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ALMATY UNIVERSITY OF POWER ENGINEERING & TELECOMMUNICATIONS

Department of Theoretical Bases of Electrical Engineering

THEORETICAL FUNDAMENTALS OF ELECTRICAL ENGINEERING

Transient process calculation in a linear electric circuit

Methodological guidelines and assignments for Course Paper
for the 5B070200 – Automation and Control baccalaureate specialty students

Almaty 2016

AUTHORS: A. S. Baimaganov, S. J. Kreslina. Theoretical Fundamentals of Electrical Engineering. Transient process calculation in a linear electric circuit. Methodological guidelines and assignments for Course Paper for the 5B070200 – Automation and Control baccalaureate specialty students. – Almaty: AUPET, 2016. – 16 p.

Methodological guidelines and assignments for Course Paper on “Theoretical Fundamentals of Electrical Engineering” discipline for the next sections: “Classical calculation method of transient process in a linear electric circuit” and “Applying Laplace transform to calculation of transient process in a linear electric circuit” are provided.

Manual contains requirements on the content and design for Course Paper.

Course Paper assignments are intended for students, which are educated in English on 5B070200 – “Automation and Control” baccalaureate specialty.

10 illustrations, 5 tables, 6 items of references.

Reviewer: PhD B. I. Tuzelbaev

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Introduction

“Theoretical Fundamentals of Electrical Engineering” (TFEE) discipline is a mandatory component for the 5B070200 – “Automation and Control” baccalaureate specialty.

An objective of TFEE discipline is to master the methods of calculation of transient processes in linear electric circuits. This discipline is based on the disciplines: “Physics” and “Higher Mathematics”.

The objectives of the Course Paper “Transient process calculation in a linear electric circuit” are to apply the theoretic principles in practical calculations, to gain the skills in carrying out calculations of circuit transient process by oneself, without assistance. That ultimately contributes to the successful mastering TFEE discipline.

Course Paper (CP) (Course Work) is devoted to calculation the transient process in the linear electric circuit of second order. Course Paper consists of two parts devoted to the main discipline sections: “Classical calculation method of transient process in a linear electric circuit” and “Applying Laplace transform to calculation of transient process in a linear electric circuit”.

The task of the discipline is to train the students based on a knowledge of qualitative and quantitative aspects of the processes taking place in the various electric devices for successful and competent solving the problems that poses by special discipline.

Successful carrying out of the Course Work has an exceptional significance for the formation of a scientific outlook, helps students to check the quality of learning material of the “Theoretical Fundamentals of Electrical Engineering” discipline and develops the skills and a clear summary of his thoughts.

As a result of the carried out Course Paper, the student should master the following: classical calculation method and Laplace transform on transient process calculation.

This brochure contains the methodological guidelines and assignments for the CP on the main sections of TFEE discipline for the 5B070200 – “Automation and Control” baccalaureate specialty.

The option of original data is determined by the first letter of the surname, by the last and the penultimate digits of the credit book number and by the year of enrolment (even or odd).

Content and design requirements

Course Paper should be included the next sections:

a) title page (a template of the title page is represented on the next page);

b) contents;

c) introduction;

d) assignment;

e) main section (procedure of calculation and graphical results presentation);

f) conclusion (make conclusion about the obtained results and using methods);

g) list of references;

h) appendices (if it is necessary).

The surname and initials, credit book number and academic group code of student should be written down on the title page (see title page template on the next page).

Choose the original assignment data according to your option, assignments text should be rewritten to the explanatory note unabridged.

Each stage of the CP should be named. All tasks are carried out on only one side of a white paper sheet (sheet size A4 with margins: top is 20 mm, bottom is 25 mm, left is 25 mm, right is 18 mm).

An explanatory note should be handwritten or printed by using Microsoft Word application (font size is 14 points, line spacing is 1…1,5 line).

Not only calculation formulas and final results, but also text with explanations, needed intermediate calculations, which allow understanding and verifying your actions are to be presented in the explanatory note.

It is necessary to write appropriate units for the parameters, having a particular of dimensions.

The graphs of instantaneous values should be performed on the inserted sheets of graph or checkered paper, or by using computer applications.

It is necessary to sign coordinate axes name and the names of variable on the graph, also choose a scale so that is would be convenient to use a graph or a diagram.

The circuit diagrams should be numbered and done in pencil neatly by using a ruler, or by using computer applications.

All pages should be continually numbered, beginning with the title page. The page number should be located in the middle of the sheet bottom.

The works, which do not satisfy requirements, will not be given a permission for defense and will be returned for revision.

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ALMATY UNIVERSITY OF POWER ENGINEERING & TELECOMMUNICATIONS

Department of Theoretical Bases of Electrical Engineering

COURSE PAPER

on the “Theoretical Fundamentals of Electrical Engineering” discipline

____

(Title of the course paper)

____

5B070200 – “Automation and Control” baccalaureate specialty

Done by ___________________________ student of the ______________ group

(Student’s Surname & Initials) (Academic group code)

Checked by ____

(Teacher’s academic degree, academic rank, Surname & Initials)

___________ _________________ “_____” “_____________” 201__ y.

(Score) (Teacher’s signature) (Date)

Almaty 201__ y.

Course Paper. Transient process calculation in a linear electric circuit

Course Paper is devoted to the calculation of transient process in a linear electric circuit of second order using the classical calculation method and Laplace transform.

Course Paper consists of two parts. The option of original data is determined by the first letter of surname, by the last and penultimate digit of the number of credit book and by the enrolment year (even or odd).

1 **Part 1. Transient process calculation in a linear electric circuit using classical calculation method**

An objective of the part 1 is to get skills in applying a classical calculation method of transient processes in a linear electric circuit.

The linear electric circuit includes two energy storage elements *L* and *C*, resistors R_1, R_2, R_3 and two energy sources: sinusoidal voltage source $e(t)=E_m\sin(\omega t+\varphi_u)$ and direct current (DC) voltage source with electromotive force E_0. The options of circuits are presented in the figures 1.1…1.10.

The switch in circuit was in position 1 for a long time. Assume that it is moved to position 2 at the time moment $t = 0$, i.e. the circuit switches from the sinusoidal voltage source to a DC voltage source at the time moment $t = 0$. After commutation in the circuit occurs the transient process. It is necessary to determine the transient time function $f(t)$ for $t > 0$.

The original parameters of energy sources are given in the table 1.1 for a variety of options. The original parameters of the circuit and the desired transient time function are given in the tables 1.2 and 1.3 for a variety of options. Choose the electric circuit, original data and desired transient time function according to your option and write them at the very beginning of the explanatory note.

1.1 **Assignment**

Determine the desired transient time function *f*(t), marked in the table 1.2 for *t* > 0 by applying classical calculation method.

Table 1.1

Enrolment year	Last digit of the credit book number									
even	1	2	3	4	5	6	7	8	9	0
odd	0	1	2	3	4	5	6	7	8	9
Circuit №	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.10
U_m, V	100	110	120	150	140	160	180	200	170	130
φ_u, deg	30°	–45°	60°	–50°	45°	–60°	50°	40°	–40°	–30°
f, kHz	0,8	1,0	0,9	1,2	1,5	0,7	1,3	1,2	1,4	1,1
U_0, V	60	30	40	60	50	70	80	90	100	110

Table 1.2

Enrolment year	Penultimate digit of the credit book number									
even	1	2	3	4	5	6	7	8	9	0
odd	0	1	2	3	4	5	6	7	8	9
L, mH	15	55	20	25	30	35	40	45	50	10
C, μF	3	5	10	8	6	11	12	9	4	7
Determine	$i_L(t)$	$u_L(t)$	$i_C(t)$	$u_C(t)$	$i_{R1}(t)$	$u_{R1}(t)$	$i_{R2}(t)$	$u_{R2}(t)$	$i_{R3}(t)$	$u_{R3}(t)$

Table 1.3

Enrolment year	First letter of the surname									
even	АЛФ	БМЦ	ВНЧ	ГОШ	ДПЩ	ЕРЭ	ЖСЮ	ЗТЯ	ИУ	КХ
odd	КЦЭ	ЧХЛ	АНМ	БЮО	ВПЯ	ЗФ	ГРШ	ЕТЩ	ДСИ	УЖ
R_1, Ω	70	65	120	30	35	25	40	45	50	60

R_2, Ω	20	30	40	10	150	60	35	25	45	70
R_3, Ω	60	50	30	120	100	150	250	35	60	80

Options of circuits:

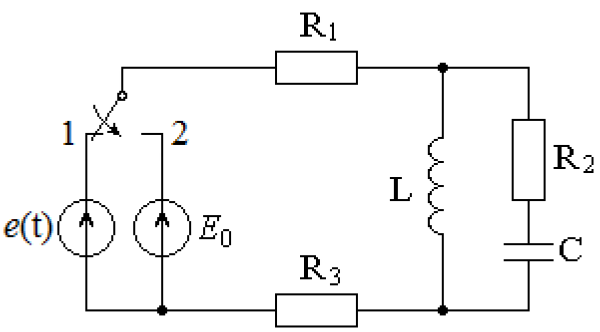


Figure 1.1

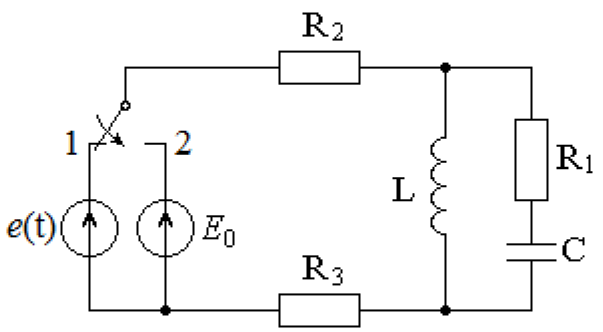


Figure 1.2

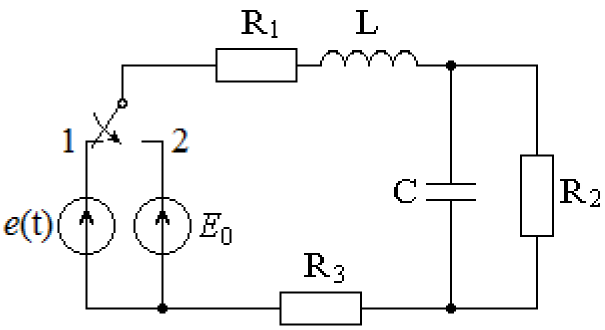


Figure 1.3

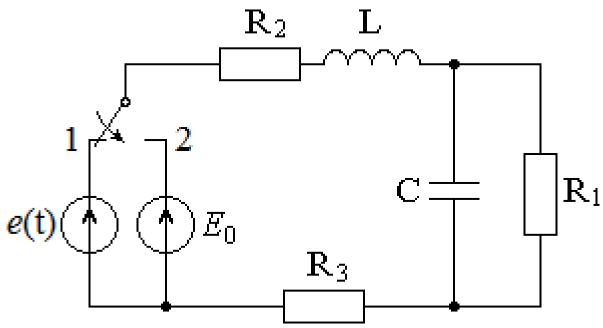


Figure 1.4

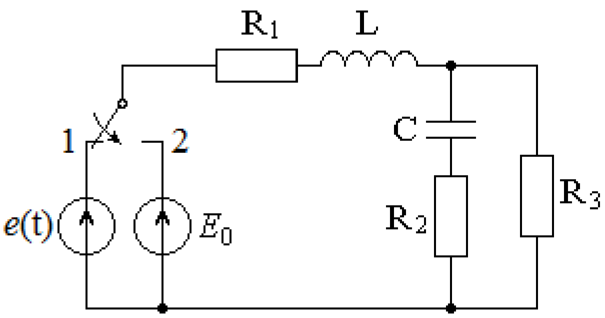


Figure 1.5

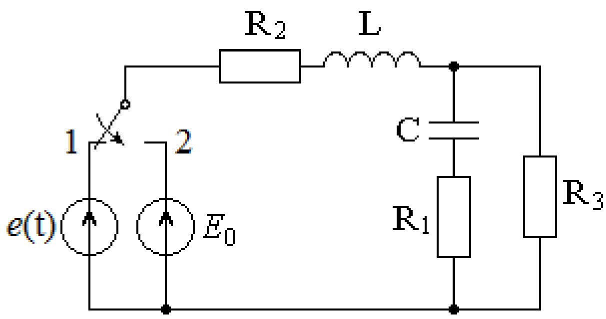


Figure 1.6

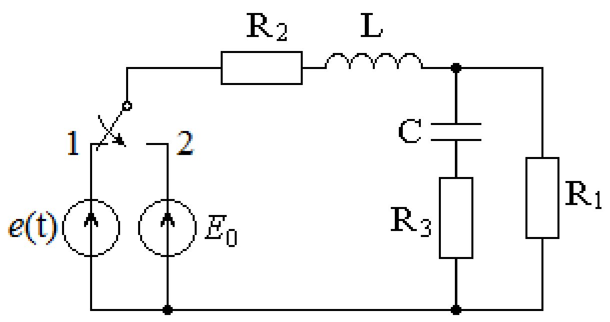


Figure 1.7

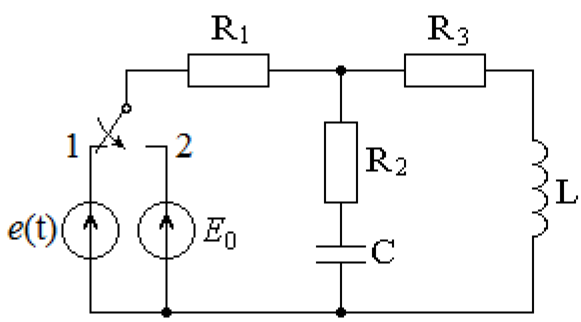


Figure 1.8

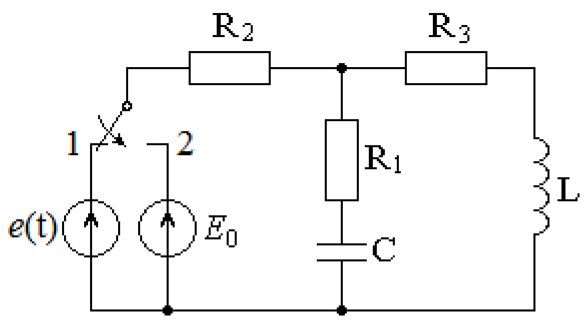


Figure 1.9

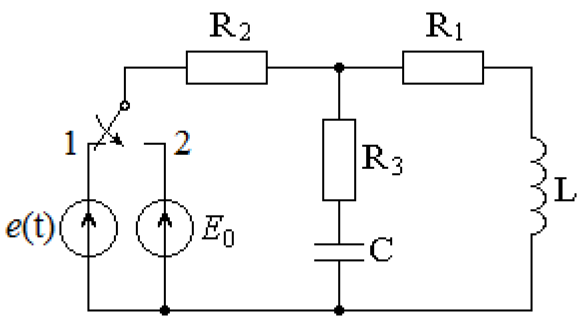


Figure 1.10

1.2 **Methodological guidelines**

Transient process calculation using classical calculation method.

Recommended procedure:

1) Calculate the independent initial conditions. The current in inductance $i_L(0+)$ and the voltage across capacitance $u_C(0+)$ at first moment after commutation are the independent initial conditions (IIC). At the first time moment after commutation (t = 0+) the IIC keep their values the same just before commutation. According to the commutation laws:

$$i_L(0+) = i_L(0-) \text{ и } u_C(0+) = u_C(0-).$$

That is why the calculation of IIC have to be carried out by using circuit before commutation, by applying the calculation methods of steady state modes.

It is necessary to remember that character of a time function at steady-state mode is fully conditioned by the character of source. If the source has a sinusoidal waveform, this means that the currents in all branches and the voltages across all elements will have the same waveform. In this case, it is recommended to use the symbolic calculation method.

At first, determine the complex amplitudes of the voltage across capacitor and current flow through the coil of inductance. Then write down their instantaneous values and calculate initial values of $u_C(0)$ and $i_L(0)$ by using obtained expressions.

2) Calculate the steady-state component of the desired time function by using the configuration of the circuit after commutation. For this purpose, use the calculation methods of a steady-state mode. At the same time, you also keep in mind that character of a time function at a steady-state mode is fully conditioned by character of source. If the waveform of source voltage (or current) is DC, this means that the currents in all branches and the voltages across elements also will be DC. Consequently, the current in capacitance $i_C = 0$ (since u_C = constant) and the voltage across inductance $u_L = 0$ (since i_L = constant).

It is recommended to make up a system of equations by using Kirchhoff's laws in differential form, to substitute i_C. = 0 and $u_L = 0$ and to calculate the steady-state component of the desired time function, for example, i_{R1ss}(t) or u_{R2ss}(t).

3) Determine the transient component f_t(t) of the desired time function, as the common solution of the homogeneous differential equation of the circuit.

At first, it is necessary to make up a characteristic equation of the circuit and find its roots. Further, according to the kind of the roots write down the general solution of the differential equation.

It is recommended to make up the characteristic equation by means of equating to zero the input impedance of the circuit after commutation relative to any open branch of the circuit, i.e. Z(p)=0. For this purpose, it is necessary to replace the circuit sources by their internal resistances. The source of EMF E must replace by jumper because it has an internal resistance r_{in} equal to zero. Branches with the current sources J mast exclude from the circuit because their internal resistances are equal infinity. Inductive and capacitive reactance are taking equal to their complex impedances in which $j\omega$ is replaced by the operator p, i.e. the inductive reactance is equal to pL, and the capacitive reactance is equal $1/pC$.

In electric circuits, which includes two energy storage elements, a square characteristic equation is obtained. It is recommended expression of the characteristic equation to reduce to the form:

$$1 \cdot p^2 + p\left(\frac{1}{R_{eq1}C} + \frac{R_{eq2}}{L}\right) + \frac{1}{LC} \cdot \frac{R_{eq3}}{R_{eq1}} = 0.$$

This will verify the correctness of equivalent transformations and use a simple formula to determine the roots of a quadratic equation:

$$\text{if } p^2 + bp + d = 0, \text{ then } p_{1,2} = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - d}.$$

The discriminant $D = \left(\frac{b}{2}\right)^2 - d$.

Depending on the discriminant value, it is possible three kinds of the roots and three kinds of the character of transient process, respectively:

a) $D > 0$ – the roots is real, different, and negative: p_1 and p_2. The transient process character is *overdamped*. The transient component f_t(t) of the desired time function is:

$$f_t(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t}.$$

b) $D = 0$ – the roots is real, equal, negative: $p_1 = p_2 = p$. The character of transient process is *critically damped*. The transient component $f_{\mathrm{t}}(t)$ of the desired time function:

$$f_{\mathrm{t}}(t) = (A_1 + A_2 \cdot t)e^{pt}.$$

c) $D < 0$ – the roots is complex conjugate: $p_{1,2} = -\alpha \pm j\omega_0$. The character of transient process is *underdamped*. The transient component f_{t} (t) of the desired time function:

$$f_{\mathrm{t}}(t) = A_1 e^{-\alpha t} \sin \omega_0 t + A_2 e^{-\alpha t} \cos \omega_0 t = A e^{-\alpha t} \sin(\omega_0 t + \psi),$$

where: $A = \sqrt{{A_1}^2 + {A_2}^2}$, $\psi = \tan^{-1}\left(\frac{A_2}{A_1}\right)$.

Write down expression for the transient component of the desired time function according to the kind of the roots of characteristic equation. An expression for the transient component is a common solution of the homogeneous differential equation. It is necessary to find two unknown constants of integration A_1 and A_2 (or A and ψ), which this expression includes.

4) Determine the constants of integration A_1 and A_2 or A and ψ.

The desired function should be presented as the sum of the steady state and transient components:

$$f(t) = f_{ss}(t) + f_t(t).$$

To determine two constants of integration at first it is necessary to find the numerical values of the function itself $f(0)$ and its first derivative df/dt(0) at the first moment of time after commutation t = 0+. For this purpose, normally make up a system of equations based on Kirchhoff's laws for the circuit configuration after commutation. Substitute the known values under commutation laws of independent initial conditions $i_{\mathrm{L}}(0+)$ and $u_{\mathrm{C}}(0+)$ and find the numerical values of the function itself and its first derivative at time moment t = 0+. Then express the function and its first derivative at time moment t = 0+ through unknown constants of integration and equate them to numeric values found before. Solving the obtained two equations gives values of two constants of integration.

2 **Part 2. Transient process calculation in a linear electric circuit using Laplace transform (operator calculation method)**

An objective of the part 2 is to get skills in applying a Laplace transform to transient processes calculation.

The option of circuits, original data and the desired time function have been elected in the part 1.

At the time moment t = 0 in the circuit a commutation (closing or opening of switch) takes place. After commutation in the circuit a transient process occurs.

2.1 **Assignment**

1. Calculate the desired transient time function of the current or voltage, marked in the table 1.2 using Laplace transform (operator calculation method).
2. Compare the expressions of the transient time function obtained by applying the classical and Laplace transform methods. Make a conclusion about advantages and disadvantages of the calculation procedures based on these both methods.
3. Plot a time diagram by using obtained time function expression on time interval $0\ldots5\tau_{max}$, where τ_{max} is the largest time constant of the circuit.

2.2 **Methodological guidelines**

Transient process calculation using Laplace transform (operator calculation method). Recommended procedure:

1) Calculate the independent initial conditions (IIC): $i_L(0+)$ and $u_C(0+)$. The configuration of electric circuit is before commutation and operation mode is steady state.

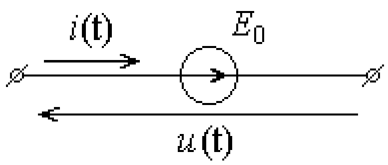
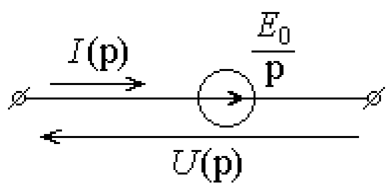
This section is the same as in the classic calculation method. Therefore, you can use the results for IIC obtained in classic calculation method.

2) Make up an equivalent replacement scheme for images of the currents and voltages. The configuration of the circuit is after commutation. Use the table 2.1.

3) Make up a system of equations and determine the image of desired transient function F(p). For this purpose, it is possible to apply known calculation methods of a steady state mode like mesh analysis and nodal analysis methods or Kirchhoff laws.

4) Determine the original of the desired transient function f(t) by the its image F(p) using decomposition theorem, see table 2.2.

Table 2.1

Originals and corresponding images of the linear electric circuit elements	
Original, f(t)	Image, F(p)
 $u(t) = E_0 = constant$	

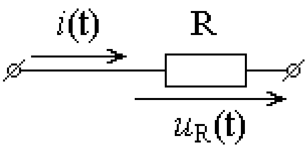
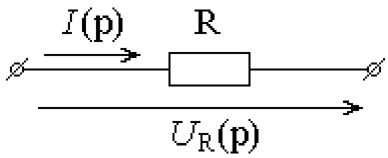
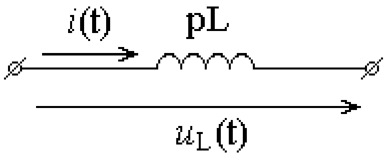
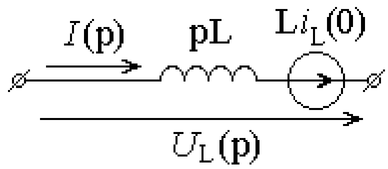
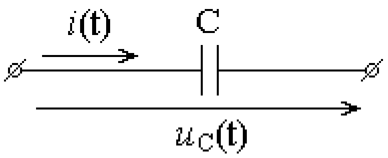
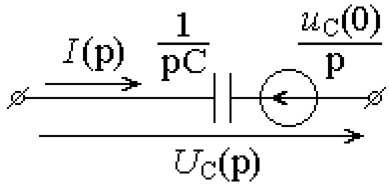
 $u_R(t) = R \cdot i(t)$	 $U_R(p) = R \cdot I(p)$
 $u_L(t) = \frac{d\psi(t)}{dt} = \frac{dLi(t)}{dt} = L\frac{di(t)}{dt}.$	 $U_L(p) = pL \cdot I(p) - L \cdot i_L(0)$
 $u_C(t) = \frac{q(t)}{C} = \frac{1}{C}\int i(t)\,dt.$	 $U_C(p) = \frac{1}{pC} \cdot I(p) + \frac{u_C(0)}{p}.$

Table 2.2

Decomposition theorem	
Expression of image must be reduced to the form of rational algebraic fraction: $\frac{F_1(p)}{F_2(p)} = \frac{a_m p^m + a_{m-1}p^{m-1} + \dots + a_1 p + a_0}{b_n p^n + b_{n-1}p^{n-1} + \dots + b_1 p + b_0}$, where $m < n$ $F_2(p) = 0$ is analogue of the characteristic equation in a classic calculation method The original is determined by using decomposition theorem	
Kind of the roots of a characteristic equation F_2(p) = 0 at n = 2	Decomposition theorem
The roots are real, different and negative: p_1 and p_2	$\frac{F_1(p)}{F_2(p)} \doteqdot f(t) = \frac{F_1(p_1)}{F_2'(p_1)}e^{p_1 t} + \frac{F_1(p_2)}{F_2'(p_2)}e^{p_2 t}$, where $F_2'(p) = dF(p)/dp$
The roots are complex conjugate: $p_{1,2} = -\alpha \pm j\omega_0$	$\frac{F_1(p)}{F_2(p)} \doteqdot f(t) = 2\,\mathrm{Re}\left[\frac{F_1(p_1)}{F_2'(p_1)}e^{p_1 t}\right]$
The denominator has a zero root: $pF_2(p)$. The roots of F_2(p) = 0 are real, different and negative: p_1 и p_2	$F(p) \doteqdot \frac{F_1(p)}{pF_2(p)}$ $f(t) = \frac{F_1(0)}{F_2(0)} + \frac{F_1(p_1)}{p_1 F_2'(p_1)}e^{p_1 t} + \frac{F_1(p_2)}{p_2 F_2'(p_2)}e^{p_2 t}$

The denominator has a zero root: $pF_2(p)$. The roots of F_2(p) = 0 are complex conjugate: $p_{1,2} = -\alpha \pm j\omega_0$	$\frac{F_1(p)}{pF_2(p)} \doteqdot f(t) = \frac{F_1(0)}{F_2(0)} + 2\,\mathrm{Re}\left[\frac{F_1(p_1)}{p_1 F_2'(p_1)} e^{p_1 t}\right]$

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