Non-Profit Joint Stock Company

ALMATY UNIVERSITY OF POWER ENGINEERING AND TELECOMMUNICATIONS

The department of Higher mathematics

MATHEMATICS 1

Methodological Guidelines and Tasks
for carrying out the calculation-graphical work for students of specialties
5B071700 «Heat power engineering»,
5B071800 «Electrical power engineering»,
5B071900 «Radio engineering, electronics and telecommunications»
Part 1

Almaty 2014

Compiler: Kim R.E., Mathematics 1. Methodological Guidelines and Tasks for carrying out the calculation-graphical work for students of specialties 5B071700 «Heat power engineering», 5B071800 «Electrical power engineering», 5B071900 «Radio engineering, electronics and telecommunications». Part 1. – Almaty, 2014 – 33 p.

Methodological Guidelines and Tasks for carrying out the calculation-graphical work contain sections of the first semester program of course of mathematics for students of AUPET: elements of linear algebra, analytic geometry and complex numbers.

The basic theoretical questions of the program and the solution of an exemplary embodiment are given.

Tables 11, figures 6, bibl. 3

Reviewer: candidate of sciences in philology, V.S. Kozlov

Printed according to the Publishing plan of Non-Profit Joint Stock Company "Almaty University of Power Engineering and Telecommunications " for 2014

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"Almaty University of Power Engineering and Telecommunications", 2014

Introduction

Mathematics plays an important role in engineering studies. It is not only a quantitative calculation device, but also mathematics is the means of accurate research and extremely precise formulation of concepts and problems. Mathematical methods have become an integral part of any technical discipline. These facts lead to the need to strengthen the applied orientation of course of mathematics and improve basic mathematical training. CGW is performed in a separate thin notebook. In the number of each task the second digit indicates the variant.

Calculation-graphical work.
Elements of linear algebra, analytic geometry and complex numbers

Purpose: to master the fundamental concepts and methods of the theory of algebra and geometry. Get the skills in calculation of determinants and operations on matrices and vectors, which are used for solving systems of equations and used in analytic geometry. By the analytical curve equations determine their geometric properties and relative location on the plane or in space.

Perform all arithmetic operations with complex numbers.

Theoretical questions

1. Determinants and their properties, the calculation.
2. Matrices, operations on them, the inverse matrix.
3. Vectors, their length, linear operations on vectors. Collinearity.
4. Scalar, vector, mixed products of vectors and their applications.
5. Different types of equation of the straight line on the plane and in space.
6. Equations of a plane.
7. Angle between the lines, planes, straight line and plane.
8. The distance from a point to the straight line and to the plane.
9. Ellipse, hyperbola, parabola. Their canonical equations.
10. Second-order surfaces.
11. Bringing of the general equations of the second-order curves and surfaces to the canonical form.
12. Various methods for solving systems of linear equations:

a) Cramer's rule;

b) by using the inverse matrix.

13. Complex numbers, the module and the main part of the complex numbers, operations over the complex numbers, De Moivre formula.

Calculated tasks

Task1. The third order determinant is given.

It is required to calculate:

a) the minor M_{23} and the algebraic complement A_{23} of the element a_{23};

b) the determinant by the triangle rule (rule of Sarrus);

c) the determinant by the method of expansion along the *i*-th row and the *j*-th column.

1.1 $\begin{vmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{vmatrix}, i=2, j=1$	1.2 $\begin{vmatrix} 2 & 3 & 2 \\ 1 & 3 & -1 \\ 4 & 1 & 3 \end{vmatrix}, i=2, j=3$	1.3 $\begin{vmatrix} 6 & 7 & 3 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{vmatrix}, i=1, j=2$
1.4 $\begin{vmatrix} 1 & 7 & 3 \\ -4 & 8 & 0 \\ 4 & 3 & 2 \end{vmatrix}, i=3, j=1$	1.5 $\begin{vmatrix} 2 & 6 & 4 \\ 1 & 3 & -2 \\ 0 & 1 & 1 \end{vmatrix}, i=1, j=2$	1.6 $\begin{vmatrix} 6 & 9 & 4 \\ -1 & -1 & 1 \\ 10 & 1 & 7 \end{vmatrix}, i=3, j=2$
1.7 $\begin{vmatrix} 1 & 0 & 3 \\ 3 & 1 & 7 \\ -2 & 1 & 8 \end{vmatrix}, i=2, j=1$	1.8 $\begin{vmatrix} 5 & 1 & -2 \\ 1 & 3 & -1 \\ 8 & 4 & -1 \end{vmatrix}, i=3, j=2$	1.9 $\begin{vmatrix} 2 & 2 & 5 \\ 3 & -3 & 6 \\ 4 & 3 & 1 \end{vmatrix}, i=3, j=1$
1.10 $\begin{vmatrix} 3 & 5 & 5 \\ -3 & 0 & 1 \\ 5 & 6 & -4 \end{vmatrix}, i=3, j=2$	1.11 $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 1 & -2 & -1 \end{vmatrix}, i=2, j=3$	1.12 $\begin{vmatrix} 4 & -3 & 2 \\ -4 & 0 & 5 \\ 3 & 2 & 3 \end{vmatrix}, i=3, j=1$
1.13 $\begin{vmatrix} 2 & -1 & -3 \\ 8 & -7 & -6 \\ 3 & 4 & 2 \end{vmatrix}, i=2, j=3$	1.14 $\begin{vmatrix} 2 & -1 & -2 \\ 3 & -5 & 4 \\ 1 & 2 & 1 \end{vmatrix}, i=1, j=2$	1.15 $\begin{vmatrix} 3 & 5 & -6 \\ 2 & 4 & 3 \\ -3 & 1 & 1 \end{vmatrix}, i=2, j=1$
1.16 $\begin{vmatrix} 2 & 8 & -5 \\ -3 & 1 & 0 \\ 4 & 5 & -3 \end{vmatrix}, i=3, j=1$	1.17 $\begin{vmatrix} 3 & 6 & 0 \\ 2 & 4 & -6 \\ 1 & -2 & 3 \end{vmatrix}, i=2, j=3$	1.18 $\begin{vmatrix} 3 & 0 & 1 \\ 0 & 2 & 7 \\ 1 & -3 & 2 \end{vmatrix}, i=3, j=3$
1.19 $\begin{vmatrix} -6 & 1 & 11 \\ 9 & 2 & 5 \\ 0 & 3 & 7 \end{vmatrix}, i=3, j=2$	1.20 $\begin{vmatrix} 1 & -2 & 5 \\ 3 & 0 & 6 \\ 4 & 3 & 4 \end{vmatrix}, i=1, j=3$	1.21 $\begin{vmatrix} -1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{vmatrix}, i=3, j=2$
1.22 $\begin{vmatrix} 5 & 4 & 2 \\ 1 & 2 & 4 \\ 3 & 0 & -5 \end{vmatrix}, i=2, j=2$	1.23 $\begin{vmatrix} 5 & 4 & -5 \\ 3 & -7 & 1 \\ 1 & 2 & 2 \end{vmatrix}, i=3, j=1$	1.24 $\begin{vmatrix} 3 & 1 & 0 \\ 4 & 3 & 2 \\ 2 & 2 & -7 \end{vmatrix}, i=2, j=1$
1.25 $\begin{vmatrix} 2 & 7 & 0 \\ 5 & 3 & 1 \\ 1 & -6 & 1 \end{vmatrix}, i=2, j=3$	1.26 $\begin{vmatrix} 8 & -1 & -1 \\ 5 & -5 & -1 \\ 10 & 3 & 2 \end{vmatrix}, i=1, j=3$	1.27 $\begin{vmatrix} 3 & 2 & 5 \\ 3 & 2 & 1 \\ -1 & 0 & 2 \end{vmatrix}, i=1, j=1$
1.28 $\begin{vmatrix} 4 & -7 & -6 \\ 3 & 2 & -1 \\ 0 & 1 & 2 \end{vmatrix}, i=2, j=3$	1.29 $\begin{vmatrix} 7 & 5 & 1 \\ 5 & 3 & -1 \\ 1 & 2 & 3 \end{vmatrix}, i=1, j=3$	1.30 $\begin{vmatrix} 1 & 2 & 1 \\ 1 & -2 & 4 \\ 3 & -5 & 7 \end{vmatrix}, i=3, j=2$

Task 2. Given matrices A, B, C.

Find:

a) the product of matrices AB or BC, or CB (if possible). Explain why if it is impossible;

b) the matrix A^{-1}, the inverse matrix for A.

2.1	$A=\begin{pmatrix}3 & -7 & 2\\ 1 & -8 & 3\\ 4 & -2 & 3\end{pmatrix},\quad B=\begin{pmatrix}2\\ -7\\ 1\end{pmatrix},\quad C=\begin{pmatrix}1 & 1 & -4\\ 0 & 1 & 3\end{pmatrix}$
2.2	$A=\begin{pmatrix}0 & 5 & -3\\ 2 & 4 & 1\\ 2 & 1 & 5\end{pmatrix},\quad B=\begin{pmatrix}3 & 1 & -1\\ -2 & 4 & 0\end{pmatrix},\quad C=\begin{pmatrix}5\\ -2\\ 4\end{pmatrix}$
2.3	$A=\begin{pmatrix}2 & -1 & -4\\ 4 & -9 & 3\\ 2 & -7 & -1\end{pmatrix},\quad B=\begin{pmatrix}-4 & 2 & 1\end{pmatrix},\quad C=\begin{pmatrix}7 & 1\\ 2 & 1\\ 3 & -3\end{pmatrix}$
2.4	$A=\begin{pmatrix}1 & 1 & -1\\ 2 & -4 & 1\\ 4 & -3 & 6\end{pmatrix},\quad B=\begin{pmatrix}-2 & 4\\ 1 & 7\\ -4 & 3\end{pmatrix},\quad C=\begin{pmatrix}4 & 3 & -2\end{pmatrix}$
2.5	$A=\begin{pmatrix}1 & 0 & -4\\ 2 & 5 & -3\\ 4 & -3 & 2\end{pmatrix},\quad B=\begin{pmatrix}3 & 1\\ 0 & -2\end{pmatrix},\quad C=\begin{pmatrix}-3 & 1 & 4 & 2\\ 7 & 1 & 5 & 6\end{pmatrix}$
2.6	$A=\begin{pmatrix}1 & 2 & 1\\ 1 & -2 & 4\\ 3 & -5 & 3\end{pmatrix},\quad B=\begin{pmatrix}-4 & 1\\ 0 & 2\\ 1 & -1\\ 6 & 5\end{pmatrix},\quad C=\begin{pmatrix}7 & 1\\ 0 & 2\end{pmatrix}$
2.7	$A=\begin{pmatrix}5 & 7 & 1\\ 5 & 3 & -1\\ 1 & 2 & 3\end{pmatrix},\quad B=\begin{pmatrix}3\\ -6\\ 2\end{pmatrix},\quad C=\begin{pmatrix}2 & 3 & -3\\ 1 & 0 & 3\end{pmatrix}$
2.8	$A=\begin{pmatrix}-3 & 4 & 0\\ 4 & 5 & 1\\ -2 & 3 & 3\end{pmatrix},\quad B=\begin{pmatrix}4 & 2 & -2\\ 3 & 5 & 1\end{pmatrix},\ C=\begin{pmatrix}6\\ -1\\ 3\end{pmatrix}$
2.9	$A=\begin{pmatrix}1 & 7 & -1\\ 0 & 2 & 6\\ 2 & -1 & 1\end{pmatrix},\quad B=\begin{pmatrix}-5 & 3 & 1\end{pmatrix},\quad C=\begin{pmatrix}8 & -1\\ 3 & 0\\ 2 & -2\end{pmatrix}$
2.10	$A=\begin{pmatrix}-1 & 0 & 2\\ 2 & 3 & 2\\ 3 & 7 & 1\end{pmatrix},\quad B=\begin{pmatrix}-3 & 1\\ 4 & 7\\ -5 & 0\end{pmatrix},\quad C=\begin{pmatrix}5 & 4 & -2\end{pmatrix}$
2.11	$A=\begin{pmatrix}3 & 0 & 1\\ -3 & 1 & 7\\ 1 & 3 & 2\end{pmatrix},\quad B=\begin{pmatrix}4 & 2\\ 1 & -3\end{pmatrix},\quad C=\begin{pmatrix}-4 & 2 & 5 & 3\\ 1 & 5 & 6 & 1\end{pmatrix}$

2.12	$A=\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}$, $B=\begin{pmatrix}-5 & 2\\ 1 & 3\\ 2 & 0\\ 7 & 4\end{pmatrix}$, $C=\begin{pmatrix}8 & 2\\ 0 & 1\end{pmatrix}$
2.13	$A=\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 1\\ 1 & 2 & -2\end{pmatrix}$, $B=\begin{pmatrix}4\\ -3\\ 1\end{pmatrix}$, $C=\begin{pmatrix}0 & 2 & -4\\ 1 & 3 & 7\end{pmatrix}$
2.14	$A=\begin{pmatrix}6 & 9 & 4\\ -1 & -1 & 1\\ 2 & 1 & 1\end{pmatrix}$, $B=\begin{pmatrix}1 & -2 & 3\\ 2 & 4 & 7\end{pmatrix}$, $C=\begin{pmatrix}5\\ -8\\ 1\end{pmatrix}$
2.15	$A=\begin{pmatrix}1 & 0 & 3\\ 2 & -1 & 8\\ 3 & 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}6 & -2 & 4\end{pmatrix}$, $C=\begin{pmatrix}3 & -6\\ 7 & 1\\ 2 & -2\end{pmatrix}$
2.16	$A=\begin{pmatrix}4 & 5 & 1\\ -1 & 0 & 3\\ 4 & 2 & 1\end{pmatrix}$, $B=\begin{pmatrix}3 & -2\\ 4 & -2\\ 7 & 5\end{pmatrix}$, $C=\begin{pmatrix}-2 & 3 & 4\end{pmatrix}$
2.17	$A=\begin{pmatrix}3 & 3 & 6\\ 4 & 3 & -4\\ 2 & 1 & -2\end{pmatrix}$, $B=\begin{pmatrix}4 & -5\\ 1 & 2\end{pmatrix}$, $C=\begin{pmatrix}2 & -3 & 1 & 8\\ 1 & -9 & 2 & 2\end{pmatrix}$
2.18	$A=\begin{pmatrix}2 & 5 & -3\\ 4 & -3 & 2\\ 1 & 0 & 3\end{pmatrix}$, $B=\begin{pmatrix}3 & -1\\ 4 & 9\\ 1 & 2\\ -2 & 3\end{pmatrix}$, $C=\begin{pmatrix}5 & -6\\ 2 & 1\end{pmatrix}$
2.19	$A=\begin{pmatrix}4 & -1 & -2\\ 4 & 3 & 7\\ 2 & 0 & 6\end{pmatrix}$, $B=\begin{pmatrix}-8\\ 1\\ 9\end{pmatrix}$, $C=\begin{pmatrix}6 & -1 & 5\\ 2 & 4 & 3\end{pmatrix}$
2.20	$A=\begin{pmatrix}3 & 0 & 6\\ 1 & 7 & 2\\ -2 & 1 & 4\end{pmatrix}$, $B=\begin{pmatrix}3 & 8 & -4\\ 4 & 1 & 1\end{pmatrix}$, $C=\begin{pmatrix}2\\ -6\\ 8\end{pmatrix}$
2.21	$A=\begin{pmatrix}3 & 1 & 7\\ 1 & 0 & 3\\ -4 & 2 & 5\end{pmatrix}$, $B=\begin{pmatrix}7 & -4 & 0\end{pmatrix}$, $C=\begin{pmatrix}2 & -3\\ 1 & 9\\ 3 & 4\end{pmatrix}$
2.22	$A=\begin{pmatrix}-3 & 4 & 1\\ -2 & 3 & 5\\ 4 & 6 & 2\end{pmatrix}$, $B=\begin{pmatrix}-6 & 1\\ -2 & 2\\ 5 & 3\end{pmatrix}$, $C=\begin{pmatrix}0 & 6 & 5\end{pmatrix}$
2.23	$A=\begin{pmatrix}1 & -5 & 3\\ -3 & 4 & 0\\ 2 & 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}3 & 9\\ -2 & 1\end{pmatrix}$, $C=\begin{pmatrix}2 & 3 & -1 & 4\\ 1 & -3 & 9 & 2\end{pmatrix}$

2.24	$A=\begin{pmatrix}3 & -6 & 1\\ 2 & 1 & -7\\ 4 & 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}7 & 2\\ -5 & 1\\ 0 & 2\\ 3 & 5\end{pmatrix}$, $C=\begin{pmatrix}6 & 2\\ -2 & 4\end{pmatrix}$
2.25	$A=\begin{pmatrix}2 & 3 & -3\\ 4 & 1 & 5\\ 2 & 7 & -1\end{pmatrix}$, $B=\begin{pmatrix}3\\ -5\\ 4\end{pmatrix}$, $C=\begin{pmatrix}2 & -4 & 1\\ 0 & 6 & -9\end{pmatrix}$
2.26	$A=\begin{pmatrix}5 & 6 & -4\\ 3 & 1 & 8\\ 2 & -2 & 3\end{pmatrix}$, $B=\begin{pmatrix}6 & -8 & 1\\ 0 & -1 & -2\end{pmatrix}$, $C=\begin{pmatrix}7\\ 6\\ 3\end{pmatrix}$
2.27	$A=\begin{pmatrix}-2 & 3 & 5\\ 1 & 7 & 2\\ -2 & 5 & 4\end{pmatrix}$, $B=\begin{pmatrix}5 & 1 & -4\end{pmatrix}$, $C=\begin{pmatrix}-9 & 1\\ 2 & 7\\ 3 & 1\end{pmatrix}$
2.28	$A=\begin{pmatrix}-5 & 1 & 3\\ 2 & 4 & 0\\ 4 & 6 & 1\end{pmatrix}$, $B=\begin{pmatrix}5 & 8\\ -4 & 1\\ 0 & 3\end{pmatrix}$, $C=\begin{pmatrix}5 & 2 & -8\end{pmatrix}$
2.29	$A=\begin{pmatrix}10 & 0 & -2\\ 3 & 1 & -1\\ 5 & 2 & 3\end{pmatrix}$, $B=\begin{pmatrix}7 & 3\\ -4 & 2\end{pmatrix}$, $C=\begin{pmatrix}2 & 3 & -7 & 1\\ 4 & 1 & 3 & 6\end{pmatrix}$
2.30	$A=\begin{pmatrix}2 & -3 & 1\\ 4 & 1 & 9\\ 3 & 2 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & -3\\ 1 & 6\\ -3 & 7\\ 2 & 3\end{pmatrix}$, $C=\begin{pmatrix}5 & 1\\ -4 & 2\end{pmatrix}$

Task 3. Given points *A* and *B*; and vectors $\bar{b}$ and $\vec{c}$.
Find:

a) the module (length) of the vector $|\vec{a}| = |\overrightarrow{AB}|$ and the midpoint of the segment *AB*;

b) the projection of the vector $\vec{a}$ on $\vec{c}$;

c) the area of the parallelogram, obtained from the vectors $\vec{b}$ and $\vec{c}$;

d) volume of the pyramid constructed from the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

3.1	A(5, -4, 3), B(1, 2, -8), $\bar{в}$=(0, 1, 4), $\bar{c}$=(5, 2, -3)	3.2	A(-3, 1, 0), B(7, 1, -5), $\bar{в}$=(7, 1, 4), $\bar{c}$=(5, 8, -3)
3.3	A(0, 4, 5), B(3, -2, 1), $\bar{в}$=(6, 4, 4), $\bar{c}$=(0, 2, -2)	3.4	A(3, -2, 5), B(4, 5, 7), $\bar{в}$=(5, 1, 4), $\bar{c}$=(5, -3, -3)
3.5	A(2, -3, 7), B(3, 2, 8), $\bar{в}$=(6, 4, 6), $\bar{c}$=(0, 6, -2)	3.6	A(2, -1, 7), B(6, 3, 4) , $\bar{в}$=(4, 5, 4), $\bar{c}$=(7, 8, 5)

3.7	A(3, 1, 7), B(2, -3, 9), $\overline{в}$=(9, 1, 4), $\overline{c}$=(8, 2, -3)	3.8	A(2, 1, -6), B(1, 4, 9) , $\overline{в}$=(1, 1, 8), $\overline{c}$=(5, -3, 9)
3.9	A(2, -4, 8), B(5, 4, 7), $\overline{в}$=(6, 3, 4), $\overline{c}$=(7, 2, -2)	3.10	A(3, 2, 5), B(4, 0, -3) , $\overline{в}$=(7, 2, 4), $\overline{c}$=(5, 8, 4)
3.11	A(2, 3, -1),B(-6, 4, 2), $\overline{в}$=(2, 9, 6), $\overline{c}$=(0, 6, 4)	3.12	A(-4, 2, 3), B(8, 7, -2), $\overline{в}$=(5, 3, 4), $\overline{c}$=(5, 3, 4)
3.13	A(5, 3, 6), B(-2, 3, 5), $\overline{в}$=(6, 2, 4), $\overline{c}$=(7, 2, -7)	3.14	A(0, 6, 0), B(5, 3, -4), $\overline{в}$=(8, 5, 3), $\overline{c}$=(7, 8, 7)
3.15	A(4, 2, 0), B(1, -7, 8), $\overline{в}$=(2, 8, 6), $\overline{c}$=(0, 6, 3)	3.16	A(4, 2, 5), B(-1, 0, 6), $\overline{в}$=(2, 1, 8), $\overline{c}$=(5, 9, 9)
3.17	A(3,-5, 8), B(6, 3, 9), $\overline{в}$=(7, 3, 4), $\overline{c}$=(3, 2, -2)	3.18	A(7, 2, 2), B(-5, 7, -7) , $\overline{в}$=(6, 1, 3), $\overline{c}$=(1, 8, 7)
3.19	A(5, -3, 1), B(2, 3, 7), $\overline{в}$=(2, 7, 6), $\overline{c}$=(7, 6, 4)	3.20	A(8, -6, 4), B(10, 5, 1), $\overline{в}$=(9, 1, 8), $\overline{c}$=(4, 9, 1)
3.21	A(5, 6,-8), B(8, 10,7), $\overline{в}$=(1, 3, 4), $\overline{c}$=(3, 6, -2)	3.22	A(1, -1, 3), B(6, 5, 8), $\overline{в}$=(7, 8, 3), $\overline{c}$=(1, 8, 9)
3.23	A(3, 5,-7), B(8, 4, 1), $\overline{в}$=(2, 5, 6), $\overline{c}$=(7, 6, 8)	3.24	A(6, -6, 5), B(4, 9, 5), $\overline{в}$=(6, 1, 4), $\overline{c}$=(0, 9, 1)
3.25	A(4,6,11), B(9,3,-4), $\overline{в}$=(3, 7, 6), $\overline{c}$=(8, 6, 4)	3.26	A(5, 7, 4),B(4,-10, 9), $\overline{в}$=(8, 1, 7), $\overline{c}$=(4, 5, 1)
3.27	A(-9, 8, 9), B(7, 1,-2), $\overline{в}$=(7, 2, 4), $\overline{c}$=(5, 2, -1)	3.28	A(5, 2, 6), B(1, 8, -2), $\overline{в}$=(4, 2, 3), $\overline{c}$=(2, 3, 7)
3.29	A(2, 8, -9), B(7, 5,-5), $\overline{в}$=(8, 1, 6), $\overline{c}$=(6, 1, 4)	3.30	A(-2, 7, 0), B(6, 3, 5), $\overline{в}$=(2, -3, 8), $\overline{c}$=(3, 9, 2)

Task 4. Given points A_1, A_2 on a plane and the equation of the line L_1. Write the equation of a straight line:

a) $L_2 = (A_1A_2)$ – passing through these points;

b) L_2 – as a general equation of straight line;

c) L_2 – in the form of equation of straight line with slope;

d) L_2 – in the form of equation of straight line in segments;

e) L_3 , passing through the point A_2 and perpendicular to L_1.

4.1	$A_1(3,1),\quad A_2(2,-3),\quad L_1: x-5y+4=0$	4.2	$A_1(-4,1),\quad A_2(6,2),\quad L_1: 2x-3y-1=0$
4.3	$A_1(-4,7),\quad A_2(2,-13),\quad L_1: 3x-5y+2=0$	4.4	$A_1(5,1),\quad A_2(-4,2),\quad L_1: x-3y-1=0$
4.5	$A_1(2,1),\quad A_2(6,-3),\quad L_1: 4x-5y+1=0$	4.6	$A_1(8,3),\quad A_2(-1,2),\quad L_1: 2x+2y-9=0$
4.7	$A_1(-1,7),\quad A_2(3,-1),\quad L_1: 3x-4y+2=0$	4.8	$A_1(-2,1),\quad A_2(-7,3),\quad L_1: 8x-3y-1=0$
4.9	$A_1(6,1),\quad A_2(5,-3),\quad L_1: x-7y+4=0$	4.10	$A_1(-4,2),\quad A_2(6,1),\quad L_1: 2x-5y+4=0$
4.11	$A_1(-4,3),\quad A_2(2,-2),\quad L_1: 3x-y+2=0$	4.12	$A_1(7,1),\quad A_2(-4,2),\quad L_1: x-y+4=0$

4.13	$A_1(-9,1),\quad A_2(1,-3),\quad L_1: x-5y+1=0$	4.14	$A_1(8,1),\quad A_2(-1,3),\quad L_1: 2x+5y+8=0$
4.15	$A_1(5,7),\quad A_2(3,-1),\quad L_1: 3x-5y+2=0$	4.16	$A_1(-2,7),\quad A_2(-7,-3),\quad L_1: 7x-6y-1=0$
4.17	$A_1(5,1),\quad A_2(7,-3),\quad L_1: x-7y+5=0$	4.18	$A_1(-4,8),\quad A_2(2,1),\quad L_1: 2x-4y-6=0$
4.19	$A_1(-4,4),\quad A_2(9,-2),\quad L_1: x-y+2=0$	4.20	$A_1(7,6),\quad A_2(-4,1),\quad L_1: 2x-y-1=0$
4.21	$A_1(-8,1),\quad A_2(1,-7),\quad L_1: 2x-5y+1=0$	4.22	$A_1(3,1),\quad A_2(-1,-3),\quad L_1: 2x-4y+9=0$
4.23	$A_1(5,1),\quad A_2(6,-1),\quad L_1: x-5y+2=0$	4.24	$A_1(-2,8),\quad A_2(-1,4),\quad L_1: 7x-y+4=0$
4.25	$A_1(-4,2),\quad A_2(9,-2),\quad L_1: 9x-y+2=0$	4.26	$A_1(2,6),\quad A_2(-2,1),\quad L_1: 5x-5y+4=0$
4.27	$A_1(-6,1),\quad A_2(1,-9),\quad L_1: 2x-5y+4=0$	4.28	$A_1(10,1),\quad A_2(-1,5),\quad L_1: x+4y-9=0$
4.29	$A_1(3,1),\quad A_2(1,-1),\quad L_1: x+5y+8=0$	4.30	$A_1(-1,8),\quad A_2(-1,5),\quad L_1: 7x-2y-1=0$

Task 5. Given points A_1, A_2, A_3. It is required to:

a) write the equation of the plane $P_1 = (A_1A_2A_3)$;

b) write P_1 as a general equation of a plane;

c) write P_1 as equation of a plane in segments;

d) write P_1 in a general equation of a plane, passing through the point A_1;

e) make a canonical equation of the straight line $L_1 = (A_2A_3)$;

f) write a parametric equation of the straight line L_1;

g) write the equation of the straight line $L_2 = (A_1N)$ perpendicular to the plane P_1.

5.1	$A_1(3,1,5),\quad A_2(2,-3,1),\quad A_3(4,-6,2)$	5.2	$A_1(7,4,1),\quad A_2(1,-4,2),\quad A_3(1,2,7)$
5.3	$A_1(3,4,5),\quad A_2(3,-9,1),\quad A_3(4,5,7)$	5.4	$A_1(-3,4,-2),\quad A_2(9,5,1),\quad A_3(4,-3,-1)$
5.5	$A_1(0,8,5),\quad A_2(3,-2,4),\quad A_3(2,5,6)$	5.6	$A_1(2,5,6),\quad A_2(3,-7,8),\quad A_3(5,4,-2)$
5.7	$A_1(-1,4,2),\quad A_2(1,-9,1),\quad A_3(4,4,7)$	5.8	$A_1(1,9,-1),\quad A_2(2,6,1),\quad A_3(2,8,-6)$
5.9	$A_1(-1,4,5),\quad A_2(3,-5,1),\quad A_3(4,5,2)$	5.10	$A_1(9,5,1),\quad A_2(4,-7,8),\quad A_3(5,2,-2)$
5.11	$A_1(3,7,5),\quad A_2(3,-9,2),\quad A_3(1,5,7)$	5.12	$A_1(1,6,-1),\quad A_2(2,7,1),\quad A_3(2,3,-6)$
5.13	$A_1(0,9,5),\quad A_2(4,-2,4),\quad A_3(2,7,6)$	5.14	$A_1(7,5,-1),\quad A_2(2,-4,1),\quad A_3(3,-2,7)$
5.15	$A_1(3,4,-1),\quad A_2(1,-7,1),\quad A_3(4,3,7)$	5.16	$A_1(-4,4,-2),\quad A_2(8,5,1),\quad A_3(4,-3,-1)$
5.17	$A_1(1,4,1),\quad A_2(1,8,1),\quad A_3(4,3,-6)$	5.18	$A_1(2,-5,6),\quad A_2(3,-2,-7),\quad A_3(-5,4,-2)$
5.19	$A_1(2,4,1),\quad A_2(3,-7,1),\quad A_3(5,3,7)$	5.20	$A_1(3,9.-2),\quad A_2(-2,6,1),\quad A_3(2,8,5)$
5.21	$A_1(1,8,-3),\quad A_2(2,8,1),\quad A_3(6,3,-6)$	5.22	$A_1(1,6,-5),\quad A_2(4,7,1),\quad A_3(1,3,-6)$
5.23	$A_1(8,4,-1),\quad A_2(1,-9,1),\quad A_3(4,2,7)$	5.24	$A_1(8,5,-1),\quad A_2(2,-3,2),\quad A_3(-3,1,7)$
5.25	$A_1(1,4,-2),\quad A_2(9,8,1),\quad A_3(4,-3,-6)$	5.26	$A_1(-4,3,2),\quad A_2(8,5,9),\quad A_3(4,-3,-5)$
5.27	$A_1(2,5,-1),\quad A_2(3,-7,8),\quad A_3(5,3,-2)$	5.28	$A_1(9,-5,6),\quad A_2(3,2,-7),\quad A_3(-5,1,-2)$
5.29	$A_1(1,8,-1),\quad A_2(2,6,1),\quad A_3(2,3,-6)$	5.30	$A_1(3,9,-1),\quad A_2(-4,6,1),\quad A_3(2,7,5)$

Task 6. Solve the system
a) by Cramer's method;
b) by matrix method (using the inverse matrix).

6.1	$\begin{cases} 2x+y=7 \\ x+3y=1 \end{cases}$	6.2	$\begin{cases} 4x+y=-3 \\ x+2y=1 \end{cases}$	6.3	$\begin{cases} 5x+y=2 \\ -x+2y=3 \end{cases}$
6.4	$\begin{cases} x+y=3 \\ 2x+3y=1 \end{cases}$	6.5	$\begin{cases} 8x+y=-3 \\ x-2y=1 \end{cases}$	6.6	$\begin{cases} 6x+y=2 \\ -x+2y=4 \end{cases}$
6.7	$\begin{cases} x+y=13 \\ 2x-3y=1 \end{cases}$	6.8	$\begin{cases} 4x+y=-3 \\ 6x+2y=11 \end{cases}$	6.9	$\begin{cases} 9x+y=2 \\ -x+3y=3 \end{cases}$
6.10	$\begin{cases} 2x+5y=7 \\ x+3y=1 \end{cases}$	6.11	$\begin{cases} 4x+y=-3 \\ x+7y=1 \end{cases}$	6.12	$\begin{cases} 7x+y=2 \\ -x+3y=4 \end{cases}$
6.13	$\begin{cases} 2x+y=7 \\ x+3y=1 \end{cases}$	6.14	$\begin{cases} 2x+y=7 \\ 6x+2y=1 \end{cases}$	6.15	$\begin{cases} 5x+6y=2 \\ -x+3y=3 \end{cases}$
6.16	$\begin{cases} x+8y=3 \\ 2x+3y=1 \end{cases}$	6.17	$\begin{cases} 8x+y=7 \\ x-2y=1 \end{cases}$	6.18	$\begin{cases} 6x-y=6 \\ -x+2y=4 \end{cases}$
6.19	$\begin{cases} 4x+3y=7 \\ x+3y=-1 \end{cases}$	6.20	$\begin{cases} -3x+y=7 \\ 6x+3y=11 \end{cases}$	6.21	$\begin{cases} 7x+y=2 \\ -x+2y=3 \end{cases}$
6.21	$\begin{cases} x-2y=13 \\ x+3y=1 \end{cases}$	6.23	$\begin{cases} 3x-y=12 \\ x+2y=6 \end{cases}$	6.24	$\begin{cases} 7x+y=2 \\ -x+2y=4 \end{cases}$
6.23	$\begin{cases} 2x+y=7 \\ x+3y=1 \end{cases}$	6.26	$\begin{cases} 7x-y=2 \\ x+2y=6 \end{cases}$	6.27	$\begin{cases} x-4y=7 \\ x+y=6 \end{cases}$
6.28	$\begin{cases} 2x+y=-3 \\ 6x+2y=1 \end{cases}$	6.29	$\begin{cases} x-y=12 \\ x+2y=6 \end{cases}$	6.30	$\begin{cases} x-y=2 \\ x+2y=5 \end{cases}$

Task 7. Solve the system of equations.
a) by Cramer's method;
b) by matrix method (using the inverse matrix).

7.1	7.2	7.3
$\begin{cases} 2x_1+x_2+3x_3=7 \\ 2x_1+3x_2+x_3=9 \\ 3x_1-2x_2+3x_3=2 \end{cases}$	$\begin{cases} 2x_1-x_2+2x_3=10 \\ x_1+x_2+2x_3=6 \\ 3x_1+x_2+4x_3=16 \end{cases}$	$\begin{cases} 3x_1-x_2+x_3=-12 \\ x_1+2x_2+4x_3=-13 \\ 5x_1-x_2+2x_3=-17 \end{cases}$

7.4 $\begin{cases} 2x_1 - x_2 + 3x_3 = 3 \\ x_1 + 3x_2 + x_3 = 9 \\ x_1 - 2x_2 - 2x_3 = 2 \end{cases}$	7.5 $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 3 \\ 3x_1 + 4x_2 - 2x_3 = -3 \\ 2x_1 - x_2 - x_3 = -15 \end{cases}$	7.6 $\begin{cases} 2x_1 + 3x_2 - 6x_3 = 13 \\ x_1 + 3x_2 - x_3 = -9 \\ 4x_1 + x_2 + 3x_3 = -9 \end{cases}$
7.7 $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 9 \\ x_1 - 7x_2 - 5x_3 = 16 \\ 2x_1 + 4x_2 + 3x_3 = 4 \end{cases}$	7.8 $\begin{cases} 4x_1 + x_2 - 3x_3 = 11 \\ x_1 + x_2 - x_3 = 4 \\ 2x_1 - 3x_2 - 6x_3 = 7 \end{cases}$	7.9 $\begin{cases} 2x_1 + 3x_2 + 4x_3 = -2 \\ 7x_1 - 5x_2 + x_3 = -4 \\ 4x_1 + x_2 + x_3 = 11 \end{cases}$
7.10 $\begin{cases} x_1 - 4x_2 - x_3 = -8 \\ 2x_1 + 5x_2 + 4x_3 = 6 \\ 3x_1 + 2x_2 + 5x_3 = -8 \end{cases}$	7.11 $\begin{cases} 3x_1 - 2x_2 + 4x_3 = -3 \\ 3x_1 - 4x_2 - 2x_3 = -1 \\ 2x_1 - x_2 + x_3 = -5 \end{cases}$	7.12 $\begin{cases} x_1 - 2x_2 - 5x_3 = -7 \\ 2x_1 + 3x_2 + 4x_3 = -7 \\ x_1 + 2x_2 + 3x_3 = -3 \end{cases}$
7.13 $\begin{cases} 4x_1 + x_2 - 4x_3 = 15 \\ 2x_1 - x_2 - 2x_3 = 9 \\ x_1 + 3x_2 + 2x_3 = 10 \end{cases}$	7.14 $\begin{cases} 2x_1 - x_2 + 3x_3 = -3 \\ 4x_1 + 3x_2 + 4x_3 = -6 \\ x_1 + x_2 + 2x_3 = -7 \end{cases}$	7.15 $\begin{cases} 2x_1 - x_2 - 2x_3 = 6 \\ x_1 + 3x_2 + 2x_3 = -5 \\ 4x_1 + x_2 - 3x_3 = 2 \end{cases}$
7.16 $\begin{cases} 2x_1 - x_2 - 3x_3 = 4 \\ 5x_1 + x_2 + x_3 = 9 \\ 3x_1 + x_2 + 2x_3 = 4 \end{cases}$	7.17 $\begin{cases} 2x_1 - x_2 + 3x_3 = -2 \\ 3x_1 + 4x_2 - 2x_3 = 6 \\ 5x_1 + x_2 + 3x_3 = 0 \end{cases}$	7.18 $\begin{cases} 3x_1 + 5x_2 + 4x_3 = 7 \\ 3x_1 + x_2 - 2x_3 = 5 \\ x_1 + 4x_2 + 2x_3 = -5 \end{cases}$
7.19 $\begin{cases} 3x_1 + x_2 + x_3 = -5 \\ 3x_1 + 5x_2 + x_3 = 8 \\ x_1 + 4x_2 - 2x_3 = -2 \end{cases}$	7.20 $\begin{cases} 3x_1 + x_2 + x_3 = 6 \\ 5x_1 + x_2 - 2x_3 = 1 \\ x_1 + 2x_2 + 4x_3 = 8 \end{cases}$	7.21 $\begin{cases} 3x_1 - x_2 + x_3 = 9 \\ 5x_1 - 2x_2 + 2x_3 = 13 \\ x_1 + 2x_2 + 4x_3 = -1 \end{cases}$
7.22 $\begin{cases} 2x_1 + 3x_2 + x_3 = 3 \\ 2x_1 + 3x_2 + 3x_3 = 7 \\ 3x_1 + 2x_2 + x_3 = 11 \end{cases}$	7.23 $\begin{cases} 4x_1 - x_2 + 5x_3 = 8 \\ 3x_1 + x_2 + 4x_3 = 7 \\ 3x_1 - x_2 + 2x_3 = 11 \end{cases}$	7.24 $\begin{cases} x_1 + 2x_2 + 5x_3 = -8 \\ 2x_1 + 3x_2 + 4x_3 = -14 \\ 3x_1 + 2x_2 + 5x_3 = -2 \end{cases}$
7.25 $\begin{cases} x_1 - 2x_2 - 3x_3 = -5 \\ 2x_1 + 3x_2 + 4x_3 = -1 \\ 3x_1 + x_2 + x_3 = -6 \end{cases}$	7.26 $\begin{cases} x_1 + 2x_2 + 6x_3 = 1 \\ 3x_1 + x_2 - 4x_3 = -5 \\ 2x_1 + 3x_2 + x_3 = 8 \end{cases}$	7.27 $\begin{cases} 2x_1 + 8x_2 - 7x_3 = -5 \\ 2x_1 + 5x_2 - 6x_3 = -1 \\ x_1 + 3x_2 - x_3 = -1 \end{cases}$

7.28 $\begin{cases} 5x_1+2x_2+4x_3=-4 \\ 2x_1+x_2-3x_3=4 \\ -2x_1-3x_2+x_3=3 \end{cases}$	7.29 $\begin{cases} 2x_1+3x_2+2x_3=-9 \\ 3x_1+4x_2-7x_3=6 \\ 5x_1+x_2-5x_3=-9 \end{cases}$	7.30 $\begin{cases} 7x_1-4x_2-3x_3=-3 \\ 3x_1+2x_2-3x_3=3 \\ 2x_1-3x_2+x_3=1 \end{cases}$

Task 8. Given point *A*; radius of the circle *R*; *a*, *b* – semiaxes of curves; equation of the straight line *D*. It is required to:

a) write the equation of a circle with center *A* and radius *R*;

б) write the canonical equation of an ellipse with semiaxes *a* and *b*. Find the coordinates of it's foci and the eccentricity;

c) write the canonical equation of the hyperbola with the real semiaxis *a* and imaginary semiaxis *b*. Find the coordinates of its foci, eccentricity, equations of the asymptotes;

d) write the canonical equation of the parabola with vertex at the origin and if *D* is its directrix. Find the coordinates of its focus, eccentricity;

e) make the drawing of ellipse, hyperbola, parabola.

8.1	A(2,-4), R=4, a=1, b=3, D: x=-5	8.2	A(-8,2), R=1, a=6, b=5, D: x=-5
8.3	A(1,-4), R=5, a=8, b=3, D: y=-6	8.4	A(5,-4), R=2, a=6, b=4, D: y=-2
8.5	A(2,-5), R=7, a=3, b=2, D: x=4	8.6	A(1,8), R=5 , a=3, b=2, D: x=-3
8.7	A(3,-4), R=9, a=7, b=6, D: y=-2	8.8	A(10,1), R=8, a=1, b=6, D: y=-4
8.9	A(5,-4), R=1, a=6, b=4, D: x=-5	8.10	A(6,3), R=8 , a=2, b=3, D: x=-5
8.11	A(1,-3), R=5 , a=8, b=2, D: y=6	8.12	A(5,5), R=2, a=1, b=3, D: y=-7
8.13	A(2,-6), R=7, a=3, b=4, D: x=5	8.14	A(12,6), R=7, a=6, b=2, D: x=-5
8.15	A(3,4), R=9 , a=2, b=6, D: y=-8	8.16	A(0,5), R=4, a=6, b=4, D: y=8
8.17	A(2,-9), R=7, a=5, b=2, D: x=6	8.18	A(-5,0), R=7, a=4, b=5, D: x=1
8.19	A(8,4), R=6 , a=8, b=5, D: y=2	8.20	A(5,1), R=2, a=9, b=1, D: x=-1
8.21	A(5,-4), R=4, a=6, b=4, D: x=1	8.22	A(-3,2), R=4, a=8, b=4, D: y=1
8.23	A(1,8), R=5, a=9, b=4, D: y=-6	8.24	A(9,1), R=6, a=4, b=7, D: x=-3
8.25	A(2,-5), R=7, a=7, b=4, D: x=9	8.26	A(-9,2), R=7, a=1, b=8, D: y=7
8.27	A(7,4), R=5, a=1, b=7, D: y=8	8.28	A(11,-4), R=2, a=2, b=4, D: x=8
8.29	A(-2,5), R=5, a=7, b=1, D: x=8	8.30	A(12,4), R=7, a=3, b=5, D: y=-9

Task 9. Determine the type (name) of the second order surface and make a schematic drawing:

9.1	$a)\frac{x^2}{3}+\frac{y^2}{4}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}-\frac{z^2}{6}=3y$	9.2	$a)\frac{x^2}{16}-\frac{y^2}{16}-\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}+\frac{z^2}{3}=1$
9.3	$a)\frac{x^2}{3}+\frac{y^2}{16}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{3}+\frac{z^2}{4}=y$	9.4	$a)\frac{x^2}{10}+\frac{y^2}{10}+\frac{z^2}{10}=1;\quad b)\frac{x^2}{3}-\frac{z^2}{6}=1$
9.5	$a)-\frac{x^2}{3}+\frac{y^2}{4}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{1}+\frac{y^2}{9}=5z$	9.6	$a)\frac{x^2}{1}+\frac{y^2}{1}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{3}-\frac{y^2}{5}=z$

9.7	$a)\frac{x^2}{25}-\frac{y^2}{9}+\frac{z^2}{4}=1;\quad b)\frac{y^2}{3}+\frac{z^2}{4}=1$	9.8	$a)-\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{4}=-1;\quad b)\frac{y^2}{16}-\frac{z^2}{9}=6z$
9.9	$a)\frac{x^2}{12}+\frac{y^2}{9}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{3}-\frac{z^2}{6}=1$	9.10	$a)\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{1}=1;\quad b)\frac{x^2}{9}+\frac{z^2}{4}=-2y$
9.11	$a)\ \frac{x^2}{9}+\frac{y^2}{16}-\frac{z^2}{16}=1;\quad b)\frac{x^2}{9}+\frac{z^2}{4}=4y$	9.12	$a)-\frac{x^2}{12}+\frac{y^2}{9}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}+\frac{y^2}{6}=-4z$
9.13	$a)-\frac{x^2}{3}-\frac{y^2}{8}+\frac{z^2}{4}=1;\quad b)\frac{y^2}{10}-\frac{z^2}{6}=2x$	9.14	$a)\frac{x^2}{2}+\frac{y^2}{16}+\frac{z^2}{1}=1;\quad b)\frac{x^2}{4}-\frac{z^2}{4}=0$
9.15	$a)-\frac{x^2}{4}+\frac{y^2}{16}-\frac{z^2}{1}=1;\quad b)-\frac{x^2}{3}+\frac{z^2}{4}=2y$	9.16	$a)-\frac{x^2}{3}-\frac{y^2}{9}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}-\frac{y^2}{6}=0$
9.17	$a)\frac{x^2}{3}+\frac{y^2}{4}-\frac{z^2}{5}=-1;\quad b)\frac{x^2}{3}=3y$	9.18	$a)\frac{x^2}{4}+\frac{y^2}{16}-\frac{z^2}{1}=-1;\quad b)-\frac{x^2}{8}+\frac{z^2}{4}=0$
9.19	$a)-\frac{x^2}{4}+\frac{y^2}{1}-\frac{z^2}{16}=-1;\quad b)\frac{x^2}{9}-\frac{z^2}{4}=y$	9.20	$a)\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{4}=-1;\quad b)\frac{x^2}{3}-\frac{z^2}{6}=1$
9.21	$a)\frac{x^2}{3}-\frac{y^2}{16}+\frac{z^2}{9}=1;\quad b)\frac{x^2}{3}+\frac{y^2}{5}=2z$	9.22	$a)\frac{x^2}{9}-\frac{y^2}{16}+\frac{z^2}{64}=1;\quad b)\frac{x^2}{3}=-8y$
9.23	$a)\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{12}-\frac{z^2}{4}=2y$	9.24	$a)\frac{x^2}{12}+\frac{y^2}{4}-\frac{z^2}{4}=1;\quad b)-\frac{z^2}{6}=4y$
9.25	$a)-\frac{x^2}{9}-\frac{y^2}{25}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{3}-\frac{z^2}{9}=1$	9.26	$a)\frac{x^2}{16}+\frac{y^2}{16}+\frac{z^2}{16}=1;\quad b)\frac{x^2}{16}+\frac{z^2}{16}=-2y$
9.27	$a)\frac{x^2}{25}-\frac{y^2}{16}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{4}+\frac{z^2}{4}=1$	9.28	$a)\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{4}=1;\quad b)\frac{x^2}{4}-\frac{z^2}{4}=2y$
9.29	$a)-\frac{x^2}{9}+\frac{y^2}{1}+\frac{z^2}{25}=1;\quad b)\frac{y^2}{3}-\frac{x^2}{6}=4z$	9.30	$a)-\frac{x^2}{16}+\frac{y^2}{16}-\frac{z^2}{1}=1;\quad b)\frac{x^2}{4}+\frac{z^2}{4}=1$

Task 10. Lead to the canonical form the equation of the second order, construct this curve.

10.1	$x^2+8x+2y+20=0$	10.2	$9x^2+4y^2-54x-32y+109=0$
10.3	$x^2+8x-2y-16=0$	10.4	$2x^2-2y^2+2x=0$
10.5	$x^2+2y^2-2x+8y+7=0$	10.6	$-5y^2+x+20y-6=0$
10.7	$4x^2+8x-6y+28=0$	10.8	$x^2+6x-2y+5=0$
10.9	$x^2+9y^2-40x+36y+100=0$	10.10	$16x^2-9y^2-64x-18y+199=0$
10.11	$4y^2-2x+8y+28=0$	10.12	$9x^2+10y^2+40y-50=0$
10.13	$3x^2-4y^2+18x+20=0$	10.14	$x^2+6x-2y+30=0$
10.15	$y^2+4x+8y+16=0$	10.16	$4y^2+8x+12y+10=0$

10.17	$4x^2+8x+2y+30=0$	10.18	$x^2-4y^2-2x-24y-64=0$
10.19	$y^2-8x+8y+32=0$	10.20	$x^2+y^2-2x+8y+9=0$
10.21	$5x^2+9y^2+30x+18y+9=0$	10.22	$4y^2+8x+16y+30=0$
10.23	$-y^2+8x-2y-9=0$	10.24	$9x^2-y^2-18x-6y-10=0$
10.25	$9x^2-16y^2-36x-64y-127=0$	10.26	$-3y^2+2x-12y+30=0$
10.27	$-y^2+12x-2y-25=0$	10.28	$x^2-y^2-4y-4=0$
10.29	$x^2+y^2+8x-12y+20=0$	10.30	$-16y^2-4x-64y-64=0$

Task 11. Given complex numbers z_1 and z_2. It is required to find:

a) the module of the complex numbers z_1;

b) the argument of the complex number z_1;

c) the representation of the complex number z_1 in the trigonometric and exponential forms;

d) the sum of complex numbers z_1 and z_2 analytically and graphically;

e) $(z_2)^5$;

f) multiplication of $z_1 \cdot z_2$ in trigonometric form;

g) all complex roots $\sqrt[3]{z_2}$ (for versions with even numbers) and $\sqrt[4]{z_2}$ (for versions with odd numbers). Show the solution in the drawing.

11.1	$z_1=8i-8;$ $z_2=8\sqrt{2}(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})$	11.2	$z_1=5i+5\sqrt{3};$ $z_2=5(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$
11.3	$z_1=-7i-7;$ $z_2=7\sqrt{2}(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2})$	11.4	$z_1=-10;$ $z_2=16(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$
11.5	$z_1=-9+9\sqrt{3}i;$ $z_2=18(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4})$	11.6	$z_1=3-3i;$ $z_2=3\sqrt{2}(\cos\frac{6\pi}{4}+i\sin\frac{6\pi}{4})$
11.7	$z_1=2i-2\sqrt{3};$ $z_2=4(\cos\pi+i\sin\pi)$	11.8	$z_1=27i-27;$ $z_2=27(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6})$
11.9	$z_1=-4+4i;$ $z_2=4\sqrt{2}(\cos(-\frac{3\pi}{4})+i\sin(-\frac{3\pi}{4}))$	11.10	$z_1=6+6\sqrt{3}i;$ $z_2=12(\cos\frac{2\pi}{6}+i\sin\frac{2\pi}{6})$

11.11	$z_1 = -9\sqrt{3} - 9i;$ $z_2 = 18(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$	11.12	$z_1 = 4 - 4\sqrt{3}i;$ $z_2 = 8(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$
11.13	$z_1 = -4\sqrt{3} - 4i;$ $z_2 = 8(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3}))$	11.14	$z_1 = -5 - 5i;$ $z_2 = 5\sqrt{2}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$
11.15	$z_1 = -9i;$ $z_2 = 9(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$	11.16	$z_1 = 3i;$ $z_2 = 3(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$
11.17	$z_1 = \sqrt{3} - 3i;$ $z_2 = 2\sqrt{3}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$	11.18	$z_1 = \sqrt{2} - \sqrt{2}i;$ $z_2 = 2(\cos\frac{6\pi}{3} + i\sin\frac{6\pi}{3})$
11.19	$z_1 = 3 - \sqrt{3}i;$ $z_2 = 2\sqrt{3}(\cos\frac{9\pi}{6} + i\sin\frac{9\pi}{6})$	11.20	$z_1 = 8 - 8i;$ $z_2 = 8\sqrt{2}(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3}))$
11.21	$z_1 = \sqrt{3}i;$ $z_2 = \sqrt{3}(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$	11.22	$z_1 = -64;$ $z_2 = 64(\cos\pi + i\sin\pi)$
11.23	$z_1 = 64i;$ $z_2 = 64(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$	11.24	$z_1 = 3\sqrt{3} - 3i;$ $z_2 = 6(\cos\frac{6\pi}{8} + i\sin\frac{6\pi}{8})$
11.25	$z_1 = 6 + 6i;$ $z_2 = 6\sqrt{2}(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})$	11.26	$z_1 = -5i;$ $z_2 = 5(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$
11.27	$z_1 = \sqrt{3} - \sqrt{3}i;$ $z_2 = 2\sqrt{3}(\cos\frac{8\pi}{6} + i\sin\frac{8\pi}{6})$	11.28	$z_1 = 8 - 8i;$ $z_2 = 8\sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4}))$
11.29	$z_1 = \sqrt{3} - i;$ $z_2 = 2(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4})$	11.30	$z_1 = \sqrt{3}i - 1;$ $z_2 = \sqrt{3}(\cos\frac{10\pi}{6} + i\sin\frac{10\pi}{6})$

Solution of an exemplary embodiment

Task1. The third order determinant is given $\begin{vmatrix} 3 & 4 & -1 \\ 0 & 2 & 8 \\ 1 & 5 & -2 \end{vmatrix}$, $i=2, j=3$.

It is required to calculate:

a) the minor M_{23} and the algebraic complement A_{23} of the element a_{23};

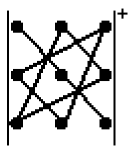
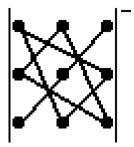
b) the determinant by the triangle rule (rule of Sarrus);

c) the determinant by the method of expansion along the 2-nd row and the 3-rd column.

Solution:

a) the minor M_{ij} of the element a_{ij} equals to the determinant, obtained from given by deleting the *i*-th row and *j*-th column. Thus cross out second row and the third column in our determinant, we obtain $M_{23} = \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} = 3 \cdot 5 - 1 \cdot 4 = 11$. The algebraic complement of the element is calculated by formula $A_{ij} = (-1)^{i+j} M_{ij}$.

So, $A_{23} = (-1)^{2+3}\, 11 = -11$;

b) the triangle rule: the third order determinant is the sum of six terms; terms with a plus sign are obtained by multiplication of three elements of the determinant taken by the scheme , terms with a negative sign – by the scheme .

So, $\begin{vmatrix} 3 & 4 & -1 \\ 0 & 2 & 8 \\ 1 & 5 & -2 \end{vmatrix} = 3 \cdot 2 \cdot (-2) + 4 \cdot 8 \cdot 1 + 0 \cdot 5 \cdot (-1) - 1 \cdot 2 \cdot (-1) -$

$$- 3 \cdot 8 \cdot 5 - 4 \cdot 0 \cdot (-2) = -12 + 32 + 2 - 120 = -98;$$

c) expansion formula along the third column has the form:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33},$$

$$\begin{vmatrix} 3 & 4 & -1 \\ 0 & 2 & 8 \\ 1 & 5 & -2 \end{vmatrix} = (-1) \cdot (-1)^{1+3} \cdot \begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} + 8 \cdot (-1)^{2+3} \cdot \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} + (-2) \cdot (-1)^{3+3} \cdot \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} =$$

$$= 2 - 88 - 12 = -98;$$

similarly calculate the determinant expanding along the second row:

$$\begin{vmatrix} 3 & 4 & -1 \\ 0 & 2 & 8 \\ 1 & 5 & -2 \end{vmatrix} = 0\cdot(-1)^{2+1}\cdot\begin{vmatrix} 4 & -1 \\ 5 & -2 \end{vmatrix} + 2\cdot(-1)^{2+2}\cdot\begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} + 8\cdot(-1)^{2+3}\cdot\begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} =$$

$$= 0 - 10 - 88 = -98.$$

Task 2. Given matrices $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 \end{pmatrix}$, $C = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$.

Find:

a) the product of matrices *AB* or *BC*, or *CB* (if possible). Explain why if it is impossible;

b) the matrix A^{-1}, the inverse matrix for *A*.

Solution:

a) the product of matrices *AB* is possible if the number of columns of matrix *A* equals the number of rows of the matrix *B*. Matrix sizes: $A_{3\times3}$, $B_{2\times4}$, $C_{4\times1}$. So, $A_{3\times3}\cdot B_{2\times4} = |3 \neq 2|$ – multiplication is impossible. $B_{2\times4}\cdot C_{4\times1} = |4 = 4|$ – multiplication is possible. $C_{4\times1}\cdot B_{2\times4} = |1 \neq 2|$ – multiplication is impossible. Matrix multiplication *BC* is a matrix *E*, the number of rows is equal to the number of rows of the matrix *B*, the number of columns equals the number of columns of the matrix *C*: $B_{m\times k}\cdot C_{k\times n} = E_{m\times n}$. Element e_{ij} of the matrix *E* is equal to the sum of products of *i*-th row of the matrix *B* for the *j*-th column of the matrix *C*.

So, $BC = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1\cdot1+1\cdot2+1\cdot3+1\cdot4 \\ 3\cdot1+3\cdot2+3\cdot3+3\cdot4 \end{pmatrix} = \begin{pmatrix} 10 \\ 30 \end{pmatrix}$,

where $\left(B_{2\times4}\cdot C_{4\times1} = E_{2\times1}\right)$.

And $B\cdot C = E = \begin{pmatrix} 10 \\ 30 \end{pmatrix}$;

b) an inverse matrix exists for a square matrix *A*, if the determinant of the matrix is not equal to zero; and an inverse matrix does not exist if $|A|$=0.

The inverse matrix A^{-1} for matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is obtained by the formula $A^{-1} = \dfrac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$, where $|A|$ – the determinant of the matrix A; A_{ij} – the algebraic complement of the elements a_{ij}.

Let us find the determinant of the matrix A:

$$|A| = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & -1 \end{vmatrix} = -30 \neq 0, \text{ so, } A^{-1} \text{ exists.}$$

Let us find the algebraic complements of all elements of the matrix A:

$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & -4 \\ -2 & -1 \end{vmatrix} = -11;$ $A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -4 \\ 3 & -1 \end{vmatrix} = -10;$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -13;$ $A_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 3 \\ -2 & -1 \end{vmatrix} = -8;$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix} = -10;$ $A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -2 \\ 3 & -2 \end{vmatrix} = -4;$

$A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 3 \\ 3 & -4 \end{vmatrix} = -1;$ $A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} = 10;$

$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = 7.$

Let us form A^{-1} by the above formula

$$A^{-1} = \frac{1}{-30}\begin{pmatrix} -11 & -8 & -1 \\ -10 & -10 & 10 \\ -13 & -4 & 7 \end{pmatrix};$$

and then it is need to check:

$$AA^{-1} = A^{-1}A \equiv E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Task 3. Given points $A(7, -9, 3)$ and $B(1, 0, -5)$; and vectors $\bar{b} = (1, 0, 5), \bar{c} = (6, 1, -2)$.

Find:

a) the module (length) of the vector $\overrightarrow{a} = |\overrightarrow{AB}|$ and the midpoint of the segment *AB*;

b) the projection of the vector $\overrightarrow{a}$ on $\overrightarrow{c}$;

c) the area of the parallelogram, obtained from the vectors $\vec{b}$ and $\overrightarrow{c}$;

d) volume of the pyramid constructed from the vectors $\overrightarrow{a}$, $\vec{b}$, $\overrightarrow{c}$.

Solution:

a) for points $A\ (x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ coordinates of the vector $\bar{a} = \overline{AB}$ are obtained by the formula $\bar{a} = \overline{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$.

So, $\bar{a} =$ (1–7, 0– (–9), –5–3) = (–6, 9, –8);

The module (length) of the vector $\bar{a}$: $|\bar{a}| = \sqrt{x_1^2 + y_1^2 + z_1^2}$.

So, $|\overline{AB}| = \sqrt{(-6)^2 + 9^2 + (-8)^2} = \sqrt{181}$;

the midpoint of the segment AB has coordinates

$$C\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = C\left(\frac{7+1}{2}, \frac{0-9}{2}, \frac{-5+3}{2}\right) = C(4, -9/2, -1);$$

b) the projection of the vector $\overrightarrow{a}$ on $\overrightarrow{c}$ is $\Pr_c \vec{a} = \dfrac{\vec{a} \cdot \vec{c}}{|\bar{c}|}$;

where $\bar{a} \cdot \bar{c} = x_1 \cdot x_3 + y_1 \cdot y_3 + z_1 \cdot z_3$ – scalar product of vectors $\bar{a} = (x_1, y_1, z_1)$, $\bar{c} = (x_3, y_3, z_3)$.

If vectors $\overrightarrow{a}$ and $\overrightarrow{c}$ are orthogonal, then $\bar{a} \cdot \bar{c} = 0$.

In our case $\bar{a} \cdot \bar{c} = (-6) \cdot 6 + 9 \cdot 1 + 8 \cdot (-2) = -43$, as $\bar{a} \cdot \bar{c} \neq 0$, so the vectors $\bar{a}$ and $\bar{c}$ are not orthogonal; calculate more

$$\Pr_c \vec{a} = \frac{\vec{a} \cdot \vec{c}}{|\bar{c}|} = \frac{-43}{\sqrt{6^2 + 1^2 + (-2)^2}} = \frac{-43}{41} \approx -1{,}049;$$

c) the area of the parallelogram, obtained from the vectors $\bar{b} = (x_2, y_2, z_2)$, $\bar{c} = (x_3, y_3, z_3)$, is the module of the vector, received from their vector product: $|\vec{d}| = |\vec{b} \times \vec{c}|$; at first find the coordinates:

$$\bar{b} \times \bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = \begin{vmatrix} y_2 & z_2 \\ y_3 & z_3 \end{vmatrix} \cdot \bar{i} - \begin{vmatrix} x_2 & z_2 \\ x_3 & z_3 \end{vmatrix} \cdot \bar{j} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \cdot \bar{k} = \left(\begin{vmatrix} y_2 & z_2 \\ y_3 & z_3 \end{vmatrix}, -\begin{vmatrix} x_2 & z_2 \\ x_3 & z_3 \end{vmatrix}, \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \right);$$

$$\bar{b} \times \bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 0 & 5 \\ 6 & 1 & -2 \end{vmatrix} = \begin{vmatrix} 0 & 5 \\ 1 & -2 \end{vmatrix} \cdot \bar{i} - \begin{vmatrix} 1 & 5 \\ 6 & -2 \end{vmatrix} \cdot \bar{j} + \begin{vmatrix} 1 & 0 \\ 6 & 1 \end{vmatrix} \cdot \bar{k} = (-5, 32, 1);$$

further find module of the vector: $\left|\vec{d}\right| = \left|\vec{b}\times\vec{c}\right| = |(-5,32,1)| = \sqrt{(-5)^2+32^2+1^2} = \sqrt{1050}$;

d) the volume of pyramid constructed from vectors $\bar{a}$, $\bar{b}$, $\bar{c}$, can be found from a module of their mixed product

$$V = \frac{1}{6}\left|\overline{a}\,\overline{b}\,\overline{c}\right| = \frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} =$$

$$= \frac{1}{6}\begin{vmatrix} -6 & 9 & 8 \\ 1 & 0 & 5 \\ 6 & 1 & -2 \end{vmatrix} = \frac{1}{6}(0+270+8-0+30+18) = \frac{1}{6}\cdot 326 = 54\frac{1}{3}.$$

As $\overline{a}\,\overline{b}\,\overline{c} \neq 0$, these vectors are not coplanar.

Task 4. Given points $A_1(4, -2)$, $A_2(8, 1)$ on a plane and the equation of the line L_1: $-x + 4y + 5 = 0$.

Write the equation of a straight line:

a) $L_2 = (A_1A_2)$ – passing through these points;

b) L_2 – as a general equation of straight line;

c) L_2 – in the form of equation of straight line with slope;

d) L_2 – in the form of equation of straight line in segments;

e) L_3, passing through the point A_2 and perpendicular to L_1.

Solution:

a) the equation of the straight line passing through two points $M_1(x_1, y_1)$, $M_2(x_2, y_2)$, is obtained by the formula $\dfrac{x-x_1}{x_2-x_1} = \dfrac{y-y_1}{y_2-y_1}$.

So, equation of the straight line L_2 is $\dfrac{x-4}{8-4} = \dfrac{y-(-2)}{1-(-2)}$ or $\dfrac{x-4}{4} = \dfrac{y+2}{3}$;

b) let us write the equation of the straight line L in the general form $Ax + By + C = 0$:

$\dfrac{x-4}{4} = \dfrac{y+2}{3} \rightarrow 3x-12 = 4y+8 \rightarrow 3x-4y-20=0$, ($A = 3$, $B = -4$, $C = -20$).

The geometric meaning of the coefficients: A, B – are coordinates of the normal (perpendicular) vector of the straight line L, ie $\overline{N} = (A, B) = (3, -4) \perp L_2$;

c) L_2 in the form of equation of straight line with slope $y = kx + m$:

$$\frac{x-4}{4}=\frac{y+2}{3} \to \frac{3}{4}(x-4)=y+2 \to y=\frac{3}{4}x-5 \text{ , } (k = \frac{3}{4}, m = -5);$$

d) L_2 in the form of equation of straight line in segments $\frac{x}{a}+\frac{y}{b}=1$:

$$3x-4y = 20 \to \frac{x}{20/3}+\frac{y}{-5}=1, \ (a = 20/3, b = -5);$$

e) the equation of the straight line passing through the point (x_1, y_1) and parallel to the vector $\bar{b}=(m,n)$, has the form: $\frac{x-x_1}{m}=\frac{y-y_1}{n}$.

If vector $\overline{N}_1 = (A;B) = (-1;4)$ is perpendicular to the straight line L_1 and the straight line L_3 is also perpendicular to L_1, then we find in our case that the directional vector for the straight line L_3 is $\bar{b}=(m,n)=\overline{N}_1=(-1;4)$ and the equation has the form $L_3 \perp L_1:$ $\frac{x-8}{-1}=\frac{y-1}{4}$.

Task 5. Given points $A_1(1, 2, -1)$, $A_2(3, 3, 2)$, $A_3(2, -3, 7)$. It is required to:

a) write the equation of the plane $P_1 = (A_1A_2A_3)$;

b) write P_1 as a general equation of a plane;

c) write P_1 as equation of a plane in segments;

d) write P_1 in a general equation of a plane, passing through the point A_1;

e) make a canonical equation of the straight line $L_1 = (A_2A_3)$;

f) write a parametric equation of the straight line L_1;

g) write the equation of the straight line $L_2 = (A_1N)$ perpendicular to the plane P_1.

Solution:

a) the equation of the plane passing through the points $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$, $M_3(x_3, y_3, z_3)$ has the form: $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$.

Here P_1: $\begin{vmatrix} x-1 & y-2 & z+1 \\ 3-1 & 3-2 & 2+1 \\ 2-1 & -3-2 & 7+1 \end{vmatrix}=0 \to \begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & 1 & 3 \\ 1 & -5 & 8 \end{vmatrix}=0$ (*);

b) expanding the determinant in the right-hand side of (*), we obtain the equation of the plane P_1 in the general form $Ax + By + Cz + D = 0$:

$$23x - 13y - 11z - 8 = 0, \ (A = 23, B = -13, C = -11, D = -8).$$

The geometrical sense of coefficients: A, B, C – are coordinates of a normal (perpendicular) vector of the plane, ie vector $\overline{N} = (23,-13,-11) \perp P_1$;

c) let us move free term –8 in the general equation of plane on the right side and divide both sides by 8. Obtain the equation of the plane in segments $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$:

$$\frac{x}{8/23}+\frac{y}{-8/13}+\frac{z}{-8/11}=1,$$

where a = 8/23, b= –8/13, c = –8/11 – are sizes of the segments cut off by the plane on coordinate axes, considering from the origin of coordinates;

d) let us expand the determinant on the left side of (*) along the first row

$$(x-1)\cdot\begin{vmatrix} 1 & 3 \\ -5 & 8 \end{vmatrix} - (y-2)\cdot\begin{vmatrix} 2 & 3 \\ 1 & 8 \end{vmatrix} + (z+1)\cdot\begin{vmatrix} 2 & 1 \\ 1 & -5 \end{vmatrix} = 0 \to$$

$$\to 23\cdot(x-1) - 13\cdot(y-2) - 11\cdot(z+1) = 0.$$

We obtain the equation of plane in the form $A(x\text{-}x_0) + B(y\text{-}y_0) + C(z\text{-}z_0) = 0$ $(x_0 = 1, y_0 = 2, z_0 = -1)$;

e) the canonical equation of the straight line passing through two points $M_1(x_1, y_1, z_1)$ and $M_2(x_2, y_2, z_2)$, has the form $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$. So, the equation of the straight line L_1 has the form $\frac{x-3}{2-3}=\frac{y-3}{-3-3}=\frac{z-2}{7-2}$ or $\frac{x-3}{-1}=\frac{y-3}{-6}=\frac{z-2}{5}$;

f) the equation of the straight line in the form $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ is called canonical, where $a = (m, n, p)$ – is the direction vector. In the previous section we have already found the canonical equations of straight lines. The equations of the straight line in the form $\begin{cases} x = mt + x_0 \\ y = nt + y_0 \\ z = pt + z_0 \end{cases}$ are called parametric. To get the parametric equations of the straight line, we equate the canonical equations to the parameter t, and from these equalities we find x, y, z.

$$L_1: \quad \frac{x-3}{-1}=\frac{y-3}{-6}=\frac{z-2}{5}= t \to \begin{cases} \frac{x-3}{-1} = t \to x = -t+3 \\ \frac{y-3}{-6} = t \to y = -6t+3 \\ \frac{z-2}{5} = t \to z = 5t+2 \end{cases}.$$

So, the parametric equations of $L_1:\begin{cases} x=-t+3 \\ y=-6t+32 \\ z=5t+2 \end{cases}$;

g) as $(A_1N)\perp P_1$, so vector $\overline{N}=(23,-13,-11)\perp P_1$ is the direction vector of the straight line $L_2=(A_1N)$. Then the canonical equation of the straight line $L_2=(A_1N)$ has the form: $\frac{x-1}{23}=\frac{y-2}{-13}=\frac{z+1}{-11}$.

Task 6. Solve the system $\begin{cases} x+2y=1 \\ -3x+4y=3 \end{cases}$:

a) by Cramer's method;
b) by matrix method (using the inverse matrix).

Solution:

a) the solution of the system $\begin{cases} a_{11}x+a_{12}y=b_1 \\ a_{21}x+a_{22}y=b_2 \end{cases}$ by Cramer's rule has the form $x=\frac{\Delta_1}{\Delta}$, $y=\frac{\Delta_2}{\Delta}$, where $\Delta=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ – determinant of the system, $\Delta_1=\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}$, $\Delta_2=\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$ – auxiliary determinants obtained from the determinant of the system by replacing the first and second columns by column of free terms.

In this case $\Delta=\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix}=10$, $\Delta_1=\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}=-2$, $\Delta_2=\begin{vmatrix} 1 & 1 \\ -3 & 3 \end{vmatrix}=6$.

So, $x=-1/5$, $y=3/5$.

The answer can be written as a vector: $\overline{X}=\begin{pmatrix} -1/5 \\ 3/5 \end{pmatrix}$;

b) in matrix form solution of the system $\begin{cases} a_{11}x+a_{12}y=b_1 \\ a_{21}x+a_{22}y=b_2 \end{cases}$ is written as:

$X=A^{-1}\cdot B$, where $X=\begin{pmatrix} x \\ y \end{pmatrix}$ – column matrix of unknowns, $B=\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ – column matrix of free terms, A^{-1} – inverse matrix for the matrix of the system $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$.

Let us find the inverse matrix (look example 2b) for the matrix of the system $A=\begin{pmatrix}1 & 2\\ -3 & 4\end{pmatrix}$. As the determinant of the system $|A|=\Delta=10\neq 0$, then the inverse matrix exists and is equal to $A^{-1}=\frac{1}{10}\begin{pmatrix}4 & -2\\ 3 & 1\end{pmatrix}$.

Then $\begin{pmatrix}x\\ y\end{pmatrix}=X=A^{-1}\cdot B=\frac{1}{10}\begin{pmatrix}4 & -2\\ 3 & 1\end{pmatrix}\cdot\begin{pmatrix}1\\ 3\end{pmatrix}=\begin{pmatrix}-1/5\\ 3/5\end{pmatrix}$. Or the answer in the usual form: $x = -1/5$, $y = 3/5$.

Task 7. Solve the system of equations. $\begin{cases}x_1-x_2+5x_3=3\\ 2x_1-3x_2+4x_3=2\\ 3x_1-3x_2-x_3=-7\end{cases}$:

a) by Cramer's method;
b) by matrix method (using the inverse matrix).

Solution:

a) the coefficient matrix of the system is $A=\begin{pmatrix}1 & -1 & 5\\ 2 & -3 & 4\\ 3 & -3 & -1\end{pmatrix}$, column of free terms – $B=\begin{pmatrix}3\\ 2\\ -7\end{pmatrix}$.

Then the solution of the system by Cramer's rule has the form:

$$x_1=\frac{\Delta_1}{\Delta},\, x_2=\frac{\Delta_2}{\Delta},\, x_3=\frac{\Delta_3}{\Delta},$$

where $\Delta=\begin{vmatrix}1 & -1 & 5\\ 2 & -3 & 4\\ 3 & -3 & -1\end{vmatrix}=3-12-30+45+12-2=16$ is the determinant of the system,

$$\Delta_1=\begin{vmatrix}3 & -1 & 5\\ 2 & -3 & 4\\ -7 & -3 & -1\end{vmatrix}=-64,\quad \Delta_2=\begin{vmatrix}1 & -3 & 5\\ 2 & 2 & 4\\ 3 & -7 & -1\end{vmatrix}=-32,\quad \Delta_3=\begin{vmatrix}1 & -1 & -3\\ 2 & -3 & 2\\ 3 & -3 & -7\end{vmatrix}=16$$

– auxiliary determinants obtained from the determinant of the system by replacing the first, second and third column by column of free terms. Finally, we obtain the solution by Cramer's method:

$x_1 = \frac{\Delta_1}{\Delta} = \frac{-64}{16} = -4, \quad x_2 = \frac{\Delta_2}{\Delta} = \frac{-32}{16} = -2, \quad x_3 = \frac{\Delta_3}{\Delta} = \frac{16}{16} = 1$ or

$$\overline{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix};$$

b) in matrix form the solution of the system is written:

$$\overline{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 5 \\ 2 & -3 & 4 \\ 3 & -3 & -1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 3 \\ 2 \\ -7 \end{pmatrix}.$$

Let us find the inverse matrix (see example 2b) , as $|A| = \Delta = 16 \neq 0$, the inverse matrix exists and has the form

$$A^{-1} = \begin{pmatrix} 1 & -1 & 5 \\ 2 & -3 & 4 \\ 3 & -3 & -1 \end{pmatrix}^{-1} = \frac{1}{16}\begin{pmatrix} 15 & -16 & 11 \\ 14 & -16 & 6 \\ 3 & 0 & -1 \end{pmatrix}.$$

So,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^{-1} \cdot B = \frac{1}{16}\begin{pmatrix} 15 & -16 & 11 \\ 14 & -16 & 6 \\ 3 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -7 \end{pmatrix} = \frac{1}{16}\begin{pmatrix} 45-32-77 \\ 42-32-42 \\ 9+7 \end{pmatrix} = \frac{1}{16}\begin{pmatrix} -64 \\ -32 \\ 16 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}.$$

Task 8. Given point $A(3, -7)$; radius of the circle $R = 6$; $a = 2$; $b = 3$ – semi-axes of curves; D: $y = -3$ – equation of the straight line. It is required to:

a) write the equation of a circle with center A and radius R;

b) write the canonical equation of an ellipse with semiaxes $a = 2$ and $b = 3$. Find the coordinates of it's foci and the eccentricity;

c) write the canonical equation of the hyperbola with the real semiaxis $a = 2$ and imaginary semiaxis $b = 3$. Find the coordinates of it's foci, eccentricity, equations of the asymptotes;

d) write the canonical equation of the parabola with vertex at the origin and if D: $y = -3$ is its directrix. Find the coordinates of its focus, eccentricity;

e) make the drawing of ellipse, hyperbola, parabola.

Solution:

a) the equation of a circle centered at (x_0, y_0) with the radius R has the form $(x-x_0)^2 + (y-y_0)^2 = R^2$, So, in our version: $(x-3)^2 + (y+7)^2 = 36$;

b) the canonical equation of an ellipse with semiaxes a and b has the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; So, in our version: $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

If $a > b$, then $c = \sqrt{a^2 - b^2}$ and points $F_1(-c,0),\ F_2(c,0)$ are foci of the ellipse, and the eccentricity ε of the ellipse is $\varepsilon = \dfrac{c}{a}$.

If $b < a$, then $c = \sqrt{b^2 - a^2}$ and $F_1(0,-c),\ F_2(0,c)$, $\varepsilon = \dfrac{c}{b}$.

Since $b < a$ in our version then $c = \sqrt{b^2 - a^2} = \sqrt{9-4} = \sqrt{5}$ and the eccentricity equals $\varepsilon = \dfrac{c}{b} = \dfrac{\sqrt{5}}{3}$, ($\varepsilon < 1$). Foci lie on the *y*-axis: $F_1(0,-\sqrt{5}),\ F_2(0,\sqrt{5})$.

Let us draw the ellipse on the coordinate plane (see Figure 1), where A_1, A_2, B_1, B_2 – vertices of the ellipse.

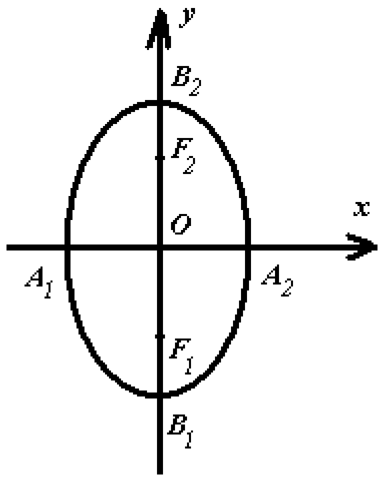


Figure 1

c) the canonical equation of hyperbola with real semiaxis a, imaginary semiaxis b has the form $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$; with the real semiaxis b and the imaginary semiaxis *a*: $-\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = -1$. For hyperbola with real semiaxis *a*, eccentricity equals $\varepsilon = \dfrac{c}{a}$, where $c = \sqrt{a^2 + b^2}$; asymptotes of the hyperbola equation have the form $y = \pm\dfrac{b}{a}x$; foci are points $F_1(-c,0),\ F_2(c,0)$, located on the real axis.

By hypothesis *a* =2, *b* =3, so the canonical equation of the hyperbola with the real semiaxis *a* is $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$. For it half-foci distance $c = \sqrt{4+9} = \sqrt{13}$;

eccentricity equals $\varepsilon = \frac{c}{a} = \frac{\sqrt{13}}{2}, (\varepsilon > 1)$; foci: $F_1(-\sqrt{13}, 0)$, $F_2(\sqrt{13}, 0)$; equations of the asymptotes: $y = \pm\frac{3}{2}x$.

It is easy to construct a hyperbola as follows: construct a rectangle with sides $x = \pm a,\ y = \pm b$ (in our case $x = \pm 2,\ y = \pm 3$). The diagonals of a rectangle are the asymptotes of the hyperbola, the point of intersection of the rectangle with the real axis of the hyperbola – vertices of the hyperbola (see Figure 2):

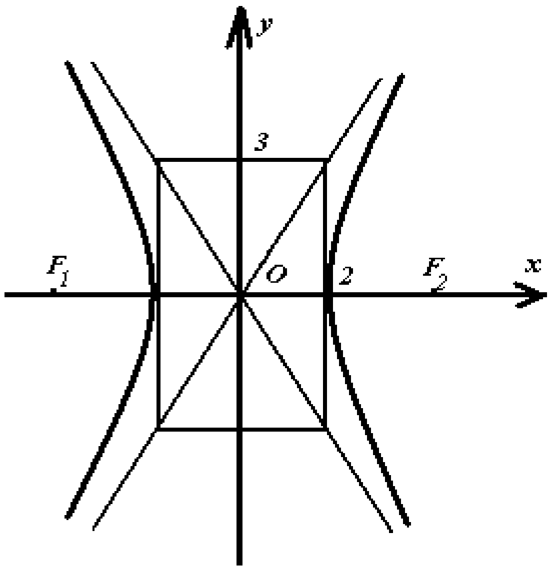


Figure 2

d) by hypothesis, if the directrix of the parabola *y* = –*p*/2, then the axis of symmetry of the parabola with vertex at the origin is the *y*-axis. Hence, its canonical equation is $x^2 = 2py$; if the directrix of the parabola *x* = –*p*/2, then the axis of symmetry of the parabola with vertex at the origin is the *x*-axis. Hence, its canonical equation has the form $y^2 = 2px$. As the directrix of the parabola has the equation *y* = –3, then $-\frac{p}{2} = -3 \to p = 6$ and the equation of the parabola is: $x^2 = 2 \cdot 6 \cdot y \to x^2 = 12y$. The focus of the parabola is the point $F(0, \frac{p}{2})$, lying on the axis of symmetry. In our case, the focus is $F(0,3)$. Let us construct the parabola (see Figure 3).

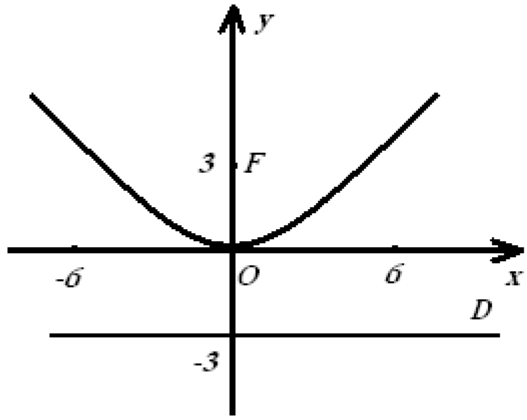


Figure 3

Task 9. Determine the type (name) of the second order surface and make a schematic drawing:

a) $-\frac{x^2}{12}+\frac{y^2}{2}+\frac{z^2}{4}=1$; b) $\frac{x^2}{3}+\frac{y^2}{2}-\frac{z^2}{4}=0$.

Solution:

a) $-\frac{x^2}{12}+\frac{y^2}{2}+\frac{z^2}{4}=1$ is the canonical equation of one-sheeted hyperboloid with the symmetry axis *Ox*. Here its schematic drawing (see Figure 4);

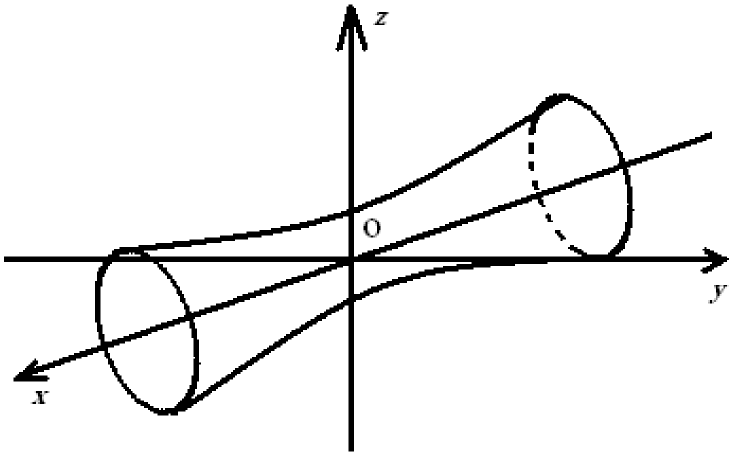


Figure 4

b) $\frac{x^2}{3}+\frac{y^2}{2}-\frac{z^2}{4}=0$ is the canonical equation of the cone of the second order with the symmetry axis *Oz* and vertex at the origin. Here its schematic drawing (see Figure 5);

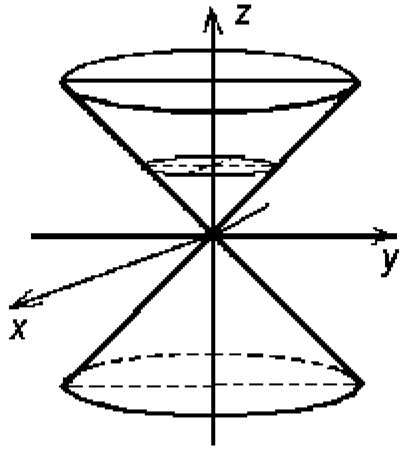


Figure 5

Task 10. Lead to the canonical form the equation of the second order $9x^2 - 16y^2 + 18x - 64y - 19 = 0$ and construct this curve.

Solution: the equation of the form

$$\frac{(x-x_0)^2}{a^2} \pm \frac{(y-y_0)^2}{b^2} = 1$$

defines, respectively, the ellipse or the hyperbola.

The equation of the form:

$$(x-x_0)^2 = 2p(y-y_0), \quad (y-y_0)^2 = 2p(x-x_0)^2$$

defines the parabola. These curves have symmetry center (for the ellipse and hyperbola) or vertex of the parabola at the point $C(x_0, y_0)$. To determine the type of the curve, we apply the method of allocating of perfect squares:

$$(9x^2 + 18x) + (-16y^2 - 64y) - 19 = 0 \rightarrow 9(x^2 + 2x) - 16(y^2 + 4y) - 19 = 0.$$

Supplement the terms containing *x*, and the terms containing *y*, to complete the squares:

$$9(x^2 + 2x + 1 - 1) - 16(y^2 + 4y + 4 - 4) - 19 = 0,$$

$$9(x^2 + 2x + 1) - 16(y^2 + 4y + 4) - 19 - 9 \cdot 1 - 16 \cdot (-4) = 0,$$

$$9(x+1)^2 - 16(y+2)^2 = -36,$$

$$\frac{(x+1)^2}{-36/9} + \frac{(y+2)^2}{-36/(-16)} = 1, \quad -\frac{(x+1)^2}{4} + \frac{(y+2)^2}{9/4} = 1,$$

so we have the hyperbola with center $C(-1; -2)$, real semi-axis $b = 3/2$ (along the axis Oy) and the imaginary semi-axis $a = 2$. Let us construct its schematic drawing (see Figure 6)

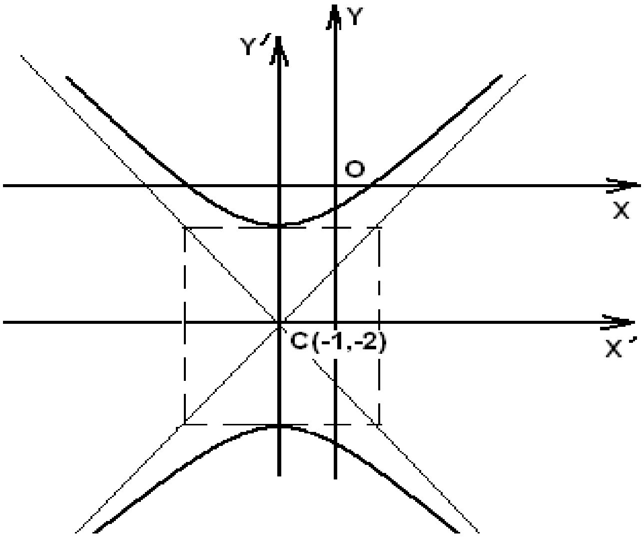


Figure 6

Task 11. Given complex numbers:

$$z_1 = 9 - 9i; \quad z_2 = 9(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}).$$

It is required to find:

a) the module of the complex numbers z_1;

b) the argument of the complex number z_1;

c) the representation of the complex number z_1 in the trigonometric and exponential forms;

d) the sum of complex numbers z_1 and z_2 analytically and graphically;

e) $(z_2)^5$;

f) multiplication of $z_1 \cdot z_2$ in trigonometric form;

g) all complex roots $\sqrt[3]{z_2}$ using De Moivre formula.

Solution:

a) if the complex number in algebraic form is:

$$z = \alpha + i\beta = (\alpha, \beta) \Rightarrow z_1 = 9 - 9i = (9, -9), \textit{ where } \quad \alpha = 9; \beta = -9,$$

then the module of the complex number z_1:

$$|z| = \sqrt{\alpha^2 + \beta^2} \quad or \quad |z_1| = \sqrt{9^2 + (-9)^2} = 9\sqrt{2};$$

b) the argument of the complex number z_1:

$$\varphi = \arg(z) = arctg\frac{\beta}{\alpha} \quad \Rightarrow \varphi_1 = \arg(z_1) = arctg\frac{-9}{9} = -arctg1 = -\frac{\pi}{4} \; (\alpha = 9 > 0; \beta = -9 < 0);$$

c) in the trigonometric and exponential form complex number has the form:

$$z = \alpha + i\beta = |z|(\cos\varphi + \sin\varphi) = |z|e^{i\varphi};$$

$$z_1 = 9 - 9i = 9\sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})) = 9\sqrt{2}e^{-i\frac{\pi}{4}};$$

d) addition of complex numbers z_1 и z_2:

$$z_2 = 9(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = 9(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}),$$

$$z_1 + z_2 = 9 - 9i + 9(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) = (9 + 9\frac{\sqrt{2}}{2}) + i(9\frac{\sqrt{2}}{2} - 9) \approx 15{,}36 - 2{,}63i;$$

e) for calculating $(z_2)^5$ we use De Moivre formula:

$$z^n = |z|^n(\cos n\varphi + \sin n\varphi) = |z|^n e^{i\varphi n};$$

$$z_2^5 = \left(9\sqrt{2}\right)^5(\cos(\frac{5\pi}{4}) + i\sin(\frac{5\pi}{4})) = \left(9\sqrt{2}\right)^5(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2});$$

f) the multiplication of $z_1 \cdot z_2$ in algebraic form is:

$$z_1 \cdot z_2 = (9 - 9i)\cdot 9(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) = 81\frac{\sqrt{2}}{2} + i81\frac{\sqrt{2}}{2} - i81\frac{\sqrt{2}}{2} - i^2 81\frac{\sqrt{2}}{2} =$$

$$= 81\frac{\sqrt{2}}{2} - (\sqrt{-1})^2 81\frac{\sqrt{2}}{2} = 81\sqrt{2};$$

but multiplication $z_1 \cdot z_2$ in trigonometric form is easily calculated:

$$z_1 \cdot z_2 = |z_1|\cdot|z_2|(\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)),$$

$$z_1 \cdot z_2 = 9\sqrt{2}\cdot 9(\cos(-\frac{\pi}{4} + \frac{\pi}{4}) + i\sin(-\frac{\pi}{4} + \frac{\pi}{4})) = 81\sqrt{2}(\cos 0 + i\sin 0) = 81\sqrt{2};$$

g) all complex roots of the n-th degree are calculated by De Moivre formula:

$$\sqrt[n]{z} = \sqrt[n]{|z|}\left(\cos\frac{\varphi + 2\pi k}{n} + i\sin\frac{\varphi + 2\pi k}{n}\right), \quad k = 0,1,2,3,\ldots,n-1,$$

$$\sqrt[3]{z_2} = \sqrt[3]{9\sqrt{2}}\left(\cos\frac{\frac{\pi}{4} + 2\pi k}{3} + i\sin\frac{\frac{\pi}{4} + 2\pi k}{3}\right), k = 0,1,2.$$

Write all the roots obtained for all k:

$$k = 0: \quad z_1 = \sqrt[3]{9\sqrt{2}}(\cos\frac{\frac{\pi}{4}}{3} + i\sin\frac{\frac{\pi}{4}}{3}) = \sqrt[3]{9\sqrt{2}}(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}));$$

$$k = 1: \quad z_1 = \sqrt[3]{9\sqrt{2}}(\cos\frac{\frac{\pi}{4} + 2\pi}{3} + i\sin\frac{\frac{\pi}{4} + 2\pi}{3})) = \sqrt[3]{9\sqrt{2}}(\cos\frac{9\pi}{12} + i\sin\frac{9\pi}{12}));$$

$$k = 2: \quad z_1 = \sqrt[3]{9\sqrt{2}}(\cos\frac{\frac{\pi}{4} + 4\pi}{3} + i\sin\frac{\frac{\pi}{4} + 4\pi}{3})) = \sqrt[3]{9\sqrt{2}}(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12})).$$

On the graph all the cube roots are located at the vertices of an equilateral triangle.

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Signed in print _______
Circulation 25 copies
Volume 2,1 ed. sheets

Format 60x84 1/16
Typographical paper
Ordering_____ Price 1050

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