+4}}{3x^2}$	3.21 $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+4}-2}{\sqrt{x^2+16}-4}$
3.22 $\lim\limits_{x\to 0}\dfrac{3x}{\sqrt{5-x}-\sqrt{5+x}}$	3.23 $\lim\limits_{x\to 9}\dfrac{\sqrt{2x+7}-5}{3-\sqrt{x}}$	3.24 $\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{\sqrt{6x+1}-5}$
3.25 $\lim\limits_{x\to 3}\dfrac{x^3-27}{\sqrt{3x}-x}$	3.26 $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x^2}-1}{x^3+x^2}$	3.27 $\lim\limits_{x\to -4}\dfrac{\sqrt{x+20}-4}{x^3+64}$
3.28 $\lim\limits_{x\to 1}\dfrac{3x^2-3}{\sqrt{x+8}-3}$	3.29 $\lim\limits_{x\to 0}\dfrac{\sqrt{x+9}-3}{x^2+x}$	3.30 $\lim\limits_{x\to 2}\dfrac{\sqrt{4x+1}-3}{x^3-8}$

Task 4. Find the limits.

4.1 а) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{2x}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^x$	4.2 а) $\lim\limits_{x\to 0}\dfrac{9x}{\sin x}$, б) $\lim\limits_{x\to 0}(1+3x)^{\frac{1}{x}}$	4.3 а) $\lim\limits_{x\to 0}\dfrac{tg2x}{5x}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{4}{x}\right)^{2x}$
4.4 а) $\lim\limits_{x\to 0}\dfrac{7x}{tgx}$, б) $\lim\limits_{x\to\infty}\left(1-\dfrac{5}{x}\right)^x$	4.5 а) $\lim\limits_{x\to 0}\dfrac{\sin 8x}{x}$, б) $\lim\limits_{x\to 0}(1+6x)^{\frac{3}{x}}$	4.6 а) $\lim\limits_{x\to 0}\dfrac{arctg3x}{4x}$, б) $\lim\limits_{x\to 0}(1-5x)^{\frac{1}{x}}$
4.7 а) $\lim\limits_{x\to 0}\dfrac{9x}{\sin 3x}$, б) $\lim\limits_{x\to 0}(1-4x)^{7/x}$	4.8 а) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\sin x}$, б) $\lim\limits_{x\to 0}(1-9x)^{\frac{2}{x}}$	4.9 а) $\lim\limits_{x\to 0}\dfrac{4x^2}{1-\cos 4x}$, б) $\lim\limits_{x\to\infty}\left(1-\dfrac{7}{x}\right)^{3x}$

4.10 а) $\lim\limits_{x\to 0}\dfrac{1-\cos 8x}{x^2}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{5}{x}\right)^{x}$	4.11 а) $\lim\limits_{x\to 0}\dfrac{10x^2}{1-\cos x}$, б) $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x}\right)^{7x}$	4.12 а) $\lim\limits_{x\to 0}\dfrac{7x}{tg5x}$, б) $\lim\limits_{x\to\infty}\left(\dfrac{x-2}{x}\right)^{9x}$
4.13 а) $\lim\limits_{x\to 0}\dfrac{7x}{\sin 5x}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{5}{x}\right)^{7x}$	4.14 а) $\lim\limits_{x\to 0}\dfrac{3x}{tgx}$, б) $\lim\limits_{x\to\infty}\left(1-\dfrac{4}{x}\right)^{2x}$	4.15 а) $\lim\limits_{x\to 0}\dfrac{7x}{tg9x}$, б) $\lim\limits_{x\to\infty}\left(1-\dfrac{7}{x}\right)^{x/6}$
4.16 а) $\lim\limits_{x\to 0}\dfrac{arctg8x}{3x}$, б) $\lim\limits_{x\to 0}\left(1+\dfrac{3x}{2}\right)^{1/x}$	4.17 а) $\lim\limits_{x\to 0} xctg7x$, б) $\lim\limits_{x\to 0}(1-11x)^{2/x}$	4.18 а) $\lim\limits_{x\to 0}\dfrac{\sin 8x}{\sin 3x}$, б) $\lim\limits_{x\to 0}(1-8x)^{\frac{7}{x}}$
4.19 а) $\lim\limits_{x\to 0}\sin 5xctg3x$, б) $\lim\limits_{x\to 0}(1+x)^{7/x}$	4.20 а) $\lim\limits_{x\to 0}\dfrac{\arcsin 3x}{9x}$, б) $\lim\limits_{x\to 0}(1+x)^{2/x}$	4.21 а) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\sin 9x}$, б) $\lim\limits_{x\to 0}(1+7x)^{\frac{2}{x}}$
4.22 а) $\lim\limits_{x\to 0}\dfrac{3x}{arctg7x}$, б) $\lim\limits_{x\to 0}\left(1+\dfrac{2x}{5}\right)^{\frac{2}{x}}$	4.23 а) $\lim\limits_{x\to 0}\dfrac{\sin 8x}{3x}$, б) $\lim\limits_{x\to 0}(1-2x)^{9/x}$	4.24 а) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{tg9x}$, б) $\lim\limits_{x\to 0}(1+6x)^{\frac{2}{x}}$
4.25 а) $\lim\limits_{x\to 0}\dfrac{x}{\sin 5x}$, б) $\lim\limits_{x\to 0}(1-4x)^{7/x}$	4.26 а) $\lim\limits_{x\to 0}\dfrac{3x}{tg7x}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{5x/2}$	4.27 а) $\lim\limits_{x\to 0}\dfrac{3x}{\sin 5x}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^{7x}$
4.28 а) $\lim\limits_{x\to 0}\dfrac{x}{\sin 2x}$, б) $\lim\limits_{x\to\infty}\left(1-\dfrac{6}{x}\right)^{7x}$	4.29 а) $\lim\limits_{x\to 0}\dfrac{3x}{tg9x}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{5x}$	4.30 а) $\lim\limits_{x\to 0}\dfrac{7x}{tg3x}$, б) $\lim\limits_{x\to\infty}\left(1-\dfrac{3}{2x}\right)^{2x}$

Task 5. Investigate given functions for continuity.

5.1 а) $f(x)=2^{\frac{1}{x-3}}$,	5.2 а) $f(x)=3^{\frac{1}{x-2}}$,	5.3 а) $f(x)=5^{\frac{1}{x+1}}$,

б) $g(x)=\dfrac{7}{2+x}$	б) $g(x)=\dfrac{1}{4-x}$	б) $g(x)=\dfrac{1}{x-12}$
5.4 а) $f(x)=7^{\frac{1}{x-4}}$, б) $g(x)=\dfrac{1}{x+12}$	5.5 а) $f(x)=7^{\frac{1}{5+x}}$, б) $g(x)=\dfrac{6}{x-4}$	5.6 а) $f(x)=8^{\frac{1}{x-3}}$, б) $g(x)=\dfrac{5}{x-22}$
5.7 а) $f(x)=9^{\frac{1}{5-x}}$, б) $g(x)=\dfrac{7}{2x+1}$	5.8 а) $f(x)=9^{\frac{1}{x-7}}$, б) $g(x)=\dfrac{7}{8-2x}$	5.9 а) $f(x)=9^{\frac{1}{2-x}}$, б) $g(x)=\dfrac{3}{5x-25}$
5.10 а) $f(x)=16^{\frac{1}{x-2}}$, б) $g(x)=\dfrac{3}{9-3x}$	5.11 а) $f(x)=4^{\frac{1}{3-x}}$, б) $g(x)=\dfrac{6}{x-4}$	5.12 а) $f(x)=3^{\frac{1}{x-4}}$, б) $g(x)=\dfrac{9}{25-5x}$
5.13 а) $f(x)=12^{\frac{1}{x}}$, б) $g(x)=\dfrac{4}{9-x}$	5.14 а) $f(x)=7^{\frac{1}{x-3}}$, б) $g(x)=\dfrac{2}{8-4x}$	5.15 а) $f(x)=3^{\frac{1}{4-x}}$, б) $g(x)=\dfrac{7}{5-x}$
5.16 а) $f(x)=8^{\frac{1}{x-2}}$, б) $g(x)=\dfrac{3}{4-2x}$	5.17 а) $f(x)=8^{\frac{1}{5-x}}$, б) $g(x)=\dfrac{10}{x+5}$	5.18 а) $f(x)=18^{\frac{1}{x}}$, б) $g(x)=\dfrac{12}{8-8x}$
5.19 а) $f(x)=10^{\frac{1}{7-x}}$, б) $g(x)=\dfrac{12}{4x-2}$	5.20 а) $f(x)=11^{\frac{1}{5-x}}$, б) $g(x)=\dfrac{9}{8+2x}$	5.21 а) $f(x)=14^{\frac{1}{6-x}}$, б) $g(x)=\dfrac{3}{2+x}$
5.22 а) $f(x)=15^{\frac{3}{4-x}}$, б) $g(x)=\dfrac{1}{6-2x}$	5.23 а) $f(x)=15^{\frac{1}{8-x}}$, б) $g(x)=\dfrac{10}{x-3}$	5.24 а) $f(x)=19^{\frac{4}{2+x}}$, б) $g(x)=\dfrac{7}{8-2x}$
5.25 а) $f(x)=11^{\frac{1}{4+x}}$, б) $g(x)=\dfrac{1}{9-x}$	5.26 а) $f(x)=5^{\frac{7}{8-x}}$, б) $g(x)=\dfrac{11}{x-15}$	5.27 а) $f(x)=13^{\frac{1}{5+x}}$, б) $g(x)=\dfrac{5}{3-x}$

5.28 а) $f(x)=6^{\frac{9}{4-x}}$, б) $g(x)=\frac{1}{2x+5}$	5.29 а) $f(x)=4^{\frac{1}{x-5}}$, б) $g(x)=\frac{3}{8-x}$	5.30 а) $f(x)=11^{\frac{2}{x-3}}$, б) $g(x)=\frac{16}{x+15}$

Task 6. Find the derivatives of functions.

6.1 а) $f(x)=tg(8x^2+x-5)$, б) $g(x)=\left(e^{\cos x}+3\right)^2$	6.2 а) $f(x)=\cos(3x^2-2)$, б) $g(x)=\frac{4x^2}{\cos^2 x}$	6.3 а) $f(x)=e^{7x+x^2}$, б) $g(x)=\frac{5x}{tg^2 2x}$
6.4 а) $f(x)=\left(4x+2x^3\right)^{10}$, б) $g(x)=\frac{\sin^3 x}{2+3\cos^2 x}$	6.5 а) $f(x)=\sqrt[3]{x^5+10x^2}$, б) $g(x)=2tg^3\left(x^2+1\right)$	6.6 а) $f(x)=arctg\left(x^5+3x\right)$, б) $g(x)=2tg^3 x+\lg\cos x$
6.7 а) $f(x)=tg(4x^2+x+5)$, б) $g(x)=\left(e^{\sin x}+2\right)^4$	6.8 а) $f(x)=\sin(3x^2-2x+1)$, б) $g(x)=\frac{3x^2}{\cos^4 x}$	6.9 а) $f(x)=e^{2x-x^3}$, б) $g(x)=\frac{tg^2 2x}{x-5}$
6.10 а) $f(x)=\left(3x^3+7x\right)^{15}$, б) $g(x)=\frac{4x^7}{arctg^3 x}$	6.11 а) $f(x)=\sqrt[3]{x^8+5x^3}$, б) $g(x)=3ctg^3\left(x^2+1\right)$	6.12 а) $f(x)=arcctg\left(x^4+x\right)$, б) $g(x)=\frac{1}{3}tg^2 x+\lg\sin x$
6.13 а) $f(x)=tg(3x^2+2x-7)$, б) $g(x)=\left(6^{\cos x}+3\right)^2$	6.14 а) $f(x)=\cos(7x^2-6x+6)$, б) $g(x)=\frac{2x^7}{\cos^3 x}$	6.15 а) $f(x)=4^{7x+x^2}$, б) $g(x)=\frac{1}{2}\sin^2 x+\ln\cos x$
6.16 а) $f(x)=\left(3x^2+8x\right)^{18}$, б) $g(x)=\frac{3x^8}{arcctg^3 x}$	6.17 а) $f(x)=\sqrt[5]{x^5+10x^2-3}$ б) $g(x)=\frac{(7+3x)^4}{\arcsin x}$	6.18 а) $f(x)=arctg\left(x^4+6x\right)$, б) $g(x)=\frac{1}{2}\cos^2 x+\ln\sin x$
6.19 а) $f(x)=\sin(4x^2+x+5)$, б) $g(x)=\left(e^{tgx}+1\right)^4$	6.20 а) $f(x)=ctg(3x^2-7x)$, б) $g(x)=4tg^5 3x^2$	6.21 а) $f(x)=13^{5x^3+2}$, б) $g(x)=\frac{(1+3x)^3}{tg2x}$

6.22 а) $f(x)=\left(9x+3x^5\right)^{12}$, б) $g(x)=\dfrac{\cos^5 x}{1+\sin x}$	6.23 а) $f(x)=\sqrt[7]{x+5x^3}$ б) $g(x)=9\cos^3\left(x^2+6\right)$	6.24 а) $f(x)=\arcsin\left(x^4+7x\right)$, б) $g(x)=ctg^5 x+\ln\sin x$
6.25 а) $f(x)=\ln\left(6x^2-x^3\right)$, б) $g(x)=\left(e^{\sqrt{x}}+8\right)^{12}$	6.26 а) $f(x)=\cos\left(\sqrt[3]{x}+2\right)$, б) $g(x)=\dfrac{\left(x+3x^5\right)^2}{\cos x}$	6.27 а) $f(x)=e^{2x+9x^3}$, б) $g(x)=\sqrt[6]{(\sin x+2x)^5}$
6.28 а) $f(x)=\sqrt{2x+\cos 5x}$, б) $g(x)=\dfrac{x^2}{\ln\left(5x^3+1\right)}$	6.29 а) $f(x)=\sqrt[3]{\ln x}$, б) $g(x)=2tg(\arcsin 5x)$	6.30 а) $f(x)=arctg(\sin x+2)$, б) $g(x)=3x+\lg(\cos x+4)$

Task 7. Find the derivative of the function given by the parametric equations.

7.1 $\begin{cases}x=(2t+3)\cos t,\\ y=3t^3\end{cases}$	7.2 $\begin{cases}x=2\cos t,\\ y=3\sin^2 t\end{cases}$	7.3 $\begin{cases}x=6\cos^3 t,\\ y=2\sin t\end{cases}$
7.4 $\begin{cases}x=1/(t+2),\\ y=t^2/(t+2)\end{cases}$	7.5 $\begin{cases}x=e^{-2t},\\ y=3te^{4t}\end{cases}$	7.6 $\begin{cases}x=\sqrt{t},\\ y=\sqrt[5]{t}\end{cases}$
7.7 $\begin{cases}x=2t/\left(t^3+1\right),\\ y=t^2/\left(t^2+1\right)\end{cases}$	7.8 $\begin{cases}x=\sqrt{t^2-1},\\ y=(t+1)/\sqrt{t^2-1}\end{cases}$	7.9 $\begin{cases}x=4t+2t^2,\\ y=5\sqrt{t^3}-3t^2\end{cases}$
7.10 $\begin{cases}x=(\ln t)/t,\\ y=\ln t\end{cases}$	7.11 $\begin{cases}x=e^t\cos t,\\ y=e^t\sin t\end{cases}$	7.12 $\begin{cases}x=t^4,\\ y=t\ln t\end{cases}$
7.13 $\begin{cases}x=5\cos^2 t,\\ y=3\sin t\end{cases}$	7.14 $\begin{cases}x=5\cos t,\\ y=3\sin^2 t\end{cases}$	7.15 $\begin{cases}x=arctgt,\\ y=\ln\left(1+t^2\right)\end{cases}$
7.16 $\begin{cases}x=\arcsin t,\\ y=\sqrt{1-t^2}\end{cases}$	7.17 $\begin{cases}x=3(t-\sin t),\\ y=3(1-\cos t)\end{cases}$	7.18 $\begin{cases}x=3(\sin t-\cos t),\\ y=3(\cos t+\sin t)\end{cases}$
7.19 $\begin{cases}x=\sin 2t,\\ y=\cos^2 t\end{cases}$	7.20 $\begin{cases}x=e^{3t},\\ y=\sqrt{t-1}\end{cases}$	7.21 $\begin{cases}x=(\ln t)/t,\\ y=t^2\ln t\end{cases}$
7.22 $\begin{cases}x=\arccos t,\\ y=\sqrt{1-t^2}\end{cases}$	7.23 $\begin{cases}x=1/(t+1),\\ y=t^2/(t+1)\end{cases}$	7.24 $\begin{cases}x=5\sin 3t,\\ y=3\cos^3 t\end{cases}$

7.25 $\begin{cases} x = e^{-2t}, \\ y = e^{8t} / t \end{cases}$	7.26 $\begin{cases} x = \sqrt[3]{(t-1)^2}, \\ y = \sqrt{t-1} \end{cases}$	7.27 $\begin{cases} x = \ln^2 t, \\ y = t + \ln t \end{cases}$
7.28 $\begin{cases} x = te^t, \\ y = t / e^t \end{cases}$	7.29. $\begin{cases} x = 6t^2 - 4, \\ y = 3\sqrt{t^5} \end{cases}$	7.30 $\begin{cases} x = \arcsin t, \\ y = \ln t \end{cases}$

Task 8. Find the value of the second derivative of the function $y = f(x)$ at the point x_0.

8.1 $y = \sin^2 x,$ $x_0 = \pi/4$	8.2 $y = arctgx,$ $x_0 = 1$	8.3 $y = \ln(2 + x^2),$ $x_0 = 0$
8.4 $y = e^x \cos x,$ $x_0 = 0$	8.5 $y = e^x \sin 2x,$ $x_0 = 0$	8.6 $y = e^{-x} \cos x,$ $x_0 = 0$
8.7 $y = \sin 2x,$ $x_0 = \pi$	8.8 $y = (2x+1)^5,$ $x_0 = 1$	8.9 $y = \ln(1 + 2x),$ $x_0 = 2$
8.10 $y = \frac{1}{2} e^x x^2,$ $x_0 = 0$	8.11 $y = \arcsin x,$ $x_0 = 0$	8.12 $y = (5x - 4)^5,$ $x_0 = 2$
8.13 $y = x \sin x,$ $x_0 = \frac{\pi}{2}$	8.14 $y = x^2 \ln x,$ $x_0 = \frac{1}{3}$	8.15 $y = x \sin 2x,$ $x_0 = -\frac{\pi}{4}$
8.16 $y = x \cos 2x,$ $x_0 = \frac{\pi}{12}$	8.17 $y = x^4 \ln x,$ $x_0 = 1$	8.18 $y = x + arctgx,$ $x_0 = 1$
8.19 $y = \cos^2 x,$ $x_0 = \frac{\pi}{4}$	8.20 $y = \ln(x^2 - 4),$ $x_0 = 3$	8.21 $y = x^2 \cos x,$ $x_0 = \frac{\pi}{2}$
8.22 $y = x \arccos x,$ $x_0 = \frac{\sqrt{3}}{2}$	8.23 $y = (x+1)\ln(x+1),$ $x_0 = -\frac{1}{2}$	8.24 $y = \ln^3 x,$ $x_0 = 1$
8.25 $y = 2^{7x+5},$ $x_0 = 1$	8.26 $y = (4x - 3)^5,$ $x_0 = 1$	8.27 $y = x \arcsin x,$ $x_0 = 2$

8.28 $y=(7x-4)^6$, $x_0=1$	8.29 $y=x\sin 2x$, $x_0=\frac{\pi}{4}$	8.30 $y=\sin(x^3+\pi)$, $x_0=\sqrt[3]{\pi}$

Task 9. For research of function $y=f(x)$ determine:

a) domain and the points of discontinuity;

b) asymptote of the graph of function;

c) points of intersection of the graph with the coordinate axes;

d) parity, oddness.

9.1 $y=\frac{x^3+4}{x^2}$	9.2 $y=\frac{x^2-x+1}{x-1}$	9.3 $y=\frac{2}{x^2+2x}$	9.4 $y=\frac{4x^2}{3+x^2}$	9.5 $y=\frac{12x}{x^2+9}$
9.6 $y=\frac{x^2-3x+3}{x-1}$	9.7 $y=\frac{4-x^3}{x^2}$	9.8 $y=\frac{x^2-4x+1}{x-4}$	9.9 $y=\frac{2x^3+1}{x^2}$	9.10 $y=\frac{(x-1)^2}{x^2}$
9.11 $y=\frac{x^2}{(x-1)^2}$	9.12 $y=\left(x+\frac{1}{x}\right)^2$	9.13 $y=\frac{12-3x^2}{12+x^2}$	9.1 4 $y=\frac{9+6x-3x^2}{x^2-2x+13}$	9.15 $y=\frac{-8x}{x^2+4}$
9.16 $y=\frac{x-1}{(x+1)^2}$	9.17 $y=\frac{3x^4+1}{x^3}$	9.18 $y=\frac{4x}{(x+1)^2}$	9.19 $y=\frac{8(x-1)}{(x+1)^2}$	9.20 $y=\frac{1-2x^3}{x^2}$
9.21 $y=\frac{4}{x^2+2x-3}$	9.22 $y=\frac{4}{3+2x-x^2}$	9.23 $y=\frac{x^2+2x-7}{x^2+2x-3}$	9.24 $y=\frac{1}{x^4-1}$	9.25 $y=-\left(\frac{x}{x+2}\right)^2$
9.26 $y=\frac{x^3-32}{x^2}$	9.27 $y=\frac{4(x+1)^2}{x^2+2x+4}$	9.28 $y=\frac{x^3-27x+54}{x^3}$	9.29 $y=\frac{x^2-6x+9}{(x-1)^2}$	9.30 $y=\frac{3x-2}{x^3}$

Task 10. Find the limits.

10.1 $\lim\limits_{x\to\infty}\left(\frac{2x+3}{5x+7}\right)^{x+1}$	10.2 $\lim\limits_{x\to\infty}\left(\frac{2x+1}{x-1}\right)^{x}$	10.3 $\lim\limits_{x\to\infty}\left(\frac{x+1}{2x-1}\right)^{3x}$	10.4 $\lim\limits_{x\to-\infty}\left(\frac{2x-1}{4x+1}\right)^{3x-1}$
10.5 $\lim\limits_{x\to\infty}\left(\frac{5x+8}{x-2}\right)^{x+4}$	10.6 $\lim\limits_{x\to-\infty}\left(\frac{x+1}{3x-1}\right)^{2x+1}$	10.7 $\lim\limits_{x\to-\infty}\left(\frac{2x+1}{x-1}\right)^{5x}$	10.8 $\lim\limits_{x\to\infty}\left(\frac{x+1}{2x-1}\right)^{4x}$

10.9 $\lim\limits_{x\to-\infty}\left(\frac{x+3}{2x-4}\right)^{x+2}$	10.10 $\lim\limits_{x\to-\infty}\left(\frac{2x+1}{3x-1}\right)^{x-1}$	10.11 $\lim\limits_{x\to\infty}\left(\frac{5x-3}{x+4}\right)^{x+3}$	10.12 $\lim\limits_{x\to-\infty}\left(\frac{2x-3}{7x+4}\right)^{x}$
10.13 $\lim\limits_{x\to-\infty}\left(\frac{x-5}{3x+4}\right)^{2x}$	10.14 $\lim\limits_{x\to\infty}\left(\frac{x+3}{4x-5}\right)^{2x}$	10.15 $\lim\limits_{x\to-\infty}\left(\frac{x-2}{3x+1}\right)^{5x}$	10.16 $\lim\limits_{x\to-\infty}\left(\frac{3x-4}{x+6}\right)^{x-1}$
10.17 $\lim\limits_{x\to\infty}\left(\frac{x-2}{3x+10}\right)^{3x}$	10.18 $\lim\limits_{x\to-\infty}\left(\frac{2x-3}{x+4}\right)^{6x+1}$	10.19 $\lim\limits_{x\to-\infty}\left(\frac{x+3}{3x-1}\right)^{2x}$	10.20 $\lim\limits_{x\to\infty}\left(\frac{6x+5}{x-10}\right)^{5x}$
10.21 $\lim\limits_{x\to-\infty}\left(\frac{3x+7}{x+4}\right)^{4x}$	10.22 $\lim\limits_{x\to\infty}\left(\frac{x-1}{4x+5}\right)^{3x}$	10.23 $\lim\limits_{x\to-\infty}\left(\frac{5x-7}{x+6}\right)^{2x}$	10.24 $\lim\limits_{x\to\infty}\left(\frac{3-4x}{2-x}\right)^{6x}$
10.25 $\lim\limits_{x\to\infty}\left(\frac{1-2x}{3-x}\right)^{-x}$	10.26 $\lim\limits_{x\to-\infty}\left(\frac{4+3x}{5+x}\right)^{7x}$	10.27 $\lim\limits_{x\to-\infty}\left(\frac{3x-1}{2x+5}\right)^{3x}$	10.28 $\lim\limits_{x\to\infty}\left(\frac{1-x}{2-10x}\right)^{5x}$
10.29 $\lim\limits_{x\to\infty}\left(\frac{x+3}{9x-4}\right)^{2x}$	10.30 $\lim\limits_{x\to-\infty}\left(\frac{x+5}{4x-2}\right)^{3x}$		

Task 11. Find the specified limits, using L'Hopital rule.

11.1 $\lim\limits_{x\to\infty}\frac{\ln(x+5)}{\sqrt[4]{x+3}}$	11.2 $\lim\limits_{x\to1}\frac{2^{\ln x}-x}{x-1}$	11.3 $\lim\limits_{x\to0}\frac{tgx-x}{x-\sin x}$
11.4 $\lim\limits_{x\to0}\frac{1-\cos x^2}{x^2-\sin x^2}$	11.5 $\lim\limits_{x\to0}\frac{tgx-x}{2\sin x+x}$	11.6 $\lim\limits_{x\to1}\frac{x^3-2x^2-x+2}{x^3-7x+6}$
11.7 $\lim\limits_{x\to0}\frac{x\cos x-\sin x}{x^3}$	11.8 $\lim\limits_{x\to0}\frac{e^x}{x^3}$	11.9 $\lim\limits_{x\to1}\frac{1-x}{1-\sin\left(\frac{\pi x}{2}\right)}$
11.10 $\lim\limits_{x\to\infty}\frac{\ln x}{\sqrt[3]{x}}$	11.11 $\lim\limits_{x\to0}\frac{e^{3x}}{1-\cos5x}$	11.12 $\lim\limits_{x\to0}\frac{\pi/x}{ctg(\pi x/2)}$

11.13 $\lim\limits_{x\to 0}\dfrac{\ln(\sin 2x)}{\ln(\sin 5x)}$	11.14 $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{tgx}{tg5x}$	11.15 $\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{5x}}{\sin x}$
11.16 $\lim\limits_{x\to 0}\dfrac{x-arctgx}{x^3}$	11.17 $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{1-2\sin x}{\cos 3x}$	11.18 $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\ln(1+2x)}$
11.19 $\lim\limits_{x\to 1}\dfrac{\ln x}{x^3-1}$	11.20 $\lim\limits_{x\to 0}\dfrac{\ln x}{ctgx}$	11.21 $\lim\limits_{x\to 0}\dfrac{1-\cos 3x}{1-\cos 6x}$
11.22 $\lim\limits_{x\to 4}\dfrac{x-4}{x^3-64}$	11.23 $\lim\limits_{x\to 0}\dfrac{e^{5x}-1}{\sin 4x}$	11.24 $\lim\limits_{x\to 0}\dfrac{1-e^{2x}}{tg4x}$
11.25 $\lim\limits_{x\to 0}\dfrac{2^x-4^x}{x\sqrt{1-x^2}}$	11.26 $\lim\limits_{x\to 0}\dfrac{\ln(1+x^2)}{\cos 3x-e^{-x}}$	11.27 $\lim\limits_{x\to 0}\dfrac{\ln(7+x)}{\sqrt[7]{x-3}}$
11.28 $\lim\limits_{x\to 0}\dfrac{\arcsin 4x}{5-5e^{-3x}}$	11.29 $\lim\limits_{x\to 0}\dfrac{e^{9x}-1}{\cos 5x}$	11.30 $\lim\limits_{x\to \pi/6}\dfrac{1-2\sin x}{\cos 3x}$

Task 12. Investigate the function for continuity and construct the graph.

12.1 $f(x)=\begin{cases}x+4, & x\prec -1,\\ x^2+2, & -1\le x\le 1,\\ 2x, & x\succ 1.\end{cases}$	12.2 $f(x)=\begin{cases}x+4, & x\le 0,\\ (x+1)^2, & 0\prec x\le 2,\\ -x+4, & x\succ 2.\end{cases}$	12.3 $f(x)=\begin{cases}x+2, & x\le -1,\\ x^2+1, & -1\prec x\le 1,\\ -x+3, & x\succ 1.\end{cases}$
12.4 $f(x)=\begin{cases}-x, & x\le 0,\\ -(x-1)^2, & 0\prec x\prec 2,\\ x-3, & x\ge 2.\end{cases}$	12.5 $f(x)=\begin{cases}-2(x+1), & x\le -1,\\ (x+1)^2, & -1\prec x\prec 0,\\ x, & x\ge 0.\end{cases}$	12.6 $f(x)=\begin{cases}-x, & x\le 0,\\ x^2, & 0\prec x\le 2,\\ x+1, & x\succ 2.\end{cases}$
12.7 $f(x)=\begin{cases}x^2+1, & x\le 1,\\ 2x, & 1\prec x\le 3,\\ x+2, & x\succ 3.\end{cases}$	12.8 $f(x)=\begin{cases}x-3, & x\prec 0,\\ x+1, & 0\le x\le 4,\\ x+3, & x\succ 4.\end{cases}$	12.9 $f(x)=\begin{cases}\sqrt{1-x}, & x\le 0,\\ 0, & 0\prec x\le 2,\\ x-2, & x\succ 2.\end{cases}$
12.10 $f(x)=\begin{cases}2x^2, & x\le 0,\\ x, & 0\prec x\le 1,\\ x+2, & x\succ 1.\end{cases}$	12.11 $f(x)=\begin{cases}\sin x, & x\prec 0,\\ x, & 0\le x\le 2,\\ 0, & x\succ 2.\end{cases}$	12.12 $f(x)=\begin{cases}\cos x, & x\le \pi/2,\\ 0, & \pi/2\prec x\prec \pi,\\ 2, & x\ge \pi.\end{cases}$

12.13 $f(x)=\begin{cases} x-1, & x\le 0,\\ x^2, & 0\prec x\prec 2,\\ 2x, & x\ge 2. \end{cases}$	12.14 $f(x)=\begin{cases} x+1, & x\prec 0,\\ x^2-1, & 0\le x\prec 1,\\ -x, & x\ge 1. \end{cases}$	12.15 $f(x)=\begin{cases} -x, & x\prec 0,\\ x^2+1, & 0\le x\prec 2,\\ 1+x, & x\ge 2. \end{cases}$
12.16 $f(x)=\begin{cases} x+3, & x\le 0,\\ 1, & 0\prec x\le 2,\\ x^2-2, & x\succ 2. \end{cases}$	12.17 $f(x)=\begin{cases} x-1, & x\prec 0,\\ \sin x, & 0\le x\prec \pi,\\ 3, & x\ge \pi. \end{cases}$	12.18 $f(x)=\begin{cases} 1-x, & x\prec -1,\\ x^2+1, & -1\le x\le 2,\\ 2x, & x\succ 2. \end{cases}$
12.19 $f(x)=\begin{cases} 1, & x\le 0,\\ 2^x, & 0\prec x\le 2,\\ x+3, & x\succ 2. \end{cases}$	12.20 $f(x)=\begin{cases} -x+2, & x\le -2,\\ x^3, & -2\prec x\le 1,\\ 2, & x\succ 1. \end{cases}$	12.21 $f(x)=\begin{cases} 3x+4, & x\le -1,\\ x^2-2, & -1\prec x\prec 2,\\ x, & x\ge 2. \end{cases}$
12.22 $f(x)=\begin{cases} x, & x\le 1,\\ (x-2)^2, & 1\prec x\prec 3,\\ -x+6, & x\ge 3. \end{cases}$	12.23 $f(x)=\begin{cases} x-1, & x\prec 1,\\ x^2+2, & 1\le x\le 2,\\ -2x, & x\succ 2. \end{cases}$	12.24 $f(x)=\begin{cases} x^3, & x\prec -1,\\ x-1, & -1\le x\le 3,\\ -x+5, & x\succ 3. \end{cases}$
12.25 $f(x)=\begin{cases} x, & x\le -2,\\ -x+1, & -2\prec x\le 1,\\ x^2-1, & x\succ 1. \end{cases}$	12.26 $f(x)=\begin{cases} x+3, & x\le 0,\\ -x^2+4, & 0\prec x\prec 2,\\ x-2, & x\ge 2. \end{cases}$	12.27 $f(x)=\begin{cases} 0, & x\le -1,\\ x^2-1, & -1\prec x\le 2,\\ 2x, & x\succ 2. \end{cases}$
12.28 $f(x)=\begin{cases} -1, & x\prec 0,\\ \cos x, & 0\le x\le \pi,\\ 1-x, & x\succ \pi. \end{cases}$	12.29 $f(x)=\begin{cases} 2, & x\prec -1,\\ 1-x, & -1\le x\le 1,\\ \ln x, & x\succ 1. \end{cases}$	12.30 $f(x)=\begin{cases} -x, & x\le 0,\\ x^3, & 0\prec x\le 2,\\ x+4, & x\succ 2. \end{cases}$

Task 13. Find the second derivative of the function given by the parametric equations in the task 7.

Task 14. For the function from the task 9 find:
a) the intervals of monotonicity, the extremums;
b) the intervals of concavity and convexity, the inflection points;
c) construct the graph.

2.3 Decision of an exemplary embodiment

Task 1. Find the limit $\lim\limits_{x\to -2}\dfrac{5x^2+13x+6}{3x^2+2x-8}$.

Solution:

$$\lim_{x\to -2}\frac{5x^2+13x+6}{3x^2+2x-8}=\left|\frac{0}{0}\right|=\lim_{x\to -2}\frac{(x+2)(5x+3)}{(x+2)(3x-4)}=\lim_{x\to -2}\frac{5x+3}{3x-4}=\frac{7}{10}=0{,}7.$$

Task 2. Find the limit $\lim\limits_{x\to -\infty}\dfrac{2x^5+3x^3-4x}{3x^2-4x+2}$.

Solution:

$$\lim_{x\to -\infty}\frac{2x^5+3x^3-4x}{3x^2-4x+2}=\left|\frac{\infty}{\infty}\right|=\lim_{x\to -\infty}\frac{x^5\left(2+\frac{3}{x^2}-\frac{4}{x^4}\right)}{x^2\left(3-\frac{4}{x}+\frac{2}{x^2}\right)}=$$

$$=\lim_{x\to -\infty}\frac{x^3\left(2+\frac{3}{x^2}-\frac{4}{x^4}\right)}{3-\frac{4}{x}+\frac{2}{x^2}}=\frac{-\infty}{3}=-\infty.$$

Task 3. Find the limit $\lim\limits_{x\to 4}\dfrac{\sqrt{21+x}-5}{x^3-64}$.

Solution:

$$\lim_{x\to 4}\frac{\sqrt{21+x}-5}{x^3-64}=\lim_{x\to 4}\frac{\left(\sqrt{21+x}-5\right)\left(\sqrt{21+x}+5\right)}{\left(x^3-64\right)\left(\sqrt{21+x}+5\right)}=\lim_{x\to 4}\frac{21+x-25}{\left(x^3-64\right)\left(\sqrt{21+x}+5\right)}=$$

$$=\lim_{x\to 4}\frac{x-4}{(x-4)\left(x^2+4x+16\right)\left(\sqrt{21+x}+5\right)}=\lim_{x\to 4}\frac{1}{\left(x^2+4x+16\right)\left(\sqrt{21+x}+5\right)}=\frac{1}{480}.$$

Task 4. Find the limits: a) $\lim\limits_{x\to 0}\dfrac{tg9x}{8x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{2x}{2x-3}\right)^{5x}$.

Solution:

a) for the disclosure of uncertainty the property of the first remarkable limit is used

$$\lim_{x\to 0}\frac{tg9x}{8x}=\left|\frac{0}{0}\right|=\lim_{x\to 0}\frac{\frac{\sin 9x}{\cos 9x}}{8x}=\lim_{x\to 0}\frac{\sin 9x}{8x}\cdot\frac{1}{\cos 9x}=\lim_{x\to 0}\frac{\sin 9x}{9x}\cdot\frac{9}{8\cos 9x}=1\cdot\frac{9}{8}=\frac{9}{8};$$

b) for the disclosure of uncertainty $\left|1^{\infty}\right|$ the property of the second remarkable limit is used

$$\lim_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{5x}=\left|1^{\infty}\right|=\lim_{x\to\infty}\left(1+\frac{2x}{2x-3}-1\right)^{5x}=\lim_{x\to\infty}\left(1+\frac{2x-2x+3}{2x-3}\right)^{5x}=$$

$$=\lim_{x\to\infty}\left(1+\frac{3}{2x-3}\right)^{5x}=\lim_{x\to\infty}\left(\left(1+\frac{3}{2x-3}\right)^{\frac{2x-3}{3}}\right)^{\frac{3}{2x-3}\cdot 5x}=\lim_{x\to\infty}e^{\frac{3}{2x-3}\cdot 5x}=e^{\frac{15}{2}}.$$

Task 5. Investigate given functions for continuity:

a) $f(x)=9^{\frac{5}{8-x}}$ b) $f(x)=\frac{4}{5+3x}$.

Solution:

a) $f(x)=9^{\frac{5}{8-x}}$ function is defined and continuous in the intervals $x\in(-\infty;8)\cup(8;+\infty)$. Consequently the discontinuity is possible at the point $x_0=8$. Let us find the one-sided limits of the function at this point:

$$f(8-0)=\lim_{x\to 8-0}9^{\frac{5}{8-x}}=9^{\frac{5}{8-(8-0)}}=9^{\frac{5}{+0}}=9^{+\infty}=+\infty;$$

$$f(8+0)=\lim_{x\to 8+0}9^{\frac{5}{8-x}}=9^{\frac{5}{8-(8+0)}}=9^{\frac{5}{-0}}=9^{-\infty}=\frac{1}{9^{\infty}}=\frac{1}{\infty}=0.$$

Consequently $x_0=8$ is a point of discontinuity of the second type;

b) $f(x)=\frac{4}{5+3x}$ function is defined and continuous in the intervals $x\in(-\infty;-\frac{5}{3})\cup\left(-\frac{5}{3};+\infty\right)$. Consequently the discontinuity is possible at the point $x_0=-\frac{5}{3}$. Let us find the one-sided limits of the function at this point:

$$f(-\frac{5}{3}-0)=\lim_{x\to-\frac{5}{3}-0}\frac{4}{5+3x}=\frac{4}{5+3\cdot\left(-\frac{5}{3}-0\right)}=\frac{4}{5-5-0}=\frac{4}{-0}=-\infty;$$

$$f(-\frac{5}{3}+0)=\lim_{x\to-\frac{5}{3}+0}\frac{4}{5+3x}=\frac{4}{5+3\cdot\left(-\frac{5}{3}+0\right)}=\frac{4}{5-5+0}=\frac{4}{+0}=+\infty.$$

Consequently $x_0=-\frac{5}{3}$ is a point of discontinuity of the second type.

Task 6. Find the derivatives of functions:

a) $y=\ln\left(4x^2-5x+2^{-3x}\right)$, b) $y=\frac{ctg^5 3x}{x+2}$.

Solution:

a) according to the formula of the derivative of a composite function $(\ln u)'=\frac{u'}{u}$ we find:

$$y'=\frac{\left(4x^2-5x+2^{-3x}\right)'}{4x^2-5x+2^{-3x}}=\frac{8x-5+2^{-3x}\ln 2\cdot(-3)}{4x^2-5x+2^{-3x}}=$$

$$=\frac{8x-5-3\cdot 2^{-3x}\ln 2}{4x^2-5x+2^{-3x}};$$

b) find separately the derivative of a composite function of the numerator:

$$\left(ctg^5 3x\right)'=\left|\begin{array}{l}\text{substitute:}\\ u=ctg\,3x;\\ t=3x\end{array}\right|=\left(u^5\right)'_u\cdot(ctgt)'_t\cdot(3x)'=$$

$$=5u^4\cdot\frac{-1}{\sin^2 t}\cdot 3=-5ctg^4 3x\frac{3}{\sin^2 3x};$$

$$y'=\left(\frac{ctg^5 3x}{x+2}\right)'=\frac{\left(ctg^5 3x\right)'\cdot(x+2)-(x+2)'\cdot ctg^5 3x}{(x+2)^2}=$$

$$=\frac{-5ctg^4 3x\cdot\frac{3}{\sin^2 3x}\cdot(x+2)-ctg^5 3x}{(x+2)^2}=\frac{\frac{-15(x+2)ctg^4 3x}{\sin^2 3x}-ctg^5 3x}{(x+2)^2}=$$

$$=\frac{-15(x+2)ctg^4 3x-\sin^2 3x\cdot ctg^5 3x}{(x+2)^2\sin^2 3x}.$$

Task 7. Find the derivative of the function given by the parametric equations:

$$\begin{cases} x = 3t^4 - t^2 \\ y = t^3 - 5. \end{cases}$$

Solution: the derivative of the function given by the parametric equations is found by the formula $y'_x = \frac{y'_t}{x'_t}$;

$$\begin{cases} x'_t = 12t^3 - 2t \\ y'_t = 3t^2, \end{cases} \quad \text{so} \quad y'_x = \frac{3t^2}{12t^3 - 2t} = \frac{3t^2}{2t\left(6t^2 - 1\right)} = \frac{3t}{2\left(6t^2 - 1\right)}.$$

Task 8. Find the value of the second derivative of the function $y = \ln^3 x$ at the point $x_0 = 1.$

Solution:

$$y' = \left(\ln^3 x\right)' = 3\ln^2 x \cdot \frac{1}{x} = \frac{3\ln^2 x}{x};$$

$$y'' = \left(\frac{3\ln^2 x}{x}\right)' = 3 \cdot \frac{2\ln x \cdot \frac{1}{x} \cdot x - 1 \cdot \ln^2 x}{x^2} = 3 \cdot \frac{2\ln x - \ln^2 x}{x^2} = \frac{3\ln x\left(2 - \ln x\right)}{x^2};$$

$$y''(1) = \frac{3\ln 1\left(2 - \ln 1\right)}{1^2} = 0.$$

Task 9. For research of functions $y = \frac{x^2 + 1}{3x}$ determine:

a) domain and the points of discontinuity;
b) asymptote of the graph of function;
c) points of intersection of the graph with the coordinate axes;
d) parity, oddness.

Solution:

a) $x = 0$ - the point of discontinuity, the function is continuous in its domain of definition: $x \neq 0 \Leftrightarrow D(y) : (-\infty;\ 0) \cup (0; +\infty)$. Let us find the one-sided limits at the point of discontinuity: $\lim\limits_{x \to -0} \frac{x^2 + 1}{3x} = -\infty,\ \lim\limits_{x \to +0} \frac{x^2 + 1}{3x} = +\infty;$

b) the straight line $x = x_0$ is a vertical asymptote, if $f(x_0 \pm 0) = \pm\infty \Rightarrow x = 0$ - is the vertical asymptote.

The straight line $y = kx + b$ - is a slant asymptote, where

$$k = \lim_{x\to\infty} \frac{y}{x} = \lim_{x\to\infty} \frac{x^2+1}{3x^2} = \frac{1}{3},$$

$$b = \lim_{x\to\infty}(y - x) = \lim_{x\to\infty}\left(\frac{x^2+1}{3x} - \frac{x}{3}\right) = \lim_{x\to\infty} \frac{x^2+1-x^2}{3x} = \lim_{x\to\infty} \frac{1}{3x} = 0.$$

so, $y = \frac{1}{3}x$ - is the slant asymptote;

c) at the point $x = 0$ the graph of function $y = f(x)$ intersects the axis *0y*. In our case $D(y): (-\infty;\ 0) \cup (0; +\infty)$, so the function does not cross the axis *0y*. At the point $y = 0$ the graph of function $y = f(x)$ intersects the axis *0x*. In our case $y = \frac{x^2+1}{3x} \neq 0$, so the function does not cross the axis *0x*;

d) $y(-x) = \frac{(-x)^2+1}{3(-x)} = -\frac{x^2+1}{3x} = -y(x)$, i.e. we have the odd function, the graph is symmetric with respect to the origin. If $x > 0$, then $y > 0$ therefore the graph is in the first and the third quarters. This function is non-periodic.

$\lim_{x\to\infty} \frac{x^2+1}{3x}\left(\frac{\infty}{\infty}\right) = \lim_{x\to\infty} \frac{(x^2+1)'}{(3x)'} = \lim_{x\to\infty} \frac{2x}{3} = +\infty$ - function increases indefinitely.

Task 10. Find the limit $\lim_{x\to\infty}\left(\frac{2x+3}{5x+7}\right)^{x+1}$.

Solution: at the value of $x = \infty$ the uncertainty of the form $\left|\left(\frac{\infty}{\infty}\right)^{\infty}\right|$ is obtained. At the beginning of disclosure of uncertainty at the base of degree in the $\left|\left(\frac{\infty}{\infty}\right)^{\infty}\right|$ the equivalence property of infinitely large functions is applied, and then we use the property of the exponential function: $a^{\infty} = \begin{cases} 0, & 0 < a < 1 \\ \infty, & a > 1. \end{cases}$

$$\lim_{x\to\infty}\left(\frac{2x+3}{5x+7}\right)^{x+1} = \left|\left(\frac{\infty}{\infty}\right)^{\infty}\right| = \lim_{x\to\infty}\left(\frac{2x}{5x}\right)^{x+1} = \lim_{x\to\infty}\left(\frac{2}{5}\right)^{x+1} = \left(\frac{2}{5}\right)^{\infty} = \left|\frac{2}{5} < 1\right| = 0.$$

Task 11. Find the specified limits, using L'Hopital rule:

a) $\lim\limits_{x\to 0}\dfrac{1-\cos 7x}{x\sin 7x}$; b) $\lim\limits_{x\to 0}\left(\dfrac{1}{\sin^2 x}-\dfrac{1}{x^2}\right)$.

Solution:

a) L'Hopital rule is applied to the uncertainties of the form $\left(\dfrac{0}{0}\right)$ or $\left(\dfrac{\infty}{\infty}\right)$:

$$\lim_{x\to 0}\frac{1-\cos 7x}{x\sin 7x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(1-\cos 7x)'}{(x\sin 7x)'}=\lim_{x\to 0}\frac{7\sin 7x}{\sin 7x+7x\cos 7x}=\left(\frac{0}{0}\right)=$$

$$=\lim_{x\to 0}\frac{(7\sin 7x)'}{(\sin 7x+7x\cos 7x)'}==\lim_{x\to 0}\frac{49\cos 7x}{7\cos 7x+7(\cos 7x-x\sin 7x)}=\frac{49}{14}=\frac{7}{2};$$

b) at the value of $x=0$ we obtain the indeterminacy of the form $\infty-\infty$. Then the fraction is reduced to a common denominator, and further the properties of equivalence infinitesimals are used.

$$\lim_{x\to 0}\left(\frac{1}{\sin^2 x}-\frac{1}{x^2}\right)=|\infty-\infty|=\lim_{x\to 0}\frac{x^2-\sin^2 x}{x^2\cdot\sin^2 x}=\left|\frac{0}{0}\right|=\lim_{x\to 0}\frac{x^2-\sin^2 x}{x^2\cdot x^2}=$$

$$=\lim_{x\to 0}\frac{(x^2-\sin^2 x)'}{(x^4)'}=\lim_{x\to 0}\frac{2x-\sin 2x}{4x^3}=\lim_{x\to 0}\frac{2-2\cos 2x}{12x^2}=\lim_{x\to 0}\frac{2\sin^2 x}{6x^2}=\lim_{x\to 0}\frac{2x^2}{6x^2}=\frac{1}{3}.$$

Task 12. Investigate the function $f(x)=\begin{cases}-x; & x\le 0\\ x^2; & 0<x\le 2\\ x-1; & x>2\end{cases}$ for continuity and construct the graph.

Solution: this function is defined in the interval $(-\infty;\ +\infty)$ and three components of this function are defined for all *x* from the corresponding intervals, thus, only the points at which the analytical expression of the function changes may be the points of discontinuity, i.e. $x = 0$ and $x = 2$. Let us investigate these points. Let us find the one-sided limits in $x = 0$: $f(0-0)=\lim\limits_{x\to 0-0}(-x)=0$; $f(0+0)=\lim\limits_{x\to 0+0}x^2=0$; the function value at this point $f(0)=-0=0$. Thus, $f(0-0)=f(0+0)=f(0)=0$, therefore $x = 0$ is the continuity point.

Similarly, for $x = 2$: $f(2-0)=\lim\limits_{x\to 2-0}x^2=4;\quad f(2+0)=\lim\limits_{x\to 2+0}(x-1)=1;$

$f(2)=2^2=4$. Since $f(2-0)\neq f(2+0)$, then we have the simple discontinuity at the point $x = 2$. $f(2+0)-f(2-0)=1-4=-3$ is the jump of function at this point. Graph has the form:

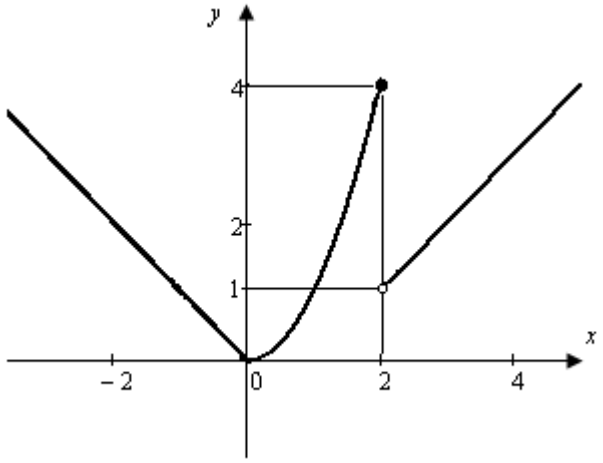


Figure 2.1

Task 13. Find the second derivative of the function given by the parametric equations $\begin{cases} x=\dfrac{1}{\cos t} \\ y=tg\,t \end{cases}$.

Solution:

the first derivative of the function given parametrically $\begin{cases} x=x(t) \\ y=y(t) \end{cases}$ is defined by the formula $y'_x=\dfrac{y'_t}{x'_t}$. The second derivative – by the formula $y''_{xx}=\dfrac{(y'_x)'_t}{x'_t}$.

For our function as $x'_t=\dfrac{\sin t}{\cos^2 t}$, $y'_t=\dfrac{1}{\cos^2 t}$,

then $y'_x=\dfrac{y'_t}{x'_t}=\dfrac{1}{\cos^2 t}\cdot\dfrac{\cos^2 t}{\sin t}=\dfrac{1}{\sin t}$;

$$y''_{xx}=\frac{(y'_x)'_t}{x'_t}=\left(\frac{1}{\sin t}\right)'\cdot\frac{\cos^2 t}{\sin t}=-\frac{\cos t}{\sin^2 t}\cdot\frac{\cos^2 t}{\sin t}=-ctg^3 t.$$

Task 14. For the function $y=\dfrac{x^2+1}{3x}$ find:

a) the intervals of monotonicity, the extremums;
b) the intervals of concavity and convexity, the inflection points;
c) construct the graph.

Solution:

a) $y' = \left(\frac{x^2+1}{3x} \right)' = \frac{1}{3} \cdot \frac{2x^2 - x^2 - 1}{x^2} = \frac{x^2-1}{3x^2}$,

$y' = 0$ at $x_1 = -1, \quad x_2 = 1$ -are stationary points. In the domain of the function y' exists. Let us investigate the change of sign of the derivative of function in each interval:

- if $0 < x < 1$, then $y' < 0$ - the function decreases;
- if $1 < x < +\infty$, then $y' > 0$ - the function increases;
- at $x = 1$ the function has a local minimum: $y(1) = \frac{1+1}{3} = \frac{2}{3}$;
- if $-\infty < x < -1$, then $y' > 0$ - the function increases;
- if $-1 < x < 0$, then $y' < 0$ - the function decreases;
- at $x = -1$ we have a local maximum: $y(-1) = -\frac{2}{3}$;

b) $y'' = \left(\frac{x^2-1}{3x^2} \right)' = \frac{1}{3} \left(1 - \frac{1}{x^2} \right)' = \frac{2}{3x^3}$, $y'' \neq 0$.

The second derivative exists in the domain of definition:

- if $x \in (0; +\infty)$, then $y'' > 0$ - the graph is concave (convex down);
- if $x \in (-\infty; 0)$, then $y'' < 0$ - the graph is convex up; no inflection points;

c) the graph of the function $y = \frac{x^2+1}{3x}$ is shown in figure 2.2.

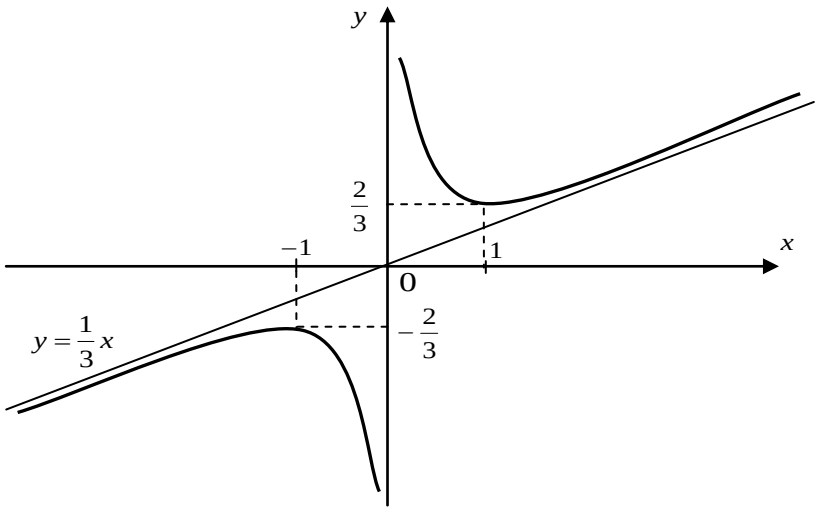


Figure 2.2

3 Calculation-graphical work №3. Integral calculus of one variable function

Purpose: teach students to possess the classical mathematical methods of integration of functions. Give students the skills of finding indefinite and definite integrals, and the ability to use the gained knowledge in the study of further mathematics course and sections of other special disciplines.

3.1 Theoretical questions

1. Antiderivative. Indefinite integral and its properties.
2. Table of integrals.
3. Decomposition of proper rational function in partial fractions. Integration of proper rational function.
4. Integration of irrational functions.
5. Integration of trigonometric functions. Universal substitution. Partial substitutions and rules for their application.
6. Definite integral as the limit of integral sums. Basic properties. The Newton-Leibniz formula. Application of the definite integral.
7. Substitution method in indefinite and definite integrals.
8. Integration by parts of indefinite and definite integrals.

3.2 Calculated tasks

Task 1. Take out the function $f(x)$ from the sign of differential.

№	$d(f(x))$	№	$d(f(x))$	№	$d(f(x))$
1.1	$d\left(\frac{1}{\sqrt{x^2-1}}\right)$	1.2	$d\left(\sqrt[4]{2x+5}\right)$	1.3	$d\left(\ln(2-x)\right)$
1.4	$d\left(\cos(3x-1)\right)$	1.5	$d\left(2^{\sin 2x}\right)$	1.6	$d(\sin(4x-5))$
1.7	$d\left(\arcsin(2x-1)\right)$	1.8	$d\left(\frac{2}{\sqrt[3]{x^7}}\right)$	1.9	$d\left(arctg(2x+5)\right)$
1.10	$d\left(e^{\cos 2x}\right)$	1.11	$d\left(\frac{1}{\sqrt{x^2+1}}\right)$	1.12	$d\left(\sqrt[3]{x+35}\right)$
1.13	$d\left(\ln(2x+7)\right)$	1.14	$d\left(2\cos(4x+1)\right)$	1.15	$d\left(3^{\sin 5x}\right)$
1.16	$d(2\sin(x+6))$	1.17	$d\left(e^{tg\,2x}\right)$	1.18	$d\left(\arcsin(x-2)\right)$
1.19	$d\left(arctg(3x+15)\right)$	1.20	$d\left(\arccos(2x-1)\right)$	1.21	$d\left(\frac{25}{\sqrt[3]{x^{10}}}\right)$

1.22	$d\left(\arcsin(2x+5)\right)$	1.23	$d\left(e^{x^2+2}\right)$	1.24	$d\left(\dfrac{2}{\sqrt{x^2-16}}\right)$
1.25	$d\left(\sqrt[5]{3x-5}\right)$	1.26	$d\left(\ln(2-3x)\right)$	1.27	$d(3\sin(4x+25))$
1.28	$d\left(5^{\sin 2x}\right)$	1.29	$d\left(4\cos(5x-2)\right)$	1.30	$d\left(\arccos(x+3)\right)$

Task 2. Introduce the function $f(x)$ under the sign of differential.

№	$f(x)$	№	$f(x)$	№	$f(x)$
2.1	$\dfrac{5}{x^3}$	2.2	$\sin(3x+5)$	2.3	3^{2x+1}
2.4	$\cos(3-2x)$	2.5	$\dfrac{1}{x\ln x}$	2.6	$tg(6x-2)$
2.7	$\dfrac{1}{\sqrt{x}}$	2.8	$ctg(x+2)$	2.9	$\dfrac{2}{\sqrt[3]{3-x}}$
2.10	$(2x-1)^7$	2.11	$\dfrac{15}{(x+2)^3}$	2.12	5^{-2x+1}
2.13	$3\sin(5x+9)$	2.14	$\dfrac{1}{(x+1)\ln(x+1)}$	2.15	$5\cos(3+4x)$
2.16	$(2+12x)^4$	2.17	$tg(6-2x)$	2.18	$\dfrac{1}{\sqrt{2x+1}}$
2.19	$-ctg(x-2)$	2.20	$\dfrac{2}{\sqrt[3]{3+2x}}$	2.21	$\dfrac{3}{(2x-1)^2}$
2.22	$\dfrac{1}{\sqrt{4-2x}}$	2.23	4^{2x-15}	2.24	$-\sin(2x-5)$
2.25	$(2x+4)^9$	2.26	$\dfrac{1}{(2x-1)\ln(2x-1)}$	2.27	$7\cos(5x-1)$
2.28	3^{5-x}	2.29	$-tg(x-2)$	2.30	$ctg(5x+2)$

Task 3. Find the indefinite integral.

№	Task	№	Task	№	Task
3.1	$\int 2^x \cdot 5^x dx$	3.2	$\int 3^x\left(5-7^{-x}\right)dx$	3.3	$\int 7^{-x}\left(\sqrt{7^{2x}}+5^x\right)dx$
3.4	$\int e^{-x}\left(2^x-3\cdot e^{x+3}\right)dx$	3.5	$\int 5^{-x}\left(4-3^{x+1}\right)dx$	3.6	$\int\left(4\cdot e^x\cdot 5^x-\pi^{2x}\right)dx$
3.7	$\int 3^{-x}\left(7+2^{x+1}\right)dx$	3.8	$\int\left(e^x+\cdot e^{2x}\right)^2 dx$	3.9	$\int\left(e^x-2\cdot e^{-x}\right)dx$
3.10	$\int e^x\cdot\sqrt{2^x}dx$	3.11	$\int 5^x\cdot 3^{1-x}dx$	3.12	$\int \pi^x\cdot e^{x+1}dx$
3.13	$\int\left(5^x-5^{-x}\right)^2 dx$	3.14	$\int 3^{-x}\left(4-5^{x+2}\right)dx$	3.15	$\int\left(5^{x+1}+5^{-x}\right)^2 dx$

3.16	$\int \frac{2^x + 3^x}{2^{x+1}} dx$	3.17	$\int \frac{3^{-x} + 2^{x+3}}{2^{x+1}} dx$	3.18	$\int \left(3^{\frac{x}{2}} - 3^{\frac{-x}{2}} \right)^2 dx$
3.19	$\int 4^x \left(7 + 2^{-x}\right) dx$	3.20	$\int e^x \left(1 - 5 \cdot e^{3-x}\right) dx$	3.21	$\int 2^{-x} \left(3 - 5^{x+2}\right) dx$
3.22	$\int \pi^{2x} \left(\frac{1}{2^x} - 3^{x+1} \right) dx$	3.23	$\int \frac{1}{3^x} \left(e^x + 2 \cdot e^{-x}\right) dx$	3.24	$\int \left(e^x - 2\right)^2 dx$
3.25	$\int \left(e^{-x} - e^x\right)^3 dx$	3.26	$\int \left(e^x + 2^{-x}\right)^2 dx$	3.27	$\int 3^x \left(7 + 2^{-x}\right) dx$
3.28	$\int \frac{3^{-x} + 2^{x+3}}{2^{x+1}} dx$	3.29	$\int \left(e^x + \cdot e^{2x}\right)^3 dx$	3.30	$\int \frac{1}{e^x} \left(e^{x+1} + 2 \cdot e^{-x}\right) dx$

Task 4. Find the indefinite integral.

№	Task	№	Task	№	Task
4.1	$\int \frac{dx}{3\sqrt{9 - x^2}}$	4.2	$\int \frac{7dx}{\sqrt{x^2 + 9}}$	4.3	$\int \frac{5dx}{\sqrt{x^2 - 9}}$
4.4	$\int \frac{dx}{\sqrt{3x - x^2}}$	4.5	$\int \frac{dx}{\sqrt{5 + x^2}}$	4.6	$\int \frac{dx}{\sqrt{x^2 + 7}}$
4.7	$\int \frac{dx}{3x^2 + 1}$	4.8	$\int \frac{dx}{3x^2 - 1}$	4.9	$\int \frac{dx}{1 - 3x^2}$
4.10	$\int \frac{dx}{4 - 5x^2}$	4.11	$\int \frac{dx}{\sqrt{x^2 - 9}}$	4.12	$\int \frac{dx}{\sqrt{16 - x^2}}$
4.13	$\int \frac{dx}{\sqrt{x^2 - 3}}$	4.14	$\int \frac{dx}{\sqrt{x^2 + 16}}$	4.15	$\int \frac{dx}{\sqrt{1 - 9x^2}}$
4.16	$\int \frac{dx}{\sqrt{3x^2 - 27}}$	4.17	$\int \frac{dx}{\sqrt{2x^2 + 8}}$	4.18	$\int \frac{dx}{\sqrt{4 - x^2}}$
4.19	$\int \frac{dx}{\sqrt{4 + x^2}}$	4.20	$\int \frac{dx}{\sqrt{1 + 9x^2}}$	4.21	$\int \frac{dx}{\sqrt{x^2 + 5}}$
4.22	$\int \frac{dx}{\sqrt{x^2 - 5}}$	4.23	$\int \frac{\sqrt{x^2 - 3}}{x^2 - 3} dx$	4.24	$\int \frac{\sqrt{x^2 + 3}}{2x^2 + 6} dx$
4.25	$\int \frac{x^{2+}\sqrt{x^2 - 4}}{x^2 - 4} dx$	4.26	$\int \frac{\sqrt{x^2 + 9} - x^2}{x^2 + 9} dx$	4.27	$\int \left(x^2 - 5\right)^{-\frac{1}{2}} dx$
4.28	$\int \left(x^2 + 5\right)^{-\frac{1}{2}} dx$	4.29	$\int \left(3 - x^2\right)^{-\frac{1}{2}} dx$	4.30	$\int \left(x^2 + 25\right)^{-\frac{1}{2}} dx$

Task 5. Find the indefinite integral.

№	Task	№	Task	№	Task
5.1	$\int\left(\frac{5}{9+x^2}\right)dx$	5.2	$\int\left(\frac{7}{3-x^2}-\frac{4}{5+x^2}\right)dx$	5.3	$\int\frac{dx}{1-7x^2}$
5.4	$\int\frac{dx}{3x^2+2}$	5.5	$\int\frac{dx}{2x^2-3}$	5.6	$\int\frac{dx}{3+7x^2}$
5.7	$\int\frac{2}{2x^2-1}dx$	5.8	$\int\frac{dx}{5-3x^2}$	5.9	$\int\frac{dx}{3x^2-2}$
5.10	$\int\left(\frac{1}{x^2}-\frac{2}{x^2+2}\right)dx$	5.11	$\int\left(\frac{3}{x^2}-\frac{4}{x^2-2}\right)dx$	5.12	$\int\left(\frac{5}{x^2-3}-\frac{7}{x^2+3}\right)dx$
5.13	$\int\frac{dx}{4x^2+1}$	5.14	$\int\left(\frac{3}{x^2-5}-\frac{2}{x^2+3}\right)dx$	5.15	$\int\frac{dx}{4x^2-1}$
5.16	$\int\left(\frac{3}{4-x^2}-\frac{4}{1+x^2}\right)dx$	5.17	$\int\left(\frac{7}{9-x^2}+\frac{4}{25+x^2}\right)dx$	5.18	$\int\frac{dx}{4x^2+25}$
5.19	$\int\frac{dx}{16-27x^2}$	5.20	$\int\frac{2}{25x^2-1}dx$	5.21	$\int\frac{dx}{63+7x^2}$
5.22	$\int\frac{dx}{25x^2-36}$	5.23	$\int\left(\frac{5}{x^2+36}\right)dx$	5.24	$\int\frac{6dx}{36x^2-25}$
5.25	$\int\left(\frac{1}{x^2}-\frac{2}{x^2+4}\right)dx$	5.26	$\int\left(\frac{2}{9+x^2}+\frac{4}{4-x^2}\right)dx$	5.27	$\int\frac{dx}{49x^2+16}$
5.28	$\int\frac{dx}{4x^2-1}$	5.29	$\int\frac{dx}{64x^2+1}$	5.30	$\int\left(\frac{7}{81-x^2}\right)dx$

Task 6. Find the indefinite integral using the table of integrals and the property of the linear argument of the integrand.

№	Task	№	Task	№	Task
6.1	$\int(\cos 2x-3\sin x)dx$	6.2	$\int\frac{dx}{\cos^2 3x}$	6.3	$\int\frac{dx}{1-\sin^2 5x}$
6.4	$\int tg^2 2xdx$	6.5	$\int\left(e^{5x}-3e^{-x}+4e^{\frac{x}{2}}\right)dx$	6.6	$\int\left(e^{4x}-2e^{-3x}\right)^2dx$
6.7	$\int\sin(2x+3)dx$	6.8	$\int\frac{2^{3x}-3^x}{3^{2x}}dx$	6.9	$\int\frac{\left(e^{-x}+5\right)^2}{e^x}dx$
6.10	$\int\left(\sqrt[3]{e^x}-x\right)dx$	6.11	$\int\frac{dx}{\sqrt{e^{3x}}}$	6.12	$\int\frac{dx}{\sqrt{6e^{5x-1}}}$
6.13	$\int(\cos x+\sin x)^2dx$	6.14	$\int(\cos x-\sin x)^2dx$	6.15	$\int(\cos 3x+\sin 3x)^2dx$

6.16	$\int(\cos 2x-\sin 2x)^2\,dx$	6.17	$\int\frac{dx}{\sin^2 5x}$	6.18	$\int\frac{dx}{\cos^2(x/3)}$
6.19	$\int\cos(1-5x)dx$	6.20	$\int\sin^2 xdx$	6.21	$\int\cos^2 xdx$
6.22	$\int\left(\cos\frac{x}{3}-\sin\frac{x}{2}\right)dx$	6.23	$\int\cos x\cdot(1-5\cdot\cos x)dx$	6.24	$\int\frac{5^{2x}-3^{-x}}{2^x}dx$
6.25	$\int\frac{5^{2x-1}-5^{2-2x}}{4^x}dx$	6.26	$\int\frac{e^x-e^{-x}}{e^{2x}}dx$	6.27	$\int\frac{e^{x+1}+e^{1-x}}{e^x}dx$
6.28	$\int\left(\cos\frac{x}{3}-5\sin\frac{x}{2}\right)^2dx$	6.29	$\int\cos x(1-25\cos x)dx$	6.30	$\int\sin^2 xdx$

Task 7. Find the indefinite integral.

№	Task	№	Task	№	Task
7.1	$\int\frac{dx}{x-5}$	7.2	$\int\frac{x}{(x+3)^2}dx$	7.3	$\int\frac{dx}{3x+1}$
7.4	$\int\frac{dx}{5-7x}$	7.5	$\int\frac{dx}{\sqrt{x+9}}$	7.6	$\int\frac{dx}{\sqrt[3]{2-x}}$
7.7	$\int(x-a)^4dx$	7.8	$\int\sqrt[5]{(x+3)}^4dx$	7.9	$\int\frac{dx}{\sqrt[3]{(x+2)^4}}$
7.10	$\int\frac{dx}{(x+1)\cdot\sqrt{2x+2}}$	7.11	$\int\sqrt[3]{(x+2)}^5dx$	7.12	$\int\sqrt{\frac{7}{(x-2)^5}}dx$
7.13	$\int\left(\sqrt[3]{\frac{1}{x+1}}\right)^5dx$	7.14	$\int\sqrt{\frac{5}{(x-2)^7}}dx$	7.15	$\int\sqrt[4]{(x+a)^5}dx$
7.16	$\int\frac{x}{(x-3)^5}dx$	7.17	$\int\frac{x}{\sqrt{x+2}}dx$	7.18	$\int\frac{1-2x}{\sqrt{x+1}}dx$
7.19	$\int\frac{x}{(x-2)^3}dx$	7.20	$\int\frac{x}{\sqrt{x+2}}dx$	7.21	$\int\frac{x}{(x-3)^5}dx$
7.22	$\int\frac{dx}{x-5}$	7.23	$\int\frac{x}{(x-3)^2}dx$	7.24	$\int\frac{dx}{3x+1}$
7.25	$\int\frac{dx}{(x+25)\cdot\sqrt{x+25}}$	7.26	$\int\sqrt[3]{(x+2)}^5dx$	7.27	$\int\sqrt{\frac{1}{(x-21)^5}}dx$
7.28	$\int\frac{dx}{(25x+1)\cdot\sqrt{25x+1}}$	7.29	$\int\sqrt[3]{(x+25)}^7dx$	7.30	$\int\frac{3dx}{(x-5)^3}$

Task 8. Find the indefinite integral by introducing under the sign of differential.

№	Task	№	Task	№	Task
8.1	$\int e^{x^2} \cdot xdx$	8.2	$\int e^{-x^2} \cdot xdx$	8.3	$\int e^{5x^2} \cdot xdx$
8.4	$\int e^{\frac{x^2}{2}} \cdot xdx$	8.5	$\int e^{x^2+3} \cdot xdx$	8.6	$\int e^{1-3x^2} \cdot xdx$
8.7	$\int x \cdot \sqrt{x^2+1}dx$	8.8	$\int x \cdot \sqrt[3]{\left(x^2+3\right)^4}dx$	8.9	$\int \frac{xdx}{\sqrt{x^2+9}}$
8.10	$\int \frac{xdx}{x^2+10}$	8.11	$\int \frac{e^x dx}{e^x+5}$	8.12	$\int e^x \cdot \sqrt{3e^x-7}dx$
8.13	$\int \frac{e^x dx}{\sqrt{e^{2x}-5}}$	8.14	$\int \frac{e^x dx}{e^{2x}+10}$	8.15	$\int \frac{\ln^2 x}{x}dx$
8.16	$\int \frac{dx}{x \cdot \ln^3 x}$	8.17	$\int \frac{dx}{x \cdot \ln x}$	8.18	$\int tgxdx$
8.19	$\int ctgxdx$	8.20	$\int \cos x \cdot \sqrt{\sin x}dx$	8.21	$\int e^{-x^3} \cdot x^2 dx$
8.22	$\int \frac{e^x dx}{e^{2x}-4}$	8.23	$\int \frac{dx}{x \cdot \left(\ln^2 x+1\right)}$	8.24	$\int x \cdot \sqrt{\left(x^2+3\right)^5}dx$
8.25	$\int \frac{\sqrt{\ln x}}{x}dx$	8.26	$\int 4^{\cos x} \sin xdx$	8.27	$\int \frac{\cos x}{2+3\sin x}dx$
8.28	$\int \frac{x^2}{1-x^3}dx$	8.29	$\int \frac{6^x}{1-6^x}dx$	8.30	$\int \frac{e^x}{\left(e^x-1\right)^2}dx$

Task 9. Find the indefinite integral (integration of quadratic trinomial).

№	Task	№	Task	№	Task
9.1	$\int \frac{dx}{x^2+8x+5}$	9.2	$\int \frac{dx}{x^2-x+1}$	9.3	$\int \frac{dx}{\sqrt{x^2-6x+10}}$
9.4	$\int \frac{dx}{\sqrt{x^2+6x+11}}$	9.5	$\int \frac{dx}{\sqrt{x^2+3x+1}}$	9.6	$\int \frac{dx}{\sqrt{1+2x-x^2}}$
9.7	$\int \frac{dx}{\sqrt{2-2x-x^2}}$	9.8	$\int \frac{xdx}{x^2+2x+5}$	9.9	$\int \frac{(x-1)dx}{x^2-2x+5}$
9.10	$\int \frac{1-x}{x^2-x+1}dx$	9.11	$\int \frac{dx}{x^2+x+2}$	9.12	$\int \frac{dx}{x+(x+1)^2}$
9.13	$\int \frac{3x+1}{(x-2)^2+(x+1)}dx$	9.14	$\int \frac{x^2+7}{x^2+4}dx$	9.15	$\int \frac{dx}{x+x^2+3}$

9.16	$\int \frac{x+1}{x+\left(x^2+3\right)}dx$	9.17	$\int \frac{dx}{x^3+1}$	9.18	$\int \frac{dx}{x^3-8}$
9.19	$\int \frac{x}{x^2+6x-1}dx$	9.20	$\int \frac{x}{x^2-4x+1}dx$	9.21	$\int \frac{3x+5}{\sqrt{x^2+6x-5}}dx$
9.22	$\int \frac{1-2x}{x^2-6x+10}dx$	9.23	$\int \frac{dx}{\sqrt{3+2x-x^2}}$	9.24	$\int \frac{dx}{2x^2-3x+1}$
9.25	$\int \frac{dx}{5x-x^2-6}$	9.26	$\int \frac{(2x-3)dx}{4+3x-x^2}$	9.27	$\int \frac{dx}{\sqrt{x^2-4x+5}}$
9.28	$\int \frac{dx}{x^2+x-3}$	9.29	$\int \frac{2x+1}{x^2+x-3}dx$	9.30	$\int \frac{dx}{x^2+8x+7}$

Task 10. Find the indefinite integral using integration by parts.

№	Task	№	Task	№	Task
10.1	$\int x \cdot e^{2x}dx$	10.2	$\int (x-1)\cdot e^{-x}dx$	10.3	$\int (2x+5)\cdot e^{\frac{x}{3}}dx$
10.4	$\int x^2 \cdot e^{x}dx$	10.5	$\int x \cdot 3^{x}dx$	10.6	$\int x \cdot \ln x dx$
10.7	$\int x \cdot \ln 5x dx$	10.8	$\int \sqrt{x} \cdot \ln x dx$	10.9	$\int (\ln x - 2\ln^2 x)dx$
10.10	$\int \frac{\ln x}{x^2}dx$	10.11	$\int x \cdot arctgx dx$	10.12	$\int x \cdot \cos x dx$
10.13	$\int x \cdot \sin x dx$	10.14	$\int x \cdot \cos 3x dx$	10.15	$\int x \cdot \sin \frac{x}{2}dx$
10.16	$\int x \cdot \cos \frac{x}{3}dx$	10.17	$\int x \cdot \sin 5x dx$	10.18	$\int x^2 \cdot e^{-x}dx$
10.19	$\int x \cdot 5^{x}dx$	10.20	$\int \frac{x}{2^x}dx$	10.21	$\int x^2\sqrt{e^x}dx$
10.22	$\int x \cdot e^{-3x}dx$	10.23	$\int x \cdot \sin \frac{x}{3}dx$	10.24	$\int x \cdot e^{\frac{x}{2}}dx$
10.25	$\int \sqrt[3]{x}\ln x dx$	10.26	$\int (x+5)3^{x}dx$	10.27	$\int \frac{\ln x}{\sqrt[3]{x}}dx$
10.28	$\int x \cdot \sin 3x dx$	10.29	$\int \frac{x}{7^x}dx$	10.30	$\int x\sqrt{e^x}dx$

Task 11. Find the indefinite integral by selecting the whole part of improper rational function.

№	Task	№	Task	№	Task
11.1	$\int \frac{3x^2+1}{x^2+25}dx$	11.2	$\int \frac{x^2-3+2x+6}{x^2-9}dx$	11.3	$\int \frac{x^3}{x^2+9}dx$
11.4	$\int \frac{x^3}{x^2+9}dx$	11.5	$\int \frac{x^2+1}{x^2+5}dx$	11.6	$\int \frac{2x^2-2}{x^2-16}dx$
11.7	$\int \frac{x^2+1}{x^2-25}dx$	11.8	$\int \frac{x^2+2}{x^2-1}dx$	11.9	$\int \frac{x^2+2x+4}{x^2+2x-1}dx$
11.10	$\int \frac{x^2+2}{x^2-1}dx$	11.11	$\int \frac{3x^2+1}{x^2+6x+25}dx$	11.12	$\int \frac{x^2+x-2}{x^2-8x+20}dx$
11.13	$\int \frac{x^2+2}{x^2-6x+10}dx$	11.14	$\int \frac{x^2+7}{x^2+4}dx$	11.15	$\int \frac{x^4}{x^2+9}dx$
11.16	$\int \frac{x^2+2x}{x^2-8x+25}dx$	11.17	$\int \frac{x^2+1}{x^2+4x+5}dx$	11.18	$\int \frac{x^2+2}{x^2-4x+13}dx$
11.19	$\int \frac{x^3}{x^2-1}dx$	11.20	$\int \frac{x^4}{x^2+1}dx$	11.21	$\int \frac{x^3}{x^3+8}dx$
11.22	$\int \frac{x^2+3}{x^2+9}dx$	11.23	$\int \frac{3x^2+2x+1}{x^2+25}dx$	11.24	$\int \frac{x^2-4x+2}{x^2-1}dx$
11.25	$\int \frac{x^2+2}{x^2+6x-1}dx$	11.26	$\int \frac{x^2+2x+16}{x^2-9}dx$	11.27	$\int \frac{x^2+x+1}{x^2+5}dx$
11.28	$\int \frac{3x^2+1}{x^2-2x+26}dx$	11.29	$\int \frac{x^3+2x}{x^2+49}dx$	11.30	$\int \frac{x^2-3+2x}{x^2-9}dx$

Task 12. Find the indefinite integral of proper rational function.

№	Task	№	Task	№	Task
12.1	$\int \frac{dx}{(x-2)^2(x+3)}$	12.2	$\int \frac{(x+1)}{(x-3)^2(x+4)}dx$	12.3	$\int \frac{(x-1)}{(x+2)^2(x-3)}dx$
12.4	$\int \frac{(x+1)}{x^2(x+3)}dx$	12.5	$\int \frac{x-7}{(x+5)(x-4)^3}dx$	12.6	$\int \frac{(x-14)dx}{(x+2)^2(x+3)}$
12.7	$\int \frac{(2x+1)}{(x+3)^2(x+4)}dx$	12.8	$\int \frac{(2x-1)}{(x+2)^2(2x-3)}dx$	12.9	$\int \frac{(2x+11)}{x^2(x+3)}dx$
12.10	$\int \frac{(x-1)dx}{(x-2)^3(x+3)}$	12.11	$\int \frac{dx}{x^2(x+2)}$	12.12	$\int \frac{dx}{x\cdot(x+1)^2}$

12.13	$\int \frac{3x+1}{(x-2)^2 \cdot (x+1)} dx$	12.14	$\int \frac{x^2+7}{(x^2+4)(x-1)} dx$	12.15	$\int \frac{dx}{x \cdot (x^2+3)}$
12.16	$\int \frac{x^2+1}{x \cdot (x^2+3)} dx$	12.17	$\int \frac{dx}{x^3+1}$	12.18	$\int \frac{dx}{x^3-8}$
12.19	$\int \frac{x^3}{x^4-1} dx$	12.20	$\int \frac{x^2}{(x^3+1)(x+1)} dx$	12.21	$\int \frac{(4x+1)}{x^2(x+3)} dx$
12.22	$\int \frac{2x-7}{(x-5)(x-4)^3} dx$	12.23	$\int \frac{(x+12)}{(x-3)^3(x+4)} dx$	12.24	$\int \frac{(2x+1)dx}{(x-2)^3(x+3)}$
12.25	$\int \frac{(3x-1)}{(x+2)^3(x-3)} dx$	12.26	$\int \frac{(x+1)}{x(x+3)^2} dx$	12.27	$\int \frac{3x-7}{(x+5)^2(x-4)} dx$
12.28	$\int \frac{(5x+1)}{x^3(x+3)} dx$	12.29	$\int \frac{(2x-1)}{(x-3)(x+4)^2} dx$	12.30	$\int \frac{(x-1)}{(x+2)(x-3)^2} dx$

Task 13. Find the indefinite integral of proper rational function.

№	Task	№	Task
13.1	$\int \frac{3x+13}{(x-1)(x^2+2x+5)} dx$	13.2	$\int \frac{5x+3}{(x+2)(x^2+2x+4)} dx$
13.3	$\int \frac{12-6x}{(x+1)(x^2-4x+13)} dx$	13.4	$\int \frac{2x^2+2x+20}{(x-1)(x^2+2x+10)} dx$
13.5	$\int \frac{3x-6}{(x+1)(x^2+6x+13)} dx$	13.6	$\int \frac{x^2+3x+20}{(x-1)(x^2+x+1)} dx$
13.7	$\int \frac{36}{(x+2)(x^2-2x+10)} dx$	13.8	$\int \frac{3x+13}{(x+3)(x^2+6x+13)} dx$
13.9	$\int \frac{6}{(x-2)(x^2+4x+13)} dx$	13.10.	$\int \frac{4x^2+2x+17}{(x-1)(x^2+2x+5)} dx$
13.11	$\int \frac{3x+13}{(x-1)(x^2+2x+5)} dx$	13.12	$\int \frac{x^2+2x+40}{(x+2)(x^2-2x+10)} dx$
13.13	$\int \frac{4x-x^2-12}{(x-1)(x^2+2x+10)} dx$	13.14	$\int \frac{x^2-13x+40}{(x+1)(x^2-4x+13)} dx$
13.15	$\int \frac{30}{(x+2)(x^2+6x+25)} dx$	13.16	$\int \frac{7x+3}{(x-4)(x^2+8x+20)} dx$
13.17	$\int \frac{36}{(x+1)(x^2-12x+40)} dx$	13.18	$\int \frac{2x^2+2x+20}{(x-1)(x^2+6x+13)} dx$

13.19	$\int \frac{2x^2+7x+7}{(x+1)(x^2+2x+5)}dx$	13.20	$\int \frac{19ч-x^2-34}{(x+1)(x^2-4x+13)}dx$
13.21	$\int \frac{5x+10}{(x+3)(x^2+2x+10)}dx$	13.22	$\int \frac{4x^2+38}{(x+2)(x^2-2x+10)}dx$
13.23	$\int \frac{8}{(x+1)(x^2+6x+13)}dx$	13.24	$\int \frac{18}{(x-3)(x^2-6x+25)}dx$
13.25	$\int \frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}dx$	13.26	$\int \frac{x+15}{(x+5)(x^2+2x+5)}dx$
13.27	$\int \frac{13x+3}{(x-1)(x^2+12x+40)}dx$	13.28	$\int \frac{72}{(x+2)(x^2-4x+20)}dx$
13.29	$\int \frac{2x^2+2x+20}{(x+1)(x^2+4x+5)}dx$	13.30	$\int \frac{3x+13}{(x-5)(x^2+12x+40)}dx$

Task 14. Find the indefinite integral.

№	Task	№	Task	№	Task
14.1	$\int \frac{1-5x^3}{x^4}dx$	14.2	$\int \frac{\sqrt{x}-3x\cdot\sqrt[3]{x}+x^5}{x^2}dx$	14.3	$\int \frac{\left(\sqrt{x}-2\sqrt[3]{x}\right)^2}{x}dx$
14.4	$\int x\cdot\sqrt[3]{a\cdot x}dx$	14.5	$\int\left(\frac{3}{\sqrt[3]{x^2}}-\frac{x+2}{\sqrt[4]{x}}\right)dx$	14.6	$\int\left(a^{\frac{1}{3}}-x^{\frac{1}{3}}\right)^3 dx$
14.7	$\int\left(5x^3-\frac{5}{\sqrt[3]{x^5}}\right)dx$	14.8	$\int\left(\frac{x^2}{\sqrt{x}}+\frac{a}{x^4}-x\right)dx$	14.9	$\int \frac{\left(\sqrt{x^3}-2x^2+3x^4\right)}{x}dx$
14.10	$\int\left(\sqrt{x}-\frac{1}{\sqrt[3]{x}}\right)^2 dx$	14.11	$\int\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)^3 dx$	14.12	$\int\left(x\cdot\sin a-\frac{4\sqrt{x}}{x^2}\right)dx$
14.13	$\int\left(\sqrt{\frac{x}{a}}-\sqrt[3]{x}\right)^2 dx$	14.14	$\int \frac{x^3-3\sqrt{x^5}}{\sqrt{x}}dx$	14.15	$\int \frac{x^4-2x-\sqrt{x}}{x^2}dx$
14.16	$\int x^2\cdot\sqrt[4]{b\cdot x}dx$	14.17	$\int \frac{\left(\sqrt{b\cdot x}-\frac{x}{b}\right)^2}{x}dx$	14.18	$\int \frac{(x-1)^3}{x\cdot\sqrt{x}}dx$
14.19	$\int \frac{(2x+a)^3}{x^2}dx$	14.20	$\int \frac{\left(x-\frac{a}{x}\right)^2}{\sqrt{x}}dx$	14.21	$\int \frac{\left(b\cdot x+\frac{a}{x}\right)^2}{\sqrt[3]{x}}dx$
14.22	$\int \frac{\left(1-b\sqrt[3]{x}\right)^3}{x^2}dx$	14.23	$\int \frac{(2x+b)^3}{x^3}dx$	14.24	$\int \frac{x^3-2\sqrt{x}+x\sqrt[3]{x}}{x}dx$

14.25	$\int \frac{(2x+5)^3}{x^2}dx$	14.26	$\int \frac{\left(x+\frac{1}{x}\right)^2}{\sqrt[4]{x}}dx$	14.27	$\int \frac{\left(2-a\sqrt{x^3}\right)^2}{x^2}dx$
14.28	$\int \frac{\left(1-b\sqrt{x}\right)^2}{x^3}dx$	14.29	$\int \frac{x^4-2\sqrt{x}+x}{x^2}dx$	14.30	$\int \frac{\left(2-\sqrt{x^3}\right)^3}{x^2}dx$

Task 15. Find the indefinite integral.

№	Task	№	Task
15.1	$\int\left(5\cdot\cos x-\frac{7}{\sin^2 x}\right)dx$	15.2	$\int\left(3\cdot\sin x+\frac{2}{\cos^2 x}\right)dx$
15.3	$\int \frac{\sin^2 x-3\cdot\sin x-\frac{5}{\sin x}}{\sin x}dx$	15.4	$\int \frac{5\cdot\sin 2x-5\cdot tgx}{\sin x\cdot\cos x}dx$
15.5	$\int \frac{\cos 2x-5\cdot\cos^3 x}{\cos^2 x}dx$	15.6	$\int \frac{1+5\cdot\cos^3 x}{\cos^2 2x}dx$
15.7	$\int \frac{1-3\sin^3 x}{\sin^2 x}dx$	15.8	$\int \frac{2+\cos^3 x}{\cos^2 x}dx$
15.9	$\int\left(5\cdot\sin x-\frac{4}{\cos^2 x}\right)dx$	15.10	$\int \frac{\sin 2x}{\cos^3 x}dx$
15.11	$\int \frac{\sin 2x}{\sin^3 x}dx$	15.12	$\int \frac{\cos 2x}{\cos^2 x}dx$
15.13	$\int \frac{\cos 2x}{\sin^2 x}dx$	15.14	$\int(3\cdot\cos x-4\cdot\sin x)dx$
15.15	$\int \frac{\cos^2 x+5\cdot\cos x}{\cos x}dx$	15.16	$\int \frac{\cos^2 2x-\cos^2 x}{\sin^4 x}dx$
15.17	$\int \frac{\cos^2 2x-\cos^2 x}{\sin^4 x}dx$	15.18	$\int\left(\frac{\sin x}{2}+\frac{7}{\cos^2 x}\right)dx$
15.19	$\int \frac{ctgx}{\sin x\cdot\cos x}dx$	15.20	$\int \frac{tgx}{\sin 2x}dx$
15.21	$\int \frac{ctgx}{\sin 2x}dx$	15.22	$\int \frac{\cos^2 2x-\cos^2 x}{\sin^4 x}dx$
15.23	$\int ctg^2 xdx$	15.24	$\int \frac{5\sin 2x}{\cos^3 x}dx$
15.25	$\int(1+tg^2 x)dx$	15.26	$\int \frac{\cos 2x-\cos^2 x}{\sin^4 x}dx$

15.27	$\int \frac{\cos^2 x + 5 \cdot \cos x}{\cos x} dx$	15.28	$\int \frac{\cos\ x + 5 \cdot \cos x}{\cos x} dx$
15.29	$\int \frac{\cos 2x - \cos^2 x}{\sin^4 x} dx$	15.30	$\int \frac{5 \sin 2x}{\cos^3 x} dx$

Task 16. Find the indefinite integral using the universal trigonometric substitution.

№	Task	№	Task
16.1	$\int \frac{dx}{5 + 2\sin x + 3\cos x}$	16.2	$\int \frac{dx}{5 - 4\sin x + 2\cos x}$
16.3	$\int \frac{dx}{5 + 3\sin x - 5\cos x}$	16.4	$\int \frac{3\sin x - 2\cos x}{1 + \cos x} dx$
16.5	$\int \frac{dx}{3 + 2\sin x - \cos x}$	16.6	$\int \frac{dx}{10\sin x + 5\cos x}$
16.7	$\int \frac{dx}{5 - 3\cos x}$	16.8	$\int \frac{dx}{8 - 4\sin x + 7\cos x}$
16.9	$\int \frac{dx}{3 + 5\cos x}$	16.10	$\int \frac{dx}{3 + 2\sin x + 3\cos x}$
16.11	$\int \frac{dx}{5 + 4\sin x}$	16.12	$\int \frac{dx}{3\sin x - 4\cos x}$
16.13	$\int \frac{dx}{7 + 3\sin x}$	16.14	$\int \frac{dx}{8 + 4\cos x}$
16.15	$\int \frac{dx}{2 + 4\sin x + 3\cos x}$	16.16	$\int \frac{dx}{7\sin x - 3\cos x}$
16.17	$\int \frac{dx}{3 - 4\sin x}$	16.18	$\int \frac{dx}{5 + \sin x + 3\cos x}$
16.19	$\int \frac{dx}{5 + 4\sin x + 3\cos x}$	16.20	$\int \frac{7 + 6\sin x - 5\cos x}{1 + \cos x} dx$
16.21	$\int \frac{dx}{3 + \sin x + \cos x}$	16.22	$\int \frac{6\sin x + \cos x}{1 + \cos x} dx$
16.23	$\int \frac{dx}{5 - 4\sin x + 3\cos x}$	16.24	$\int \frac{dx}{5 + 3\cos x}$
16.25	$\int \frac{dx}{4\cos x + 3\sin x}$	16.26	$\int \frac{dx}{4 - 4\sin x + 3\cos x}$
16.27	$\int \frac{dx}{7 - 3\sin x + \cos x}$	16.28.	$\int \frac{dx}{3 + 5\sin x + 3\cos x}$
16.29	$\int \frac{dx}{3\sin x - \cos x}$	16.30	$\int \frac{dx}{2 + \sin x - 3\cos x}$

Task 17. Calculate the definite integral.

№	Task	№	Task	№	Task
17.1	$\int_{-1}^{2} x^4 dx$	17.2	$\int_{0}^{1} \left(\sqrt{x-1}\right)^2 dx$	17.3	$\int_{3}^{10} \sqrt[3]{x-2} dx$
17.4	$\int_{1}^{8} \frac{dx}{\sqrt[3]{x^2}}$	17.5	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin 2x dx$	17.6	$\int_{-\frac{\pi}{8}}^{0} \frac{dx}{\cos^2 2x}$
17.7	$\int_{0}^{1} \frac{x+2}{x+1} dx$	17.8	$\int_{0.5}^{1} e^{-2x+1} dx$	17.9	$\int_{1}^{e} \frac{\ln x}{x} dx$
17.10	$\int_{0}^{\frac{\pi}{4}} x \sin x^2 dx$	17.11	$\int_{-1}^{1} \frac{dx}{x^2+2x+5}$	17.12	$\int_{-2}^{-1} \frac{dx}{x^2}$
17.13	$\int_{0.5}^{1} \frac{dx}{(x+1)^2}$	17.14	$\int_{\frac{\pi}{4}}^{0} \cos^2 t dt$	17.15	$\int_{0}^{\frac{\pi}{2}} \frac{dx}{\cos^2\left(\frac{x}{2}\right)}$
17.16	$\int_{0}^{\pi} \left[\sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right)\right]^2 dx$	17.17	$\int_{-1}^{-0.5} \frac{x^2-1}{x^4} dx$	17.18	$\int_{-1}^{0} \frac{dx}{\sqrt{x+2}}$
17.19	$\int_{\frac{\pi}{4}}^{0} \sin^2 t dt$	17.20	$\int_{0}^{7} \sqrt[3]{x+1} dx$	17.21	$\int_{0}^{\frac{1}{2}} \frac{dx}{(x-1)^3}$
17.22	$\int_{\frac{\pi}{2}}^{\pi} \frac{dx}{\sin^2\left(\frac{x}{2}\right)}$	17.23	$\int_{\frac{1}{2}}^{1} \frac{x^2+1}{x^4} dx$	17.24	$\int_{0}^{16} \sqrt[4]{x^5} dx$
17.25	$\int_{\pi}^{2\pi} \frac{dx}{\cos^2\left(\frac{x}{4}\right)}$	17.26	$\int_{0}^{\frac{\pi}{3}} tg \frac{x}{2} dx$	17.27	$\int_{0.5}^{1} \frac{dx}{(2x+1)^3}$
17.28	$\int_{0.25}^{1} \frac{dx}{(4x+1)^2}$	17.29	$\int_{-2}^{2} \frac{dx}{x^2+4x+5}$	17.30	$\int_{\frac{1}{3}}^{2} \frac{x^2+1}{x^4} dx$

Task 18. Calculate the definite integral using the substitution of integrand variable.

№	Task	№	Task
18.1	$\int_0^2 x^3 \cdot e^{x^2} dx$	18.2	$\int_0^{0.5} x^3 \cdot e^{-x^4} dx$
18.3	$\int_1^{\sqrt{2}} x^3 \cdot e^{-x^2} dx$	18.4	$\int_2^3 \frac{dx}{1+\sqrt{2x-1}}$
18.5	$\int_1^e \frac{dx}{x \cdot (\ln^2 x + 1)}$	18.6	$\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\cos x}$
18.7	$\int_1^6 \frac{dx}{x+\sqrt{3x-2}}$	18.8	$\int_4^9 e^{-2\sqrt{x}} dx$
18.9	$\int_0^2 \sqrt{4-x^2} dx$	18.10	$\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x} dx$
18.11	$\int_{\ln 1}^{\frac{\ln 3}{2}} \frac{dx}{e^x + e^{-x}}$	18.12	$\int_{-1}^1 \frac{xdx}{\sqrt{5+4x}}$
18.13	$\int_e^{e^2} \frac{dx}{x \cdot \ln^3 x}$	18.14	$\int_0^3 \sqrt{9-x^2} dx$
18.15	$\int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$	18.16	$\int_1^{\ln^2 2} e^{-\sqrt{x}} dx$
18.17	$\int_0^{\frac{\pi}{2}} \frac{dx}{4+3\cos x}$	18.18	$\int_0^4 \sqrt{x} \cdot e^{-\sqrt{x}} dx$
18.19	$\int_0^1 x^3 \cdot e^{x^2} dx$	18.20	$\int_0^{\sqrt{2}} \frac{xdx}{x^4+2x^2+25}$
18.21	$\int_{\frac{1}{2}}^1 \frac{e^{\frac{1}{x}}}{x^2} dx$	18.22	$\int_e^{e^2} \frac{dx}{x \cdot \sqrt{\ln^3 x}}$
18.23	$\int_{\frac{1}{\sqrt{3}}}^1 \frac{dx}{x \cdot \sqrt{x^2+1}}$	18.24	$\int_0^{\ln 2} \frac{dx}{e^x + 2e^{-x}}$

18.25	$\int_0^1 \sqrt{1-x^2}\,dx$	18.26	$\int_0^{\sqrt{2}} \frac{x\,dx}{x^4+12x^2+225}$
18.27	$\int_{\frac{1}{\sqrt{3}}}^1 \frac{dx}{x\cdot\sqrt{x^2+4}}$	18.28	$\int_e^{e^2} \frac{dx}{x\cdot\sqrt{\ln^5 x}}$
18.29	$\int_e^{e^2} \frac{dx}{x\cdot\sqrt{\ln^5 x}}$	18.30	$\int_{\frac{1}{2}}^1 \frac{e^{\frac{1}{x}}}{x^2}\,dx$

Task 19. Calculate the definite integral using the method of integration by parts and replacing of integrand variable.

№	Task	№	Task	№	Task
19.1	$\int_2^3 x\ln(x-1)dx$	19.2	$\int_{-2}^0 x^2 e^{-x/2}dx$	19.3	$\int_0^{\pi/2} x\cos x dx$
19.4	$\int_0^{\pi} x^2\sin x dx$	19.5	$\int_{-1/2}^{1/2} \arccos 2x dx$	19.6	$\int_1^2 (x-1)\ln(x)dx$
19.7	$\int_{-1/2}^0 xe^{-2x}dx$	19.8	$\int_{-\pi}^{\pi} x\sin x\cos x dx$	19.9	$\int_{-2/3}^{-1/3} x^2 e^{-3x}dx$
19.10	$\int_1^e \frac{\ln^2 x}{x^2}dx$	19.11	$\int_1^{e^2} \sqrt{x}\ln(x)dx$	19.12	$\int_0^1 arctg\sqrt{x}dx$
19.13	$\int_0^{\pi} (x+2)\cos\frac{x}{2}dx$	19.14	$\int_0^{\pi/8} x^2\sin 4x dx$	19.15	$\int_1^3 x^2\ln(x)dx$
19.16	$\int_1^2 \frac{\ln^2(x+1)}{(x+1)^2}dx$	19.17	$\int_{3/2}^2 arctg(2x-3)dx$	19.18	$\int_0^{\pi/2} (x+3)\sin x dx$
19.19	$\int_1^e x\ln^2(x)dx$	19.20	$\int_{-3}^0 (x-2)e^{-x/3}dx$	19.21	$\int_0^{\pi/9} \frac{xdx}{\cos^2 3x}$
19.22	$\int_{1/2}^1 \arcsin(1-x)dx$	19.23	$\int_1^{\sqrt{3}} arctg\frac{1}{x}dx$	19.24	$\int_{-1}^0 x\ln(1-x)dx$
19.25	$\int_0^1 \frac{\arcsin(x/2)}{\sqrt{2-x}}dx$	19.26	$\int_1^2 \ln(3x+2)dx$	19.27	$\int_0^4 x^3\sqrt{x^2+9}dx$

19.28	$\int_{-1}^{0}(x+1)e^{-2x}dx$	19.29	$\int_{0}^{\pi/4}xtg^2xdx$	19.30	$\int_{0}^{1}xarctgxdx$

Task 20. Calculate the improper integral (or prove its divergence).

№	Task	№	Task	№	Task
20.1	$\int_{0}^{+\infty}\frac{dx}{9x^2+1}$	20.2	$\int_{0}^{+\infty}\frac{dx}{\sqrt{3x^2+5}}$	20.3	$\int_{2}^{+\infty}\frac{dx}{x\ln x}$
20.4	$\int_{0}^{+\infty}\frac{dx}{\sqrt{4-9x^2}}$	20.5	$\int_{1}^{+\infty}\frac{e^{-\sqrt{x}}dx}{\sqrt{x}}$	20.6	$\int_{1}^{+\infty}\frac{dx}{2^{3x-1}}$
20.7	$\int_{0}^{+\infty}\frac{dx}{(x+2)(2x+1)}$	20.8	$\int_{0}^{+\infty}\frac{dx}{\sqrt{x^2+25}}$	20.9	$\int_{0}^{+\infty}\frac{dx}{x^2+9}$
20.10	$\int_{2}^{+\infty}\frac{dx}{(x+1)\ln(x+1)}$	20.11	$\int_{5}^{+\infty}\frac{dx}{(x-1)(2x+4)}$	20.12	$\int_{0}^{+\infty}\frac{dx}{\sqrt{16-x^2}}$
20.13	$\int_{1}^{+\infty}\frac{dx}{3^{4x-1}}$	20.14	$\int_{2}^{+\infty}\frac{dx}{(x-1)(2x+6)}$	20.15	$\int_{1}^{+\infty}\frac{e^{-\sqrt{x+1}}dx}{\sqrt{x+1}}$
20.16	$\int_{0}^{+\infty}\frac{dx}{\sqrt{25-9x^2}}$	20.17	$\int_{0}^{+\infty}\frac{dx}{9x^2+16}$	20.18	$\int_{0}^{+\infty}\frac{dx}{\sqrt{16x^2+1}}$
20.19	$\int_{1}^{+\infty}\frac{e^{-\sqrt{2x}}dx}{\sqrt{2x}}$	20.20	$\int_{2}^{+\infty}\frac{dx}{x\ln 2x}$	20.21	$\int_{1}^{+\infty}\frac{dx}{5^{4x+1}}$
20.22	$\int_{0}^{+\infty}\frac{dx}{(x+1)(3x+7)}$	20.23	$\int_{0}^{+\infty}\frac{dx}{(5x+2)(x+3)}$	20.24	$\int_{0}^{+\infty}\frac{dx}{\sqrt{3x^2+15}}$
20.25	$\int_{0}^{+\infty}\frac{dx}{4x^2+1}$	20.26	$\int_{2}^{+\infty}\frac{dx}{\sqrt{64-49x^2}}$	20.27	$\int_{1}^{+\infty}\frac{e^{-\sqrt{3x+1}}dx}{\sqrt{3x+1}}$
20.28	$\int_{1}^{+\infty}\frac{dx}{6^{3x+11}}$	20.29	$\int_{2}^{+\infty}\frac{3dx}{(2x+1)\ln(2x+1)}$	20.30	$\int_{0}^{+\infty}\frac{dx}{\sqrt{4x^2+5}}$

Task 21. Calculate the improper integral (or prove its divergence).

№	Task	№	Task	№	Task
21.1	$\int_{0}^{1}\frac{dx}{4x^2-1}$	21.2	$\int_{0}^{64}\frac{dx}{\sqrt[4]{x}}$	21.3	$\int_{0}^{2}\frac{dx}{4x^2\cdot\sqrt[3]{x}}$
21.4	$\int_{0}^{1}\frac{dx}{x^5\sqrt{x}}$	21.5	$\int_{1}^{e}\frac{dx}{x\ln x}$	21.6	$\int_{-1}^{1}\frac{dx}{(x+1)(3x+7)}$

21.7	$\int_0^5 \frac{dx}{\sqrt{25-x^2}}$	21.8	$\int_0^2 \frac{dx}{4x^2-16}$	21.9	$\int_0^1 \frac{dx}{\sqrt[7]{x}}$
21.10	$\int_1^e \frac{dx}{x\sqrt{\ln x}}$	21.11	$\int_0^3 \frac{dx}{x^2\sqrt{x}}$	21.12	$\int_0^2 \frac{dx}{(x-1)(2x+1)}$
21.13	$\int_0^3 \frac{dx}{x^2\sqrt[3]{x}}$	21.14	$\int_0^{15} \frac{dx}{\sqrt{225-x^2}}$	21.15	$\int_0^1 \frac{dx}{\sqrt[5]{x}}$
21.16	$\int_2^4 \frac{dx}{(x-2)(5x+7)}$	21.17	$\int_1^e \frac{dx}{x\ln^2 x}$	21.18	$\int_0^2 \frac{dx}{x^2-1}$
21.19	$\int_0^3 \frac{dx}{\sqrt[3]{(x-3)^7}}$	21.20	$\int_0^8 \frac{dx}{x\sqrt[3]{x}}$	21.21	$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$
21.22	$\int_0^1 \frac{dx}{\sqrt[3]{x}}$	21.23	$\int_0^3 \frac{dx}{x\sqrt{x}}$	21.24	$\int_{-2}^0 \frac{dx}{(x+1)(3x+7)}$
21.25	$\int_1^9 \frac{dx}{4x^2-32}$	21.26	$\int_1^e \frac{\ln^{-2} x dx}{x}$	21.27	$\int_0^5 \frac{dx}{\sqrt{25-9x^2}}$
21.28	$\int_0^5 \frac{dx}{(x-4)(x+1)}$	21.29	$\int_0^{27} \frac{xdx}{\sqrt[3]{x}}$	21.30	$\int_0^3 \frac{dx}{\sqrt[3]{x-3}}$

Task 22. Calculate the area of the figure *D* bounded by lines.

№	The equations of the domain D boundaries	№	The equations of the domain D boundaries	№	The equations of the domain D boundaries
22.1	$y=x^2,\ y=4$	22.2	$y=\sqrt{x},\ y=0,$ $y=1,\ y=4$	22.3	$y=\frac{x^2}{2},\ y=2x+\frac{5}{2}$
22.4	$y^2=16x,\ y=x$	22.5	$y^2=x,\ y=x^2$	22.6	$y=x^2,\ y=1-x^2$
22.7	$y=x^2-5,\ y=x-3$	22.8	$y=\cos^2 x,$ $y=0, x=0$	22.9	$y=x^2-2x,\ y-3=0$
2.10	$y=x^2-1,\ y=x+1$	22.11	$y=tgx,\ y=\sqrt{3},$ $x=0$	22.12	$y=x^2-1,\ y=x+3$
22.13	$y=ctgx,\ y=-1,$ $x=0$	22.14	$y=-2x^2+3x+6,$ $y=x+2.$	22.15	$y=4x-x^2,\ y=x.$
22.16	$x=y^2, x=1. y=0.$	22.17	$y=(x-1)^2,$ $y=x+1.$	22.18	$y=2x^2+1,\ y=x=2,$ $y=1.5$
22.19	$y=x^2+2x,$ $y=x+2$	22.20	$y^2=4x,\ x^2=4y$	22.21	$y=-x^2,$ $y=-x-2$

22.22	$y = \frac{x^2}{9},\ y = \frac{x}{3} + 2$	22.23	$y = tgx,\ y = -1,$ $x = 0$	22.24	$y = x^2,\ y = \frac{x^2}{2},$ $y = 5.$
22.25	$y = x^2 - x,\ y = 0,$ $x = 2.$	22.26	$y = x^2 + 4x,$ $y = 2x + 4$	22.27	$y = ctgx,\ y = 1,$ $x = \frac{\pi}{2}$
22.28	$y = tg2x,\ y = 1,$ $x = 0$	22.29	$y = x^2 + x,\ y = x + 1$	22.30	$y = tgx,\ y = 1,$ $x = 0$

3.3 Methodological guidelines for solving the problems of the section «Integral calculus of one variable function». Solution of an exemplary embodiment

Task 1 is solved by the formula $d(f(x)) = f'(x)dx$.

Task 2 is solved by using the same formula for the reverse action, i.e. $f'(x)dx = d\big(f(x)\big)$ or $f(x)dx = d(F(x))$, where *F(x)* – the antiderivative function for function *f(x).*

Tasks 3 – 5 must be solved by the method of direct integration, i.e. using the rules of integration, identity transformations of integrands and table of basic integrals.

We give samples of examples for these tasks. Find the indefinite integral:

1) $\int \frac{\left(a - \sqrt[3]{x}\right)^2}{x^3} dx.$

Solution:

$$\int \frac{\left(a - \sqrt[3]{x}\right)^2}{x^3} dx = \int \frac{a^2 - 2a \cdot \sqrt[3]{x} + \sqrt[3]{x^2}}{x^3} dx = \int \left(\frac{a^2}{x^3} - \frac{2a \cdot \sqrt[3]{x}}{x^3} + \frac{\sqrt[3]{x^2}}{x^3} \right) dx =$$

$$= \int (a^2 x^{-3} dx - 2a x^{-\frac{8}{3}} + x^{-\frac{7}{3}}) dx = a^2 \int x^{-3} dx - 2a \int x^{-\frac{8}{3}} dx + \int x^{-\frac{7}{3}} dx = a^2 \frac{x^{-3+1}}{-3+1} -$$

$$- 2a \frac{x^{-\frac{8}{3}+1}}{-\frac{8}{3}+1} + \frac{x^{-\frac{7}{3}+1}}{-\frac{7}{3}+1} + C = -\frac{a^2 x^{-2}}{2} + \frac{6ax^{-\frac{5}{3}}}{5} - \frac{3x^{-\frac{4}{3}}}{4} + C.$$

2) $$\int \frac{5^{2x} - 3^x}{5^x} dx = \int \left(\frac{5^{2x}}{5^x} - \frac{3^x}{5^x} \right) dx = \int 5^x dx - \int \left(\frac{3}{5} \right)^x dx = \frac{5^x}{\ln 5} - \frac{\left(\frac{3}{5} \right)^x}{\ln \frac{3}{5}} + C.$$

3) $\int \frac{1+\cos^2 x}{1+\cos 2x}dx = \left| \begin{array}{c} Apply\ \ the\ \ formula\ \ of\ \ trigonometry \\ 1+\cos 2x = 2\cos^2 x \end{array} \right| =$

$$= \int \frac{1+\cos^2 x}{2\cos^2 x}dx = \frac{1}{2}\int \frac{dx}{\cos^2 x} + \frac{1}{2}\int dx = \frac{1}{2}tg\, x + \frac{1}{2}x + C.$$

4) $$\int \left(\frac{6}{2x^2+9} + \frac{12}{2x^2-9} \right) dx = \int \frac{6dx}{2\left(x^2 + \frac{9}{2} \right)} + \int \frac{12dx}{2\left(x^2 - \frac{9}{2} \right)} =$$

$$= 3 \cdot \frac{1}{\sqrt{\frac{9}{2}}} arctg \frac{x}{\sqrt{\frac{9}{2}}} + 6 \cdot \frac{1}{2 \cdot \sqrt{\frac{9}{2}}} \ln \left| \frac{\sqrt{\frac{9}{2}} - x}{\sqrt{\frac{9}{2}} + x} \right| + C =$$

$$= \sqrt{2} arctg \frac{\sqrt{2}x}{3} + \sqrt{2} \ln \left| \frac{3-\sqrt{2}x}{3+\sqrt{2}x} \right| + C.$$

5) $$\int \frac{\sqrt{x^2+5}}{2x^2+10}dx = \frac{1}{2}\int \frac{dx}{\sqrt{x^2+5}} = \frac{1}{2}\ln\left(x + \sqrt{x^2+5} \right) + C.$$

Tasks 6 – 7. These tasks are based on using the following special case of changing the variable (formula of integration of functions with a linear argument):

$$\int f(x)dx = F(x) + C \ \Rightarrow \int f(ax+b)dx = \frac{1}{a}F(ax+b) + C \quad a \neq 0.$$

An example of the application of this formula is the integral:

$$\int \cos(3x+8)dx = \frac{1}{3}\int \cos(3x+8)d(3x+8) = \frac{1}{3}\sin(3x+8) + C.$$

Task 8 is solved by using the formula of introducing of function under the sign of the differential, in particular, using the formula $\int \frac{u'(x)dx}{u(x)} = \ln|u(x)| + C$.

For example, $\int \frac{dx}{x+8} = \int \frac{d(x+8)}{x+8} = \ln|x+8| + C$.

In addition, methodological guidelines for the tasks 1 and 2 are used.

For example, we use this method in the following integral:

$$\int e^{x^5}x^4dx=\left|\begin{array}{c}replacement:\ \ x^5=t,\ \ d\left(x^5\right)=dt,\\ 5x^4dx=dt,\ \ x^4dx=\dfrac{dt}{5}\end{array}\right|=\int e^t\cdot\frac{dt}{5}=\frac{1}{5}e^t+C=e^{x^5}+C.$$

In Task 9 an example is solved on the theme: integration of functions, containing quadratic trinomial ([2], p.43).

We recommend using one of the known methods of allocating the total square:

$$ax^2+bx+c=a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}\ \text{ or }\ ax^2+bx+c=\left(\sqrt{a}x+\frac{b}{2\sqrt{a}}\right)^2+c-\frac{b^2}{4a}.$$

Using these formulas, we can carry out the integration of functions of the form:

$$\int\frac{dx}{ax^2+bx+c},\ \int\frac{(Ax+B)dx}{ax^2+bx+c},\ \int\frac{dx}{\sqrt{ax^2+bx+c}},\ \int\frac{(Ax+B)dx}{\sqrt{ax^2+bx+c}}.$$

In Task 10 we use the formula of integration by parts ([2], p.42).

It is recommended for integrals: $\int x^ne^{ax}dx$, $\int x^n\sin axdx$, $\int x^n\cos axdx$

take that $u=x^n$, $dv=\left\{e^{ax}dx,\ \ \sin axdx,\ \ \cos axdx\right\}$.

And for the integrals:

$\int x^n\ln^m axdx$, $\int x^n\arcsin axdx$, $\int x^n\arccos axdx$, $\int x^narctgaxdx$, $\int x^narcctgaxdx$

take that $dv=x^ndx,\ \ v=\dfrac{x^{n+1}}{n+1}$,

$$u=\left\{\ln^m ax,\ \ \arcsin ax,\ \ \arccos ax,\ \ arctgax,\ \ arcctgax\right\}.$$

For example, $\int(2x-7)\sin 5x\,dx$.

We apply the method of integration by parts.

$$\int(2x-7)\sin 5xdx=\left|\begin{array}{c}According\ \ to\ \ the\ formula\ of\ integration\ \ by\ parts\\ \int udv=uv-\int vdu,\ \ where\ \ u=2x-7,\\ du=2dx,\ \ dv=\sin 5xdx,\ \ v=-\dfrac{1}{5}\cos 5x\end{array}\right|=$$

$$= -\frac{1}{5}(2x-7)\cos 5x + \frac{2}{5}\int \cos 5x\,dx = -\frac{1}{5}(2x-7)\cos 5x + \frac{2}{25}\sin 5x + C.$$

Tasks 11, 12, 13. It is necessary to study the theme «integration of rational function» ([2], p.44).

Decomposition of the proper rational function $\frac{Q_m(x)}{P_n(x)} = \frac{b_0x^m + b_1x^{m-1} + ... + b_m}{a_0x^n + a_1x^{n-1} + ... + a_n}$, where $n > m$ into partial fractions has the form:

$$\frac{Q_m(x)}{P_n(x)} = \frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + ... + \frac{A_k}{(x-a)^k} +$$

$$... + \frac{M_1x + N_1}{(x^2 + bx + c)} + ... + \frac{M_rx + N_r}{(x^2 + bx + c)^r}$$

if $a_0x^n + ... + a_n = a_0(x-a)^k ... (x^2 + bx + c)^r ...$

First of all it is necessary to study the method of undetermined coefficients or the method of sampling points for the decomposition of the proper rational function.

Task 14. Learn from lecture notes the theme «Integration of irrational functions by rationalizing» ([2], p.46).

For example, in the integrals of the form $\int R\left(x, x^{\frac{l}{k}}, x^{\frac{s}{m}}, ...\right)dx$, it is necessary to take the replacement $x = t^n$, where n – the common denominator of the fractions $\frac{l}{k}, \frac{s}{m}, ...$

In the integrals of the form $\int R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{l}{k}}, \left(\frac{ax+b}{cx+d}\right)^{\frac{s}{m}}, ...\right)dx$, it is necessary to take the replacement $\frac{ax+b}{cx+d} = t^n$, where n – the common denominator of the fractions $\frac{l}{k}, \frac{s}{m}, ...$

Task 15. If the given integral has the form $\int R(tgx)dx$, or the integrand has the form $R(-\sin x, -\cos x) = R(\sin x, \cos x)$, then we use the replacement: $tgx = t$. Moreover, on the basis of known formulas, we have:

$$\sin x = \frac{tgx}{\sqrt{1+tg^2x}} = \frac{t}{\sqrt{1+t^2}}; \qquad \cos x = \frac{1}{\sqrt{1+tg^2x}} = \frac{1}{\sqrt{1+t^2}};$$

$$x = arctgt, \quad dx = \frac{dt}{1+t^2}.$$

If the given integral has the form $\int \sin^n x \cdot \cos^m x dx$, and n – odd number, then we use the replacement: $\cos x = t$, $\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-t^2}$, $x = \arccos t$, $dx = -\frac{dt}{\sqrt{1-t^2}}$.

If m – odd number, then we use the replacement:

$$\sin x = t, \quad \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-t^2}, \quad x = \arcsin t, \quad dx = \frac{dt}{\sqrt{1-t^2}}.$$

Example: $\int \frac{\sin^3 3x}{\sqrt[5]{\cos 3x}} dx.$

$$\int \frac{\sin^3 3x}{\sqrt[5]{\cos 3x}} dx = \int \frac{\sin^2 3x \cdot \sin 3x}{\sqrt[5]{\cos 3x}} dx = \int \frac{(1-\cos^2 3x)\sin 3x}{\sqrt[5]{\cos 3x}} dx = \left| \begin{matrix} replacement \quad \cos 3x = t, \\ dt = -3\sin 3x dx \end{matrix} \right| =$$

$$= -\frac{1}{3}\int \frac{(1-t^2)dt}{t^{\frac{1}{5}}} = -\frac{1}{3}\int \left(t^{-\frac{1}{5}} - t^{\frac{9}{5}} \right) dt = -\frac{1}{3} \cdot \frac{t^{\frac{4}{5}}}{\frac{4}{5}} + \frac{1}{3} \cdot \frac{t^{\frac{14}{5}}}{\frac{14}{5}} + C = -\frac{5}{12}\sqrt[5]{\cos^4 3x} +$$

$$+ \frac{5}{42}\sqrt[5]{\cos^{14} 3x} + C.$$

Task 16. We use the universal trigonometric substitution to the integral of the form $\int R(\sin x, \cos x) dx$. The universal substitution consists of the following:

$$tg\frac{x}{2} = t; \sin x = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2t}{1+t^2}; \cos x = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}; x = 2arctgt, \; dx = \frac{2dt}{1+t^2}.$$

Task 17 is solved by Newton-Leibniz formula ([2], p.51) and using the properties of integration of functions with a linear argument (see methodological guidelines for the tasks 6 and 7).

For example, calculate the definite integral $\int_{-1}^{2} \frac{dx}{(x+3)^3}$.

Solution:

$$\int_{-1}^{2}\frac{dx}{(x+3)^3}=\int_{-1}^{2}\frac{d(x+3)}{(x+3)^3}=\int_{-1}^{2}(x+3)^{-3}d(x+3)=\left.\frac{(x+3)^{-3+1}}{-2}\right|_{-1}^{2}=-\frac{1}{2}\left[(2+3)^{-2}-(-1+3)^{-2}\right]=$$

$$=-\frac{1}{2}\left(5^{-2}-2^{-2}\right)=-\frac{1}{2}\left(\frac{1}{25}-\frac{1}{4}\right)=\frac{21}{200}\ .$$

In the Task 18 we apply the method of replacing the variable for the definite integral and Newton-Leibniz formula ([2], p.51).

Calculate the definite integral $\int_0^1\sqrt{4-x^2}\,dx$, using the replacement of variable $x = 2\cos t$.

$$\int_0^1\sqrt{4-x^2}\,dx=\left|\begin{array}{l}x=2\cos t,\quad dx=-2\sin t\,dt,\\ x=0\ \Rightarrow t=\dfrac{\pi}{2},\\ x=1\ \Rightarrow t=\dfrac{\pi}{3}\end{array}\right|=\int_{\frac{\pi}{2}}^{\frac{\pi}{3}}\sqrt{4-4\cos^2 t}\cdot(-2\sin t)dt=$$

$$=-4\int_{\frac{\pi}{2}}^{\frac{\pi}{3}}\cdot(\sin t)^2\,dt=-4\cdot\int_{\frac{\pi}{2}}^{\frac{\pi}{3}}\frac{1-\cos 2t}{2}\cdot dt=-2\int_{\frac{\pi}{2}}^{\frac{\pi}{3}}(1-\cos 2t)dt=-2\left(t-\frac{1}{2}\sin 2t\right)_{\frac{\pi}{2}}^{\frac{\pi}{3}}=$$

$$=-2\left(\frac{\pi}{3}-\frac{1}{2}\sin\frac{2\pi}{3}-\left(\frac{\pi}{2}-\frac{1}{2}\sin\pi\right)\right)=\frac{\pi}{3}+\frac{\sqrt{3}}{2}.$$

Task 19 is solved by using the formula for integration by parts for definite integral ([2], p.51).

For example, calculate the definite integral $\int_0^1 x\cos 2x\,dx$.

$$\int_0^1 x\cos 2xdx=\left|\begin{array}{l}by\quad parts:\quad x=u,\quad dx=du,\\ \cos 2xdx=dv,\quad v=\dfrac{1}{2}\sin 2x\end{array}\right|=\left.\frac{x\sin 2x}{2}\right|_0^1-\int_0^1\frac{1}{2}\sin 2xdx=$$

$$=\frac{\sin 2}{2}+\frac{1}{4}\cos 2x\Big|_0^1=\frac{\sin 2}{2}+\frac{1}{4}(\cos 2-\cos 0)=\frac{\sin 2}{2}+\frac{1}{4}(\cos 2-1).$$

Task 20. It is recommended to solve the task on the model of example:

Calculate the improper integral (with infinite limits)

$\int_{-\infty}^{\infty} \frac{dx}{x^2+8x+17}$ (or prove its divergence).

$$\int_{-\infty}^{\infty} \frac{dx}{x^2+8x+17} = \int_{-\infty}^{\infty} \frac{dx}{(x+4)^2+1} = \int_{-\infty}^{\infty} \frac{d(x+4)}{(x+4)^2+1} = arctg(x+4)\Big|_{-\infty}^{\infty} =$$

$$= arctg(\infty) - arctg(-\infty) = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi .$$

Conclusion: the improper integral converges and is equal to the number π.

Tasks 21. It is recommended to solve the task on the model of example:

Calculate the improper integral (the integrand contains the function having a break point of the second kind in the area of integration):

$\int_{-1}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}}$ (or prove its divergence);

$$\int_{-1}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}} = \int_{-1}^{1} \frac{dx}{\sqrt[3]{(x-1)^2}} +$$

$$\int_{1}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}} = \lim_{\alpha \to 1-0} \int_{-1}^{\alpha} (x-1)^{-\frac{2}{3}} d(x-1) + \lim_{\beta \to 1+0} \int_{\beta}^{2} (x-1)^{-\frac{2}{3}} d(x-1) =$$

$$= 3 \lim_{\alpha \to 1-0} (x-1)^{\frac{1}{3}} \Big|_{-1}^{\alpha} + 3 \lim_{\beta \to 1+0} (x-1)^{\frac{1}{3}} \Big|_{\beta}^{2} = 3 \lim_{\alpha \to 1-0} \left[(\alpha-1)^{\frac{1}{3}} + 2^{\frac{1}{3}} \right] + 3 \lim_{\beta \to 1+0} \left[1 - (\beta-1)^{\frac{1}{3}} \right] =$$

$$= 3 \cdot \sqrt[3]{2} + 3 = 3\left(\sqrt[3]{2} + 1\right) .$$

Conclusion: the improper integral converges and is equal to the specified number.

Task 22. Calculate the area of the figure *D* bounded by lines.

$$D: \quad y = \cos x, \quad y = -2\cos x, \qquad -\frac{\pi}{2} \le x \le \frac{\pi}{2}.$$

Solution:

$$S = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\cos x - (-2\cos x)] dx = \quad = 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 3 \sin x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$$

$$= 3\left(\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2} \right) \right) = 3\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2} \right) = 6\sin\frac{\pi}{2} = 6.$$

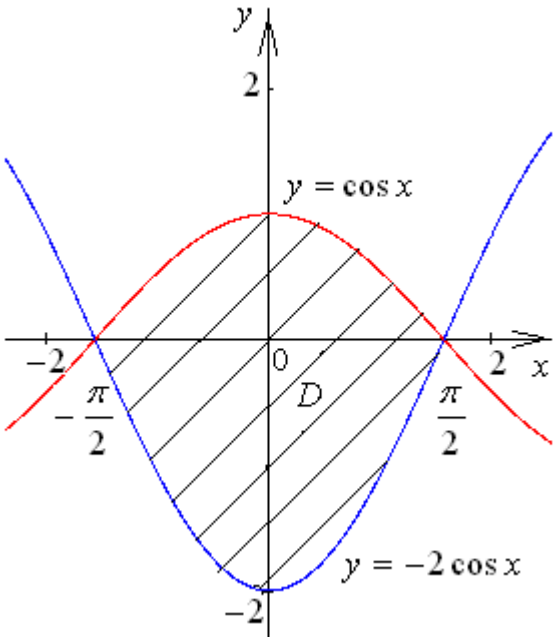


Figure 3.1

3.4 Reference material

Table of integrals

1. $\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \quad (\alpha \neq -1)$

2. $\int \frac{dx}{x} = \ln|x| + C$

3. $\int \sin x dx = -\cos x + C$

4 $\int \cos x dx = \sin x + C$

5. $\int \frac{dx}{\cos^2 x} = tgx + C$

6. $\int \frac{dx}{\sin^2 x} = -ctgx + C$

7. $\int tgx dx = -\ln|\cos x| + C$

8. $\int ctgx dx = \ln|\sin x| + C$

9. $\int e^x dx = e^x + C$

10. $\int a^x dx = \frac{a^x}{\ln a} + C$

11. $\int \frac{dx}{1+x^2} = arctgx + C$

11′. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} arctg \frac{x}{a} + C$

12. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C$

13. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$

13′. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$

14. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

15. $\int shx dx = chx + C$

16. $\int chx dx = shx + C$

17. $\int \frac{dx}{ch^2 x} = thx + C$

18. $\int \frac{dx}{sh^2 x} = -cthx + C$

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Contents

Introduction ……..... 3
1 Calculation-graphical work №1. Elements of linear algebra,
analytic geometry and complex numbers……..... 3
1.1 Theoretical questions……..... 3
1.2 Calculated tasks……..... 3
1.3 Solution of an exemplary embodiment…….. 16
2 Calculation-graphical work №2. Introduction to analysis. Differential
calculus of one-variable function…… 32
2.1 Theoretical questions……..... 32
2.2 Calculated tasks……..... 32
2.3 Solution of an exemplary embodiment……...……... 43
3 Calculation-graphical work №3. Integral calculus of one variable function…..... 52
3.1 Theoretical questions……..... 52
3.2 Calculated tasks…… 52
3.3 Solution of an exemplary embodiment…….. 69
3.4 Reference material……... 77
Bibliography……..…. 78