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ALMATY UNIVERSITY OF
POWER ENGINEERING AND
TELECOMMUNICATIONS

The department of

Higher mathematics

MATHEMATICS 1

Methodological Guidelines and Tasks
for carrying out the calculation-graphical work for students of specialties
5B071700 «Heat power engineering»,
5B071800 «Electrical power engineering»,

5B071900 «Radio engineering, electronics and telecommunications»
Part 2

Almaty 2014

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Methodological Guidelines and Tasks for carrying out the calculation-graphical work contain sections of the first semester program of course of mathematics for students of AUPET: introduction to analysis, differential calculus of one-variable functions.

The basic theoretical questions of the program and the solution of an exemplary embodiment are given.

Tables 12, figures 2, bibl. 3

Reviewer: candidate of sciences in philology, V.S. Kozlov

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Introduction

Mathematics plays an important role in engineering studies. It is not only a quantitative calculation device, but also mathematics is the means of accurate research and extremely precise formulation of concepts and problems. Mathematical methods have become an integral part of any technical discipline. These facts lead to the need to strengthen the applied orientation of mathematics course and improve basic mathematical training. CGW is performed in a separate thin notebook. In the number of each task the second digit indicates the variant.

Calculation-graphical work. Introduction to analysis. Differential calculus of one-variable function

Purpose: teach students to possess classical mathematical methods of investigation of functions. Give students the skills of finding limits, derivatives and differentials of the first and higher orders for composite and parametrically defined functions, and be able to use the gained knowledge in the study of further mathematics course and sections of other special disciplines.

Theoretical quastions

1. Function of one variable. Properties.
2. Composite function, parametrically defined function.
3. Numerical sequences and their limits.
4. Limit of function. Remarkable limits.
5. Continuity of the function. Points of discontinuity and their classification.
6. Derivative of the function. Differentiation of parametrically defined function.
7. Disclosure of the uncertainties. L'Hopital rule.
8. Signs of monotonicity of the function. Extremum of function.
9. Convexity, concavity of the function graph. Inflection point.
10. Asymptotes of graph of the function.
11. Investigation of function using the derivative. Construction of the graph of function.

The first level calculated tasks

Task 1. Find the limits

1.1 $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x^2-12x+20}$	1.2 $\lim\limits_{x\to 0}\dfrac{x^3-x^2+2x}{x^2+x}$	1.3 $\lim\limits_{x\to 3}\dfrac{6+x-x^2}{x^3-27}$
1.4 $\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{3x^2-x-2}$	1.5 $\lim\limits_{x\to 2}\dfrac{2x^2-7x+6}{x^2-5x+6}$	1.6 $\lim\limits_{x\to 3}\dfrac{12-x-x^2}{x^3-27}$

1.7 $\lim\limits_{x\to 1/3}\dfrac{3x^2+2x-1}{27x^3-1}$	1.8 $\lim\limits_{x\to -1}\dfrac{x^2-4x-5}{x^2-2x+6}$	1.9 $\lim\limits_{x\to -1}\dfrac{3x^2+2x-1}{-x^2+x+2}$
1.10 $\lim\limits_{x\to 3}\dfrac{3x^2-11x+6}{2x^2-5x-3}$	1.11 $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2+x-6}$	1.12 $\lim\limits_{x\to -1}\dfrac{x^2-x-2}{x^3+1}$
1.13 $\lim\limits_{x\to 4}\dfrac{x^2-16}{x^2+x-20}$	1.14 $\lim\limits_{x\to -3}\dfrac{4x^2+11x-3}{x^2+2x-3}$	1.15 $\lim\limits_{x\to 3}\dfrac{3x^2-7x-6}{2x^2-7x+3}$
1.16 $\lim\limits_{x\to -2}\dfrac{4x^2+7x-2}{3x^2+8x+4}$	1.17 $\lim\limits_{x\to -1}\dfrac{5x^2+4x-1}{3x^2+x-2}$	1.18 $\lim\limits_{x\to -1}\dfrac{x^2-4x-5}{3x^2+x-2}$
1.19 $\lim\limits_{x\to -1}\dfrac{7x^2+4x-3}{2x^2+3x+1}$	1.20 $\lim\limits_{x\to 4}\dfrac{3x^2-x-64}{x^2-x-12}$	1.21 $\lim\limits_{x\to 2}\dfrac{2x^2-9x+10}{x^2+3x-10}$
1.22 $\lim\limits_{x\to 1}\dfrac{4x^2+x-5}{x^2-2x+1}$	1.23 $\lim\limits_{x\to 2}\dfrac{-5x^2+11x-2}{3x^2-x-10}$	1.24 $\lim\limits_{x\to 7}\dfrac{x^2-5x-14}{2x^2-9x-35}$
1.25 $\lim\limits_{x\to 5}\dfrac{3x^2-6x-45}{2x^2-3x-35}$	1.26 $\lim\limits_{x\to -3}\dfrac{x^2+8x+15}{x^2-6x-27}$	1.27 $\lim\limits_{x\to -5}\dfrac{x^2-2x-35}{2x^2+11x+5}$
1.28 $\lim\limits_{x\to -8}\dfrac{2x^2+15x-8}{3x^2+25x+8}$	1.29 $\lim\limits_{x\to 4}\dfrac{3x^2-2x-40}{x^2-3x-4}$	1.30 $\lim\limits_{x\to -3}\dfrac{2x^2+5x-3}{3x^2+10x+3}$

Task 2. Find the limits

2.1 $\lim\limits_{x\to\infty}\dfrac{3x^3-5x^2+2}{2x^3+5x^2-x}$	2.2 $\lim\limits_{x\to\infty}\dfrac{x^5-2x+4}{2x^4+3x^2+1}$	2.3 $\lim\limits_{x\to\infty}\dfrac{2x^2+3x-5}{7x^3-2x^2+1}$
2.4 $\lim\limits_{x\to\infty}\dfrac{4x^3+7x}{2x^3-4x^2+5}$	2.5 $\lim\limits_{x\to\infty}\dfrac{3x^4+x-5}{2x^2+x+7}$	2.6 $\lim\limits_{x\to -\infty}\dfrac{3x^3-7x+2}{x^4+2x-4}$
2.7 $\lim\limits_{x\to\infty}\dfrac{5x^4-3x^2+7}{x^4+2x^3+1}$	2.8 $\lim\limits_{x\to -\infty}\dfrac{3x^2+7x-4}{x^5+2x-1}$	2.9 $\lim\limits_{x\to\infty}\dfrac{7x^4-3x+4}{3x^2-2x+1}$
2.10 $\lim\limits_{x\to\infty}\dfrac{7x^3-2x^2+4x}{2x^3+5}$	2.11 $\lim\limits_{x\to\infty}\dfrac{3x-x^6}{x^2-2x+5}$	2.12 $\lim\limits_{x\to -\infty}\dfrac{2x^2-x+7}{3x^4-5x^2+10}$
2.11 $\lim\limits_{x\to\infty}\dfrac{x^3-4x^2+28x}{5x^3+3x^2-1}$	2.14 $\lim\limits_{x\to\infty}\dfrac{2x^3+7x-1}{3x^4+2x+5}$	2.15 $\lim\limits_{x\to -\infty}\dfrac{4x^3-2x^2+x}{3x^2-x}$
2.16 $\lim\limits_{x\to\infty}\dfrac{3x^2+10x+3}{2x^2+5x-3}$	2.17 $\lim\limits_{x\to -\infty}\dfrac{2x^3-7x^2+4}{x^4+5x-1}$	2.18 $\lim\limits_{x\to\infty}\dfrac{3x^4-2x+1}{3x^2+2x-5}$
2.19 $\lim\limits_{x\to\infty}\dfrac{-3x^4+x^2+x}{x^4+3x-2}$	2.20 $\lim\limits_{x\to -\infty}\dfrac{3x^6-5x^2+2}{2x^3+4x-5}$	2.21 $\lim\limits_{x\to\infty}\dfrac{2x^2-5x+2}{x^4+3x^2-9}$

2.22 $\lim\limits_{x\to\infty}\frac{2x^2+7x+3}{5x^2-3x+4}$	2.23 $\lim\limits_{x\to\infty}\frac{x^7+5x^2-4x}{3x^2+11x-7}$	2.24 $\lim\limits_{x\to-\infty}\frac{5x^2-4x+2}{4x^3+2x-5}$
2.25 $\lim\limits_{x\to\infty}\frac{-x^2+3x+1}{3x^2+x-5}$	2.26 $\lim\limits_{x\to-\infty}\frac{7x^2+6x+9}{1+4x-x^3}$	2.27 $\lim\limits_{x\to\infty}\frac{2x^3-3x^2+2x}{x^2+7x+1}$
2.28 $\lim\limits_{x\to\infty}\frac{x^3-3x^2+10}{7x^3+2x+1}$	2.29 $\lim\limits_{x\to\infty}\frac{3x^4+x^2-6}{2x^2+3x+1}$	2.30 $\lim\limits_{x\to-\infty}\frac{3x^2-7x+5}{4x^5-3x^3-1}$

Task 3. Find the limits

3.1 $\lim\limits_{x\to3}\frac{x^2+x-12}{\sqrt{x-2}-\sqrt{4-x}}$	3.2 $\lim\limits_{x\to-4}\frac{\sqrt{x+12}-\sqrt{4-x}}{x^2+2x-8}$	3.3 $\lim\limits_{x\to-3}\frac{\sqrt{x+10}-\sqrt{4-x}}{2x^2-x-21}$
3.4 $\lim\limits_{x\to-2}\frac{\sqrt{2-x}-\sqrt{x+6}}{x^2-x-6}$	3.5 $\lim\limits_{x\to1}\frac{\sqrt{3+2x}-\sqrt{4+x}}{3x^2-4x+1}$	3.6 $\lim\limits_{x\to2}\frac{x^2-3x+2}{\sqrt{5-x}-\sqrt{x+1}}$
3.7 $\lim\limits_{x\to-1}\frac{3x^2+4x+1}{\sqrt{x+3}-\sqrt{5+3x}}$	3.8 $\lim\limits_{x\to4}\frac{2x^2-9x+4}{\sqrt{5-x}-\sqrt{x-3}}$	3.9 $\lim\limits_{x\to5}\frac{\sqrt{2x+1}-\sqrt{x+6}}{2x^2-7x-15}$
3.10 $\lim\limits_{x\to-5}\frac{\sqrt{3x+17}-\sqrt{2x+12}}{x^2+8x+15}$	3.11 $\lim\limits_{x\to0}\frac{\sqrt{x^2+2}-\sqrt{2}}{\sqrt{x^2+1}-1}$	3.12 $\lim\limits_{x\to0}\frac{\sqrt{7-x}-\sqrt{7+x}}{\sqrt{7x}}$
3.13 $\lim\limits_{x\to0}\frac{3x}{\sqrt{x+1}-\sqrt{1-x}}$	3.14 $\lim\limits_{x\to4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$	3.15 $\lim\limits_{x\to-1}\frac{\sqrt{5+x}-2}{\sqrt{8-x}-3}$
3.16 $\lim\limits_{x\to5}\frac{\sqrt{x+4}-3}{\sqrt{x-1}-2}$	3.17 $\lim\limits_{x\to7}\frac{\sqrt{x-3}-2}{\sqrt{x+2}-3}$	3.18 $\lim\limits_{x\to3}\frac{\sqrt{4x-3}-3}{x^2-9}$
3.19 $\lim\limits_{x\to3}\frac{\sqrt{5x+1}-4}{x^2+2x-15}$	3.20 $\lim\limits_{x\to0}\frac{2-\sqrt{x^2+4}}{3x^2}$	3.21 $\lim\limits_{x\to0}\frac{\sqrt{x^2+4}-2}{\sqrt{x^2+16}-4}$
3.22 $\lim\limits_{x\to0}\frac{3x}{\sqrt{5-x}-\sqrt{5+x}}$	3.23 $\lim\limits_{x\to9}\frac{\sqrt{2x+7}-5}{3-\sqrt{x}}$	3.24 $\lim\limits_{x\to4}\frac{2-\sqrt{x}}{\sqrt{6x+1}-5}$
3.25 $\lim\limits_{x\to3}\frac{x^3-27}{\sqrt{3x}-x}$	3.26 $\lim\limits_{x\to0}\frac{\sqrt{1+3x^2}-1}{x^3+x^2}$	3.27 $\lim\limits_{x\to-4}\frac{\sqrt{x+20}-4}{x^3+64}$
3.28 $\lim\limits_{x\to1}\frac{3x^2-3}{\sqrt{x+8}-3}$	3.29 $\lim\limits_{x\to0}\frac{\sqrt{x+9}-3}{x^2+x}$	3.30 $\lim\limits_{x\to2}\frac{\sqrt{4x+1}-3}{x^3-8}$

Task 4. Find the limits

4.1 a) $\lim\limits_{x\to0}\frac{\sin 3x}{2x}$, б) $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x$	4.2 a) $\lim\limits_{x\to0}\frac{9x}{\sin x}$, б) $\lim\limits_{x\to0}(1+3x)^{\frac{1}{x}}$	4.3 a) $\lim\limits_{x\to0}\frac{tg 2x}{5x}$, б) $\lim\limits_{x\to\infty}\left(1+\frac{4}{x}\right)^{2x}$

4.4 а) $\lim\limits_{x\to 0}\frac{7x}{tgx}$, б) $\lim\limits_{x\to\infty}\left(1-\frac{5}{x}\right)^{x}$	4.5 а) $\lim\limits_{x\to 0}\frac{\sin 8x}{x}$, б) $\lim\limits_{x\to 0}(1+6x)^{\frac{3}{x}}$	4.6 а) $\lim\limits_{x\to 0}\frac{arctg\,3x}{4x}$, б) $\lim\limits_{x\to 0}(1-5x)^{\frac{1}{x}}$
4.7 а) $\lim\limits_{x\to 0}\frac{9x}{\sin 3x}$, б) $\lim\limits_{x\to 0}(1-4x)^{7/x}$	4.8 а) $\lim\limits_{x\to 0}\frac{\sin 3x}{\sin x}$, б) $\lim\limits_{x\to 0}(1-9x)^{\frac{2}{x}}$	4.9 а) $\lim\limits_{x\to 0}\frac{4x^2}{1-\cos 4x}$, б) $\lim\limits_{x\to\infty}\left(1-\frac{7}{x}\right)^{3x}$
4.10 а) $\lim\limits_{x\to 0}\frac{1-\cos 8x}{x^2}$, б) $\lim\limits_{x\to\infty}\left(1+\frac{5}{x}\right)^{x}$	4.11 а) $\lim\limits_{x\to 0}\frac{10x^2}{1-\cos x}$, б) $\lim\limits_{x\to\infty}\left(\frac{x+5}{x}\right)^{7x}$	4.12 а) $\lim\limits_{x\to 0}\frac{7x}{tg5x}$, б) $\lim\limits_{x\to\infty}\left(\frac{x-2}{x}\right)^{9x}$
4.13 а) $\lim\limits_{x\to 0}\frac{7x}{\sin 5x}$, б) $\lim\limits_{x\to\infty}\left(1+\frac{5}{x}\right)^{7x}$	4.14 а) $\lim\limits_{x\to 0}\frac{3x}{tgx}$, б) $\lim\limits_{x\to\infty}\left(1-\frac{4}{x}\right)^{2x}$	4.15 а) $\lim\limits_{x\to 0}\frac{7x}{tg9x}$, б) $\lim\limits_{x\to\infty}\left(1-\frac{7}{x}\right)^{x/6}$
4.16 а) $\lim\limits_{x\to 0}\frac{arctg8x}{3x}$, б) $\lim\limits_{x\to 0}\left(1+\frac{3x}{2}\right)^{1/x}$	4.17 а) $\lim\limits_{x\to 0} x\,ctg7x$, б) $\lim\limits_{x\to 0}(1-11x)^{2/x}$	4.18 а) $\lim\limits_{x\to 0}\frac{\sin 8x}{\sin 3x}$, б) $\lim\limits_{x\to 0}(1-8x)^{\frac{7}{x}}$
4.19 а) $\lim\limits_{x\to 0}\sin 5x\,ctg3x$, б) $\lim\limits_{x\to 0}(1+x)^{7/x}$	4.20 а) $\lim\limits_{x\to 0}\frac{\arcsin 3x}{9x}$, б) $\lim\limits_{x\to 0}(1+x)^{2/x}$	4.21 а) $\lim\limits_{x\to 0}\frac{\sin 3x}{\sin 9x}$, б) $\lim\limits_{x\to 0}(1+7x)^{\frac{2}{x}}$
4.22 а) $\lim\limits_{x\to 0}\frac{3x}{arctg7x}$, б) $\lim\limits_{x\to 0}\left(1+\frac{2x}{5}\right)^{\frac{2}{x}}$	4.23 а) $\lim\limits_{x\to 0}\frac{\sin 8x}{3x}$, б) $\lim\limits_{x\to 0}(1-2x)^{9/x}$	4.24 а) $\lim\limits_{x\to 0}\frac{\sin 3x}{tg9x}$, б) $\lim\limits_{x\to 0}(1+6x)^{\frac{2}{x}}$

4.25 а) $\lim\limits_{x\to 0}\dfrac{x}{\sin 5x}$, б) $\lim\limits_{x\to 0}(1-4x)^{7/x}$	4.26 а) $\lim\limits_{x\to 0}\dfrac{3x}{tg7x}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{5x/2}$	4.27 а) $\lim\limits_{x\to 0}\dfrac{3x}{\sin 5x}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^{7x}$
4.28 а) $\lim\limits_{x\to 0}\dfrac{x}{\sin 2x}$, б) $\lim\limits_{x\to\infty}\left(1-\dfrac{6}{x}\right)^{7x}$	4.29 а) $\lim\limits_{x\to 0}\dfrac{3x}{tg9x}$, б) $\lim\limits_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{5x}$	4.30 а) $\lim\limits_{x\to 0}\dfrac{7x}{tg3x}$, б) $\lim\limits_{x\to\infty}\left(1-\dfrac{3}{2x}\right)^{2x}$

Task 5. Investigate given functions for continuity

5.1 а) $f(x)=2^{\frac{1}{x-3}}$, б) $g(x)=\dfrac{7}{2+x}$	5.2 а) $f(x)=3^{\frac{1}{x-2}}$, б) $g(x)=\dfrac{1}{4-x}$	5.3 а) $f(x)=5^{\frac{1}{x+1}}$, б) $g(x)=\dfrac{1}{x-12}$
5.4 а) $f(x)=7^{\frac{1}{x-4}}$, б) $g(x)=\dfrac{1}{x+12}$	5.5 а) $f(x)=7^{\frac{1}{5+x}}$, б) $g(x)=\dfrac{6}{x-4}$	5.6 а) $f(x)=8^{\frac{1}{x-3}}$, б) $g(x)=\dfrac{5}{x-22}$
5.7 а) $f(x)=9^{\frac{1}{5-x}}$, б) $g(x)=\dfrac{7}{2x+1}$	5.8 а) $f(x)=9^{\frac{1}{x-7}}$, б) $g(x)=\dfrac{7}{8-2x}$	5.9 а) $f(x)=9^{\frac{1}{2-x}}$, б) $g(x)=\dfrac{3}{5x-25}$
5.10 а) $f(x)=16^{\frac{1}{x-2}}$, б) $g(x)=\dfrac{3}{9-3x}$	5.11 а) $f(x)=4^{\frac{1}{3-x}}$, б) $g(x)=\dfrac{6}{x-4}$	5.12 а) $f(x)=3^{\frac{1}{x-4}}$, б) $g(x)=\dfrac{9}{25-5x}$
5.13 а) $f(x)=12^{\frac{1}{x}}$, б) $g(x)=\dfrac{4}{9-x}$	5.14 а) $f(x)=7^{\frac{1}{x-3}}$, б) $g(x)=\dfrac{2}{8-4x}$	5.15 а) $f(x)=3^{\frac{1}{4-x}}$, б) $g(x)=\dfrac{7}{5-x}$
5.16 а) $f(x)=8^{\frac{1}{x-2}}$, б) $g(x)=\dfrac{3}{4-2x}$	5.17 а) $f(x)=8^{\frac{1}{5-x}}$, б) $g(x)=\dfrac{10}{x+5}$	5.18 а) $f(x)=18^{\frac{1}{x}}$, б) $g(x)=\dfrac{12}{8-8x}$

5.19 a) $f(x)=10^{\frac{1}{7-x}}$, б) $g(x)=\frac{12}{4x-2}$	5.20 a) $f(x)=11^{\frac{1}{5-x}}$, б) $g(x)=\frac{9}{8+2x}$	5.21 a) $f(x)=14^{\frac{1}{6-x}}$, б) $g(x)=\frac{3}{2+x}$
5.22 a) $f(x)=15^{\frac{3}{4-x}}$, б) $g(x)=\frac{1}{6-2x}$	5.23 a) $f(x)=15^{\frac{1}{8-x}}$, б) $g(x)=\frac{10}{x-3}$	5.24 a) $f(x)=19^{\frac{4}{2+x}}$, б) $g(x)=\frac{7}{8-2x}$
5.25 a) $f(x)=11^{\frac{1}{4+x}}$, б) $g(x)=\frac{1}{9-x}$	5.26 a) $f(x)=5^{\frac{7}{8-x}}$, б) $g(x)=\frac{11}{x-15}$	5.27 a) $f(x)=13^{\frac{1}{5+x}}$, б) $g(x)=\frac{5}{3-x}$
5.28 a) $f(x)=6^{\frac{9}{4-x}}$, б) $g(x)=\frac{1}{2x+5}$	5.29 a) $f(x)=4^{\frac{1}{x-5}}$, б) $g(x)=\frac{3}{8-x}$	5.30 a) $f(x)=11^{\frac{2}{x-3}}$, б) $g(x)=\frac{16}{x+15}$

Task 6. Find the derivatives of functions

6.1 a) $f(x)=tg(8x^2+x-5)$, б) $g(x)=(e^{\cos x}+3)^2$	6.2 a) $f(x)=\cos(3x^2-2)$, б) $g(x)=\frac{4x^2}{\cos^2 x}$	6.3 a) $f(x)=e^{7x+x^2}$, б) $g(x)=\frac{5x}{tg^2 2x}$
6.4 a) $f(x)=(4x+2x^3)^{10}$, б) $g(x)=\frac{\sin^3 x}{2+3\cos^2 x}$	6.5 a) $f(x)=\sqrt[3]{x^5+10x^2}$, б) $g(x)=2tg^3(x^2+1)$	6.6 a) $f(x)=arctg(x^5+3x)$, б) $g(x)=2tg^3 x+\lg\cos x$
6.7 a) $f(x)=tg(4x^2+x+5)$, б) $g(x)=(e^{\sin x}+2)^4$	6.8 a) $f(x)=\sin(3x^2-2x+1)$, б) $g(x)=\frac{3x^2}{\cos^4 x}$	6.9 a) $f(x)=e^{2x-x^3}$, б) $g(x)=\frac{tg^2 2x}{x-5}$
6.10 a) $f(x)=(3x^3+7x)^{15}$, б) $g(x)=\frac{4x^7}{arctg^3 x}$	6.11 a) $f(x)=\sqrt[3]{x^8+5x^3}$, б) $g(x)=3ctg^3(x^2+1)$	6.12 a) $f(x)=arcctg(x^4+x)$, б) $g(x)=\frac{1}{3}tg^2 x+\lg\sin x$

6.13 а) $f(x)=tg(3x^2+2x-7)$, б) $g(x)=(6^{\cos x}+3)^2$	6.14 а) $f(x)=\cos(7x^2-6x+6)$, б) $g(x)=\dfrac{2x^7}{\cos^3 x}$	6.15 а) $f(x)=4^{7x+x^2}$, б) $g(x)=\dfrac{1}{2}\sin^2 x+\ln\cos x$
6.16 а) $f(x)=(3x^2+8x)^{18}$, б) $g(x)=\dfrac{3x^8}{arcctg^3 x}$	6.17 а) $f(x)=\sqrt[5]{x^5+10x^2-3}$ б) $g(x)=\dfrac{(7+3x)^4}{\arcsin x}$	6.18 а) $f(x)=arctg(x^4+6x)$, б) $g(x)=\dfrac{1}{2}\cos^2 x+\ln\sin x$
6.19 а) $f(x)=\sin(4x^2+x+5)$, б) $g(x)=(e^{tgx}+1)^4$	6.20 а) $f(x)=ctg(3x^2-7x)$, б) $g(x)=4tg^5 3x^2$	6.21 а) $f(x)=13^{5x^3+2}$, б) $g(x)=\dfrac{(1+3x)^3}{tg2x}$
6.22 а) $f(x)=(9x+3x^5)^{12}$, б) $g(x)=\dfrac{\cos^5 x}{1+\sin x}$	6.23 а) $f(x)=\sqrt[7]{x+5x^3}$ б) $g(x)=9\cos^3(x^2+6)$	6.24 а) $f(x)=\arcsin(x^4+7x)$, б) $g(x)=ctg^5 x+\ln\sin x$
6.25 а) $f(x)=\ln(6x^2-x^3)$, б) $g(x)=(e^{\sqrt{x}}+8)^{12}$	6.26 а) $f(x)=\cos(\sqrt[3]{x}+2)$, б) $g(x)=\dfrac{(x+3x^5)^2}{\cos x}$	6.27 а) $f(x)=e^{2x+9x^3}$, б) $g(x)=\sqrt[6]{(\sin x+2x)^5}$
6.28 а) $f(x)=\sqrt{2x+\cos 5x}$, б) $g(x)=\dfrac{x^2}{\ln(5x^3+1)}$	6.29 а) $f(x)=\sqrt[3]{\ln x}$, б) $g(x)=2tg(\arcsin 5x)$	6.30 а) $f(x)=arctg(\sin x+2)$, б) $g(x)=3x+\lg(\cos x+4)$

Task 7. Find the derivative of the function given by the parametric equations

7.1 $\begin{cases} x=(2t+3)\cos t, \\ y=3t^3 \end{cases}$	7.2 $\begin{cases} x=2\cos t, \\ y=3\sin^2 t \end{cases}$	7.3 $\begin{cases} x=6\cos^3 t, \\ y=2\sin t \end{cases}$
7.4 $\begin{cases} x=1/(t+2), \\ y=t^2/(t+2) \end{cases}$	7.5 $\begin{cases} x=e^{-2t}, \\ y=3te^{4t} \end{cases}$	7.6 $\begin{cases} x=\sqrt{t}, \\ y=\sqrt[5]{t} \end{cases}$
7.7 $\begin{cases} x=2t/(t^3+1), \\ y=t^2/(t^2+1) \end{cases}$	7.8 $\begin{cases} x=\sqrt{t^2-1}, \\ y=(t+1)/\sqrt{t^2-1} \end{cases}$	7.9 $\begin{cases} x=4t+2t^2, \\ y=5\sqrt{t^3}-3t^2 \end{cases}$

7.10 $\begin{cases} x = (\ln t)/t, \\ y = \ln t \end{cases}$	7.11 $\begin{cases} x = e^t \cos t, \\ y = e^t \sin t \end{cases}$	7.12 $\begin{cases} x = t^4, \\ y = t \ln t \end{cases}$
7.13 $\begin{cases} x = 5\cos^2 t, \\ y = 3\sin t \end{cases}$	7.14 $\begin{cases} x = 5\cos t, \\ y = 3\sin^2 t \end{cases}$	7.15 $\begin{cases} x = arctgt, \\ y = \ln(1+t^2) \end{cases}$
7.16 $\begin{cases} x = \arcsin t, \\ y = \sqrt{1-t^2} \end{cases}$	7.17 $\begin{cases} x = 3(t - \sin t), \\ y = 3(1 - \cos t) \end{cases}$	7.18 $\begin{cases} x = 3(\sin t - \cos t), \\ y = 3(\cos t + \sin t) \end{cases}$
7.19 $\begin{cases} x = \sin 2t, \\ y = \cos^2 t \end{cases}$	7.20 $\begin{cases} x = e^{3t}, \\ y = \sqrt{t-1} \end{cases}$	7.21 $\begin{cases} x = (\ln t)/t, \\ y = t^2 \ln t \end{cases}$
7.22 $\begin{cases} x = \arccos t, \\ y = \sqrt{1-t^2} \end{cases}$	7.23 $\begin{cases} x = 1/(t+1), \\ y = t^2/(t+1) \end{cases}$	7.24 $\begin{cases} x = 5\sin 3t, \\ y = 3\cos^3 t \end{cases}$
7.25 $\begin{cases} x = e^{-2t}, \\ y = e^{8t}/t \end{cases}$	7.26 $\begin{cases} x = \sqrt[3]{(t-1)^2}, \\ y = \sqrt{t-1} \end{cases}$	7.27 $\begin{cases} x = \ln^2 t, \\ y = t + \ln t \end{cases}$
7.28 $\begin{cases} x = te^t, \\ y = t/e^t \end{cases}$	7.29. $\begin{cases} x = 6t^2 - 4, \\ y = 3\sqrt{t^5} \end{cases}$	7.30 $\begin{cases} x = \arcsin t, \\ y = \ln t \end{cases}$

Task 8. Find the value of the second derivative of the function $y = f(x)$ at the point x_0

8.1 $y = \sin^2 x$, $x_0 = \pi/4$	8.2 $y = arctgx$, $x_0 = 1$	8.3 $y = \ln(2 + x^2)$, $x_0 = 0$
8.4 $y = e^x \cos x$, $x_0 = 0$	8.5 $y = e^x \sin 2x$, $x_0 = 0$	8.6 $y = e^{-x} \cos x$, $x_0 = 0$
8.7 $y = \sin 2x$, $x_0 = \pi$	8.8 $y = (2x+1)^5$, $x_0 = 1$	8.9 $y = \ln(1 + 2x)$, $x_0 = 2$
8.10 $y = \frac{1}{2} e^x x^2$, $x_0 = 0$	8.11 $y = \arcsin x$, $x_0 = 0$	8.12 $y = (5x - 4)^5$, $x_0 = 2$
8.13 $y = x \sin x$, $x_0 = \frac{\pi}{2}$	8.14 $y = x^2 \ln x$, $x_0 = \frac{1}{3}$	8.15 $y = x \sin 2x$, $x_0 = -\frac{\pi}{4}$

8.16	$y = x\cos 2x,$ $x_0 = \frac{\pi}{12}$	8.17	$y = x^4 \ln x,$ $x_0 = 1$	8.18	$y = x + arctgx,$ $x_0 = 1$
8.19	$y = \cos^2 x,$ $x_0 = \frac{\pi}{4}$	8.20	$y = \ln(x^2 - 4),$ $x_0 = 3$	8.21	$y = x^2 \cos x,$ $x_0 = \frac{\pi}{2}$
8.22	$y = x\arccos x,$ $x_0 = \frac{\sqrt{3}}{2}$	8.23	$y = (x+1)\ln(x+1),$ $x_0 = -\frac{1}{2}$	8.24	$y = \ln^3 x,$ $x_0 = 1$
8.25	$y = 2^{7x+5},$ $x_0 = 1$	8.26	$y = (4x-3)^5,$ $x_0 = 1$	8.27	$y = x\arcsin x,$ $x_0 = 2$
8.28	$y = (7x-4)^6,$ $x_0 = 1$	8.29	$y = x\sin 2x,$ $x_0 = \frac{\pi}{4}$	8.30	$y = \sin(x^3 + \pi),$ $x_0 = \sqrt[3]{\pi}$

Task 9. For research of function $y = f(x)$ determine:
a) domain and the points of discontinuity;
b) asymptote of the graph of function;
c) points of intersection of the graph with the coordinate axes;
d) parity, oddness.

9.1 $y = \frac{x^3+4}{x^2}$	9.2 $y = \frac{x^2-x+1}{x-1}$	9.3 $y = \frac{2}{x^2+2x}$	9.4 $y = \frac{4x^2}{3+x^2}$	9.5 $y = \frac{12x}{x^2+9}$
9.6 $y = \frac{x^2-3x+3}{x-1}$	9.7 $y = \frac{4-x^3}{x^2}$	9.8 $y = \frac{x^2-4x+1}{x-4}$	9.9 $y = \frac{2x^3+1}{x^2}$	9.10 $y = \frac{(x-1)^2}{x^2}$
9.11 $y = \frac{x^2}{(x-1)^2}$	9.12 $y = \left(x + \frac{1}{x}\right)^2$	9.13 $y = \frac{12-3x^2}{12+x^2}$	9.1 4 $y = \frac{9+6x-3x^2}{x^2-2x+13}$	9.15 $y = \frac{-8x}{x^2+4}$
9.16 $y = \frac{x-1}{(x+1)^2}$	9.17 $y = \frac{3x^4+1}{x^3}$	9.18 $y = \frac{4x}{(x+1)^2}$	9.19 $y = \frac{8(x-1)}{(x+1)^2}$	9.20 $y = \frac{1-2x^3}{x^2}$
9.21 $y = \frac{4}{x^2+2x-3}$	9.22 $y = \frac{4}{3+2x-x^2}$	9.23 $y = \frac{x^2+2x-7}{x^2+2x-3}$	9.24 $y = \frac{1}{x^4-1}$	9.25 $y = -\left(\frac{x}{x+2}\right)^2$

9.26	9.27	9.28	9.29	9.30
$y=\frac{x^3-32}{x^2}$	$y=\frac{4(x+1)^2}{x^2+2x+4}$	$y=\frac{x^3-27x+54}{x^3}$	$y=\frac{x^2-6x+9}{(x-1)^2}$	$y=\frac{3x-2}{x^3}$

Task 10. Find the limits

10.1 $\lim\limits_{x\to\infty}\left(\frac{2x+3}{5x+7}\right)^{x+1}$	10.2 $\lim\limits_{x\to\infty}\left(\frac{2x+1}{x-1}\right)^{x}$	10.3 $\lim\limits_{x\to\infty}\left(\frac{x+1}{2x-1}\right)^{3x}$	10.4 $\lim\limits_{x\to-\infty}\left(\frac{2x-1}{4x+1}\right)^{3x-1}$
10.5 $\lim\limits_{x\to\infty}\left(\frac{5x+8}{x-2}\right)^{x+4}$	10.6 $\lim\limits_{x\to-\infty}\left(\frac{x+1}{3x-1}\right)^{2x+1}$	10.7 $\lim\limits_{x\to-\infty}\left(\frac{2x+1}{x-1}\right)^{5x}$	10.8 $\lim\limits_{x\to\infty}\left(\frac{x+1}{2x-1}\right)^{4x}$
10.9 $\lim\limits_{x\to-\infty}\left(\frac{x+3}{2x-4}\right)^{x+2}$	10.10 $\lim\limits_{x\to-\infty}\left(\frac{2x+1}{3x-1}\right)^{x-1}$	10.11 $\lim\limits_{x\to\infty}\left(\frac{5x-3}{x+4}\right)^{x+3}$	10.12 $\lim\limits_{x\to-\infty}\left(\frac{2x-3}{7x+4}\right)^{x}$
10.13 $\lim\limits_{x\to-\infty}\left(\frac{x-5}{3x+4}\right)^{2x}$	10.14 $\lim\limits_{x\to\infty}\left(\frac{x+3}{4x-5}\right)^{2x}$	10.15 $\lim\limits_{x\to-\infty}\left(\frac{x-2}{3x+1}\right)^{5x}$	10.16 $\lim\limits_{x\to-\infty}\left(\frac{3x-4}{x+6}\right)^{x-1}$
10.17 $\lim\limits_{x\to\infty}\left(\frac{x-2}{3x+10}\right)^{3x}$	10.18 $\lim\limits_{x\to-\infty}\left(\frac{2x-3}{x+4}\right)^{6x+1}$	10.19 $\lim\limits_{x\to-\infty}\left(\frac{x+3}{3x-1}\right)^{2x}$	10.20 $\lim\limits_{x\to\infty}\left(\frac{6x+5}{x-10}\right)^{5x}$
10.21 $\lim\limits_{x\to-\infty}\left(\frac{3x+7}{x+4}\right)^{4x}$	10.22 $\lim\limits_{x\to\infty}\left(\frac{x-1}{4x+5}\right)^{3x}$	10.23 $\lim\limits_{x\to-\infty}\left(\frac{5x-7}{x+6}\right)^{2x}$	10.24 $\lim\limits_{x\to\infty}\left(\frac{3-4x}{2-x}\right)^{6x}$
10.25 $\lim\limits_{x\to\infty}\left(\frac{1-2x}{3-x}\right)^{-x}$	10.26 $\lim\limits_{x\to-\infty}\left(\frac{4+3x}{5+x}\right)^{7x}$	10.27 $\lim\limits_{x\to-\infty}\left(\frac{3x-1}{2x+5}\right)^{3x}$	10.28 $\lim\limits_{x\to\infty}\left(\frac{1-x}{2-10x}\right)^{5x}$
10.29 $\lim\limits_{x\to\infty}\left(\frac{x+3}{9x-4}\right)^{2x}$	10.30 $\lim\limits_{x\to-\infty}\left(\frac{x+5}{4x-2}\right)^{3x}$		

Task 11. Find the specified limits, using L'Hopital rule.

11.1 $\lim\limits_{x\to\infty}\dfrac{\ln(x+5)}{\sqrt[4]{x+3}}$	11.2 $\lim\limits_{x\to1}\dfrac{2^{\ln x}-x}{x-1}$	11.3 $\lim\limits_{x\to0}\dfrac{tgx-x}{x-\sin x}$
11.4 $\lim\limits_{x\to0}\dfrac{1-\cos x^2}{x^2-\sin x^2}$	11.5 $\lim\limits_{x\to0}\dfrac{tgx-x}{2\sin x+x}$	11.6 $\lim\limits_{x\to1}\dfrac{x^3-2x^2-x+2}{x^3-7x+6}$
11.7 $\lim\limits_{x\to0}\dfrac{x\cos x-\sin x}{x^3}$	11.8 $\lim\limits_{x\to0}\dfrac{e^x}{x^3}$	11.9 $\lim\limits_{x\to1}\dfrac{1-x}{1-\sin\left(\dfrac{\pi x}{2}\right)}$
11.10 $\lim\limits_{x\to\infty}\dfrac{\ln x}{\sqrt[3]{x}}$	11.11 $\lim\limits_{x\to0}\dfrac{e^{3x}}{1-\cos5x}$	11.12 $\lim\limits_{x\to0}\dfrac{\pi/x}{ctg(\pi x/2)}$
11.13 $\lim\limits_{x\to0}\dfrac{\ln(\sin2x)}{\ln(\sin5x)}$	11.14 $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{tgx}{tg5x}$	11.15 $\lim\limits_{x\to0}\dfrac{e^{3x}-e^{5x}}{\sin x}$
11.16 $\lim\limits_{x\to0}\dfrac{x-arctgx}{x^3}$	11.17 $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{1-2\sin x}{\cos3x}$	11.18 $\lim\limits_{x\to0}\dfrac{e^{2x}-1}{\ln(1+2x)}$
11.19 $\lim\limits_{x\to1}\dfrac{\ln x}{x^3-1}$	11.20 $\lim\limits_{x\to0}\dfrac{\ln x}{ctgx}$	11.21 $\lim\limits_{x\to0}\dfrac{1-\cos3x}{1-\cos6x}$
11.22 $\lim\limits_{x\to4}\dfrac{x-4}{x^3-64}$	11.23 $\lim\limits_{x\to0}\dfrac{e^{5x}-1}{\sin4x}$	11.24 $\lim\limits_{x\to0}\dfrac{1-e^{2x}}{tg4x}$
11.25 $\lim\limits_{x\to0}\dfrac{2^x-4^x}{x\sqrt{1-x^2}}$	11.26 $\lim\limits_{x\to0}\dfrac{\ln(1+x^2)}{\cos3x-e^{-x}}$	11.27 $\lim\limits_{x\to0}\dfrac{\ln(7+x)}{\sqrt[7]{x-3}}$
11.28 $\lim\limits_{x\to0}\dfrac{\arcsin4x}{5-5e^{-3x}}$	11.29 $\lim\limits_{x\to0}\dfrac{e^{9x}-1}{\cos5x}$	11.30 $\lim\limits_{x\to\pi/6}\dfrac{1-2\sin x}{\cos3x}$

The second level calculated tasks

Task 12. Investigate the function for continuity and construct the graph

12.1 $f(x)=\begin{cases}x+4, & x\prec-1,\\ x^2+2, & -1\le x\le1,\\ 2x, & x\succ1.\end{cases}$	12.2 $f(x)=\begin{cases}x+4, & x\le0,\\ (x+1)^2, & 0\prec x\le2,\\ -x+4, & x\succ2.\end{cases}$	12.3 $f(x)=\begin{cases}x+2, & x\le-1,\\ x^2+1, & -1\prec x\le1,\\ -x+3, & x\succ1.\end{cases}$
12.4 $f(x)=\begin{cases}-x, & x\le0,\\ -(x-1)^2, & 0\prec x\prec2,\\ x-3, & x\ge2.\end{cases}$	12.5 $f(x)=\begin{cases}-2(x+1), & x\le-1,\\ (x+1)^2, & -1\prec x\prec0,\\ x, & x\ge0.\end{cases}$	12.6 $f(x)=\begin{cases}-x, & x\le0,\\ x^2, & 0\prec x\le2,\\ x+1, & x\succ2.\end{cases}$

12.7 $f(x)=\begin{cases} x^2+1, & x\le 1, \\ 2x, & 1\prec x\le 3, \\ x+2, & x\succ 3. \end{cases}$	12.8 $f(x)=\begin{cases} x-3, & x\prec 0, \\ x+1, & 0\le x\le 4, \\ x+3, & x\succ 4. \end{cases}$	12.9 $f(x)=\begin{cases} \sqrt{1-x}, & x\le 0, \\ 0, & 0\prec x\le 2, \\ x-2, & x\succ 2. \end{cases}$
12.10 $f(x)=\begin{cases} 2x^2, & x\le 0, \\ x, & 0\prec x\le 1, \\ x+2, & x\succ 1. \end{cases}$	12.11 $f(x)=\begin{cases} \sin x, & x\prec 0, \\ x, & 0\le x\le 2, \\ 0, & x\succ 2. \end{cases}$	12.12 $f(x)=\begin{cases} \cos x, & x\le \pi/2, \\ 0, & \pi/2\prec x\prec \pi, \\ 2, & x\ge \pi. \end{cases}$
12.13 $f(x)=\begin{cases} x-1, & x\le 0, \\ x^2, & 0\prec x\prec 2, \\ 2x, & x\ge 2. \end{cases}$	12.14 $f(x)=\begin{cases} x+1, & x\prec 0, \\ x^2-1, & 0\le x\prec 1, \\ -x, & x\ge 1. \end{cases}$	12.15 $f(x)=\begin{cases} -x, & x\prec 0, \\ x^2+1, & 0\le x\prec 2, \\ 1+x, & x\ge 2. \end{cases}$
12.16 $f(x)=\begin{cases} x+3, & x\le 0, \\ 1, & 0\prec x\le 2, \\ x^2-2, & x\succ 2. \end{cases}$	12.17 $f(x)=\begin{cases} x-1, & x\prec 0, \\ \sin x, & 0\le x\prec \pi, \\ 3, & x\ge \pi. \end{cases}$	12.18 $f(x)=\begin{cases} 1-x, & x\prec -1, \\ x^2+1, & -1\le x\le 2, \\ 2x, & x\succ 2. \end{cases}$
12.19 $f(x)=\begin{cases} 1, & x\le 0, \\ 2^x, & 0\prec x\le 2, \\ x+3, & x\succ 2. \end{cases}$	12.20 $f(x)=\begin{cases} -x+2, & x\le -2, \\ x^3, & -2\prec x\le 1, \\ 2, & x\succ 1. \end{cases}$	12.21 $f(x)=\begin{cases} 3x+4, & x\le -1, \\ x^2-2, & -1\prec x\prec 2, \\ x, & x\ge 2. \end{cases}$
12.22 $f(x)=\begin{cases} x, & x\le 1, \\ (x-2)^2, & 1\prec x\prec 3, \\ -x+6, & x\ge 3. \end{cases}$	12.23 $f(x)=\begin{cases} x-1, & x\prec 1, \\ x^2+2, & 1\le x\le 2, \\ -2x, & x\succ 2. \end{cases}$	12.24 $f(x)=\begin{cases} x^3, & x\prec -1, \\ x-1, & -1\le x\le 3, \\ -x+5, & x\succ 3. \end{cases}$
12.25 $f(x)=\begin{cases} x, & x\le -2, \\ -x+1, & -2\prec x\le 1, \\ x^2-1, & x\succ 1. \end{cases}$	12.26 $f(x)=\begin{cases} x+3, & x\le 0, \\ -x^2+4, & 0\prec x\prec 2, \\ x-2, & x\ge 2. \end{cases}$	12.27 $f(x)=\begin{cases} 0, & x\le -1, \\ x^2-1, & -1\prec x\le 2, \\ 2x, & x\succ 2. \end{cases}$

12.28	12.29	12.30
$f(x)=\begin{cases}-1, & x\prec 0,\\ \cos x, & 0\le x\le\pi,\\ 1-x, & x\succ\pi.\end{cases}$	$f(x)=\begin{cases}2, & x\prec -1,\\ 1-x, & -1\le x\le 1,\\ \ln x, & x\succ 1.\end{cases}$	$f(x)=\begin{cases}-x, & x\le 0,\\ x^3, & 0\prec x\le 2,\\ x+4, & x\succ 2.\end{cases}$

Task 13. Find the second derivative of the function given by the parametric equations in the task 7.

Task 14. For the function from the task 9 find:
a) the intervals of monotonicity, the extremums;
b) the intervals of concavity and convexity, the inflection points;
c) construct the graph.

Decision of an exemplary embodiment

Task 1. Find the limit $\lim\limits_{x\to -2}\dfrac{5x^2+13x+6}{3x^2+2x-8}$.

Solution:

$$\lim_{x\to -2}\frac{5x^2+13x+6}{3x^2+2x-8}=\left|\frac{0}{0}\right|=\lim_{x\to -2}\frac{(x+2)(5x+3)}{(x+2)(3x-4)}=\lim_{x\to -2}\frac{5x+3}{3x-4}=\frac{7}{10}=0{,}7.$$

Task 2. Find the limit $\lim\limits_{x\to -\infty}\dfrac{2x^5+3x^3-4x}{3x^2-4x+2}$.

Solution:

$$\lim_{x\to -\infty}\frac{2x^5+3x^3-4x}{3x^2-4x+2}=\left|\frac{\infty}{\infty}\right|=\lim_{x\to -\infty}\frac{x^5\left(2+\frac{3}{x^2}-\frac{4}{x^4}\right)}{x^2\left(3-\frac{4}{x}+\frac{2}{x^2}\right)}=$$

$$=\lim_{x\to -\infty}\frac{x^3\left(2+\frac{3}{x^2}-\frac{4}{x^4}\right)}{3-\frac{4}{x}+\frac{2}{x^2}}=\frac{-\infty}{3}=-\infty.$$

Task 3. Find the limit $\lim\limits_{x\to 4}\dfrac{\sqrt{21+x}-5}{x^3-64}$.

Solution:

$$\lim_{x\to 4}\frac{\sqrt{21+x}-5}{x^3-64}=\lim_{x\to 4}\frac{\left(\sqrt{21+x}-5\right)\left(\sqrt{21+x}+5\right)}{\left(x^3-64\right)\left(\sqrt{21+x}+5\right)}=\lim_{x\to 4}\frac{21+x-25}{\left(x^3-64\right)\left(\sqrt{21+x}+5\right)}=$$

$$=\lim_{x\to 4}\frac{x-4}{(x-4)\left(x^2+4x+16\right)\left(\sqrt{21+x}+5\right)}=\lim_{x\to 4}\frac{1}{\left(x^2+4x+16\right)\left(\sqrt{21+x}+5\right)}=\frac{1}{480}.$$

Task 4. Find the limits: a) $\lim\limits_{x\to 0}\frac{tg9x}{8x}$; b) $\lim\limits_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{5x}$.

Solution:

a) for the disclosure of uncertainty the property of the first remarkable limit is used

$$\lim_{x\to 0}\frac{tg9x}{8x}=\left|\frac{0}{0}\right|=\lim_{x\to 0}\frac{\frac{\sin 9x}{\cos 9x}}{8x}=\lim_{x\to 0}\frac{\sin 9x}{8x}\cdot\frac{1}{\cos 9x}=\lim_{x\to 0}\frac{\sin 9x}{9x}\cdot\frac{9}{8\cos 9x}=1\cdot\frac{9}{8}=\frac{9}{8};$$

b) for the disclosure of uncertainty $\left|1^{\infty}\right|$ the property of the second remarkable limit is used

$$\lim_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{5x}=\left|1^{\infty}\right|=\lim_{x\to\infty}\left(1+\frac{2x}{2x-3}-1\right)^{5x}=\lim_{x\to\infty}\left(1+\frac{2x-2x+3}{2x-3}\right)^{5x}=$$

$$=\lim_{x\to\infty}\left(1+\frac{3}{2x-3}\right)^{5x}=\lim_{x\to\infty}\left(\left(1+\frac{3}{2x-3}\right)^{\frac{2x-3}{3}}\right)^{\frac{3}{2x-3}\cdot 5x}=\lim_{x\to\infty}e^{\frac{3}{2x-3}\cdot 5x}=e^{\frac{15}{2}}.$$

Task 5. Investigate given functions for continuity:

a) $f(x)=9^{\frac{5}{8-x}}$ b) $f(x)=\frac{4}{5+3x}$.

Solution:

a) $f(x)=9^{\frac{5}{8-x}}$ function is defined and continuous in the intervals $x\in(-\infty;8)\cup(8;+\infty)$. Consequently the discontinuity is possible at the point $x_0=8$. Let us find the one-sided limits of the function at this point:

$$f(8-0)=\lim_{x\to 8-0} 9^{\frac{5}{8-x}}=9^{\frac{5}{8-(8-0)}}=9^{\frac{5}{+0}}=9^{+\infty}=+\infty;$$

$$f(8+0)=\lim_{x\to 8+0} 9^{\frac{5}{8-x}}=9^{\frac{5}{8-(8+0)}}=9^{\frac{5}{-0}}=9^{-\infty}=\frac{1}{9^{\infty}}=\frac{1}{\infty}=0.$$

Consequently $x_0=8$ is a point of discontinuity of the second type;

b) $f(x)=\dfrac{4}{5+3x}$ function is defined and continuous in the intervals $x\in(-\infty;-\frac{5}{3})\cup\left(-\frac{5}{3};+\infty\right)$. Consequently the discontinuity is possible at the point $x_0=-\dfrac{5}{3}$. Let us find the one-sided limits of the function at this point:

$$f(-\frac{5}{3}-0)=\lim_{x\to -\frac{5}{3}-0}\frac{4}{5+3x}=\frac{4}{5+3\cdot\left(-\frac{5}{3}-0\right)}=\frac{4}{5-5-0}=\frac{4}{-0}=-\infty;$$

$$f(-\frac{5}{3}+0)=\lim_{x\to -\frac{5}{3}+0}\frac{4}{5+3x}=\frac{4}{5+3\cdot\left(-\frac{5}{3}+0\right)}=\frac{4}{5-5+0}=\frac{4}{+0}=+\infty.$$

Consequently $x_0=-\dfrac{5}{3}$ is a point of discontinuity of the second type.

Task 6. Find the derivatives of functions:

a) $y=\ln\left(4x^2-5x+2^{-3x}\right)$, b) $y=\dfrac{ctg^5 3x}{x+2}$.

Solution:

a) according to the formula of the derivative of a composite function $\left(\ln u\right)'=\dfrac{u'}{u}$ we find:

$$y'=\frac{\left(4x^2-5x+2^{-3x}\right)'}{4x^2-5x+2^{-3x}}=\frac{8x-5+2^{-3x}\ln 2\cdot(-3)}{4x^2-5x+2^{-3x}}=$$

$$=\frac{8x-5-3\cdot 2^{-3x}\ln 2}{4x^2-5x+2^{-3x}};$$

b) find separately the derivative of a composite function of the numerator:

$$\left(ctg^5 3x\right)' = \left|\begin{array}{l}\text{substitute:}\\ u = ctg\,3x;\\ t = 3x\end{array}\right| = \left(u^5\right)'_u \cdot (ctg t)'_t \cdot (3x)' =$$

$$= 5u^4 \cdot \frac{-1}{\sin^2 t} \cdot 3 = -5ctg^4 3x \frac{3}{\sin^2 3x};$$

$$y' = \left(\frac{ctg^5 3x}{x+2}\right)' = \frac{\left(ctg^5 3x\right)' \cdot (x+2) - (x+2)' \cdot ctg^5 3x}{(x+2)^2} =$$

$$= \frac{-5ctg^4 3x \cdot \frac{3}{\sin^2 3x} \cdot (x+2) - ctg^5 3x}{(x+2)^2} = \frac{\frac{-15(x+2)ctg^4 3x}{\sin^2 3x} - ctg^5 3x}{(x+2)^2} =$$

$$= \frac{-15(x+2)ctg^4 3x - \sin^2 3x \cdot ctg^5 3x}{(x+2)^2 \sin^2 3x}.$$

Task 7. Find the derivative of the function given by the parametric equations:

$$\begin{cases} x = 3t^4 - t^2 \\ y = t^3 - 5. \end{cases}$$

Solution: the derivative of the function given by the parametric equations is found by the formula $y'_x = \frac{y'_t}{x'_t}$;

$$\begin{cases} x'_t = 12t^3 - 2t \\ y'_t = 3t^2, \end{cases} \quad \text{so} \quad y'_x = \frac{3t^2}{12t^3 - 2t} = \frac{3t^2}{2t\left(6t^2 - 1\right)} = \frac{3t}{2\left(6t^2 - 1\right)}.$$

Task 8. Find the value of the second derivative of the function $y = \ln^3 x$ at the point $x_0 = 1.$

Solution:

$$y' = \left(\ln^3 x\right)' = 3\ln^2 x \cdot \frac{1}{x} = \frac{3\ln^2 x}{x};$$

$$y'' = \left(\frac{3\ln^2 x}{x}\right)' = 3 \cdot \frac{2\ln x \cdot \frac{1}{x} \cdot x - 1 \cdot \ln^2 x}{x^2} = 3 \cdot \frac{2\ln x - \ln^2 x}{x^2} = \frac{3\ln x(2 - \ln x)}{x^2};$$

$$y''(1) = \frac{3\ln 1(2 - \ln 1)}{1^2} = 0.$$

Task 9. For research of functions $y=\frac{x^2+1}{3x}$ determine:

a) domain and the points of discontinuity;
b) asymptote of the graph of function;
c) points of intersection of the graph with the coordinate axes;
d) parity, oddness.

Solution:

a) $x=0$ - the point of discontinuity, the function is continuous in its domain of definition: $x\neq 0 \Leftrightarrow D(y):(-\infty;\ 0)\cup(0;+\infty)$. Let us find the one-sided limits at the point of discontinuity: $\lim\limits_{x\to -0}\frac{x^2+1}{3x}=-\infty,\ \lim\limits_{x\to +0}\frac{x^2+1}{3x}=+\infty;$

b) the straight line $x=x_0$ is a vertical asymptote, if $f(x_0\pm 0)=\pm\infty \Rightarrow x=0$ - is the vertical asymptote.

The straight line $y=kx+b$ - is a slant asymptote, where

$$k=\lim_{x\to\infty}\frac{y}{x}=\lim_{x\to\infty}\frac{x^2+1}{3x^2}=\frac{1}{3},$$

$$b=\lim_{x\to\infty}(y-x)=\lim_{x\to\infty}\left(\frac{x^2+1}{3x}-\frac{x}{3}\right)=\lim_{x\to\infty}\frac{x^2+1-x^2}{3x}=\lim_{x\to\infty}\frac{1}{3x}=0.$$

so, $y=\frac{1}{3}x$ - is the slant asymptote;

c) at the point $x=0$ the graph of function $y=f(x)$ intersects the axis *0y*. In our case $D(y):(-\infty;\ 0)\cup(0;+\infty)$, so the function does not cross the axis *0y*. At the point $y=0$ the graph of function $y=f(x)$ intersects the axis *0x*. In our case $y=\frac{x^2+1}{3x}\neq 0$, so the function does not cross the axis *0x*;

d) $y(-x)=\frac{(-x)^2+1}{3(-x)}=-\frac{x^2+1}{3x}=-y(x)$, i.e. we have the odd function, the graph is symmetric with respect to the origin. If $x>0$, then $y>0$ therefore the graph is in the first and the third quarters. This function is non-periodic. $\lim\limits_{x\to\infty}\frac{x^2+1}{3x}\left(\frac{\infty}{\infty}\right)=\lim\limits_{x\to\infty}\frac{(x^2+1)'}{(3x)'}=\lim\limits_{x\to\infty}\frac{2x}{3}=+\infty$ - function increases indefinitely.

Task 10. Find the limit $\lim\limits_{x\to\infty}\left(\frac{2x+3}{5x+7}\right)^{x+1}$.

Solution: at the value of $x=\infty$ the uncertainty of the form $\left|\left(\frac{\infty}{\infty}\right)^{\infty}\right|$ is obtained. At the beginning of disclosure of uncertainty at the base of degree in the $\left|\left(\frac{\infty}{\infty}\right)^{\infty}\right|$ the equivalence property of infinitely large functions is applied, and then we use the property of the exponential function: $a^{\infty}=\begin{cases}0, & 0<a<1\\ \infty, & a>1.\end{cases}$

$$\lim_{x\to\infty}\left(\frac{2x+3}{5x+7}\right)^{x+1}=\left|\left(\frac{\infty}{\infty}\right)^{\infty}\right|=\lim_{x\to\infty}\left(\frac{2x}{5x}\right)^{x+1}=\lim_{x\to\infty}\left(\frac{2}{5}\right)^{x+1}=\left(\frac{2}{5}\right)^{\infty}=\left|\frac{2}{5}<1\right|=0.$$

Task 11. Find the specified limits, using L'Hopital rule:

a) $\lim\limits_{x\to 0}\frac{1-\cos 7x}{x\sin 7x}$; b) $\lim\limits_{x\to 0}\left(\frac{1}{\sin^2 x}-\frac{1}{x^2}\right)$.

Solution:

a) L'Hopital rule is applied to the uncertainties of the form $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$:

$$\lim_{x\to 0}\frac{1-\cos 7x}{x\sin 7x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(1-\cos 7x)'}{(x\sin 7x)'}=\lim_{x\to 0}\frac{7\sin 7x}{\sin 7x+7x\cos 7x}=\left(\frac{0}{0}\right)=$$

$$=\lim_{x\to 0}\frac{(7\sin 7x)'}{(\sin 7x+7x\cos 7x)'}==\lim_{x\to 0}\frac{49\cos 7x}{7\cos 7x+7(\cos 7x-x\sin 7x)}=\frac{49}{14}=\frac{7}{2};$$

b) at the value of $x=0$ we obtain the indeterminacy of the form $\infty-\infty$. Then the fraction is reduced to a common denominator, and further the properties of equivalence infinitesimals are used.

$$\lim_{x\to 0}\left(\frac{1}{\sin^2 x}-\frac{1}{x^2}\right)=|\infty-\infty|=\lim_{x\to 0}\frac{x^2-\sin^2 x}{x^2\cdot\sin^2 x}=\left|\frac{0}{0}\right|=\lim_{x\to 0}\frac{x^2-\sin^2 x}{x^2\cdot x^2}=$$

$$=\lim_{x\to 0}\frac{(x^2-\sin^2 x)'}{(x^4)'}=\lim_{x\to 0}\frac{2x-\sin 2x}{4x^3}=\lim_{x\to 0}\frac{2-2\cos 2x}{12x^2}=\lim_{x\to 0}\frac{2\sin^2 x}{6x^2}=\lim_{x\to 0}\frac{2x^2}{6x^2}=\frac{1}{3}.$$

Task 12. Investigate the function $f(x)=\begin{cases}-x; & x\le 0\\ x^2; & 0<x\le 2\\ x-1; & x>2\end{cases}$ for continuity and construct the graph.

Solution:

this function is defined in the interval $(-\infty;\ +\infty)$ and three components of this function are defined for all x from the corresponding intervals, thus, only the points at which the analytical expression of the function changes may be the points of discontinuity, i.e. $x = 0$ and $x = 2$. Let us investigate these points. Let us find the one-sided limits in $x = 0$: $f(0-0)=\lim\limits_{x\to 0-0}(-x)=0$; $f(0+0)=\lim\limits_{x\to 0+0}x^2=0$; the function value at this point $f(0)=-0=0$. Thus, $f(0-0)=f(0+0)=f(0)=0$, therefore $x = 0$ is the continuity point.

Similarly, for $x = 2$: $f(2-0)=\lim\limits_{x\to 2-0}x^2=4$; $f(2+0)=\lim\limits_{x\to 2+0}(x-1)=1$;

$f(2)=2^2=4$. Since $f(2-0)\ne f(2+0)$, then we have the simple discontinuity at the point $x = 2$. $f(2+0)-f(2-0)=1-4=-3$ is the jump of function at this point. Graph has the form:

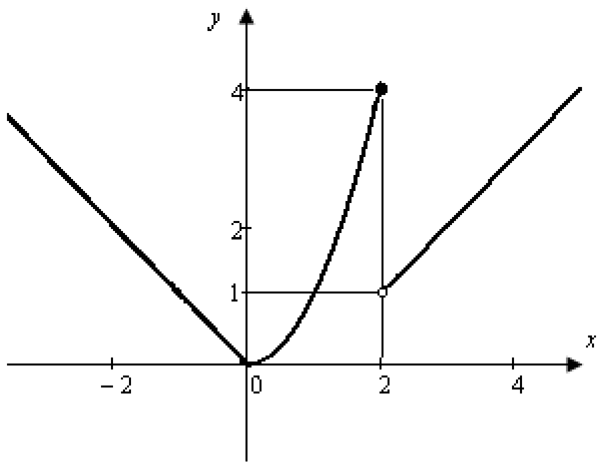

Figure 1

Task 13. Find the second derivative of the function given by the parametric equations $\begin{cases} x=\dfrac{1}{\cos t}\\ y=tg\,t\end{cases}$.

Solution:

the first derivative of the function given parametrically $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ is defined by the formula $y'_x = \frac{y'_t}{x'_t}$. The second derivative – by the formula $y''_{xx} = \frac{(y'_x)'_t}{x'_t}$.

For our function as $x'_t = \frac{\sin t}{\cos^2 t}$, $y'_t = \frac{1}{\cos^2 t}$,

then $y'_x = \frac{y'_t}{x'_t} = \frac{1}{\cos^2 t} \cdot \frac{\cos^2 t}{\sin t} = \frac{1}{\sin t}$;

$$y''_{xx} = \frac{(y'_x)'_t}{x'_t} = \left(\frac{1}{\sin t}\right)' \cdot \frac{\cos^2 t}{\sin t} = -\frac{\cos t}{\sin^2 t} \cdot \frac{\cos^2 t}{\sin t} = -ctg^3 t.$$

Task 14. For the function $y = \frac{x^2+1}{3x}$ find:

a) the intervals of monotonicity, the extremums;
b) the intervals of concavity and convexity, the inflection points;
c) construct the graph.

Solution:

a) $y' = \left(\frac{x^2+1}{3x}\right)' = \frac{1}{3} \cdot \frac{2x^2 - x^2 - 1}{x^2} = \frac{x^2-1}{3x^2}$,

$y' = 0$ at $x_1 = -1$, $x_2 = 1$ -are stationary points. In the domain of the function y' exists. Let us investigate the change of sign of the derivative of function in each interval:

if $0 < x < 1$, then $y' < 0$ - the function decreases;

if $1 < x < +\infty$, then $y' > 0$ - the function increases;

at $x = 1$ the function has a local minimum: $y(1) = \frac{1+1}{3} = \frac{2}{3}$;

if $-\infty < x < -1$, then $y' > 0$ - the function increases;

if $-1 < x < 0$, then $y' < 0$ - the function decreases;

at $x = -1$ we have a local maximum: $y(-1) = -\frac{2}{3}$;

b) $y'' = \left(\dfrac{x^2-1}{3x^2}\right)' = \dfrac{1}{3}\left(1-\dfrac{1}{x^2}\right)' = \dfrac{2}{3x^3}$, $y'' \neq 0$.

The second derivative exists in the domain of definition:
if $x \in (0; +\infty)$, then $y'' > 0$ - the graph is concave (convex down);
if $x \in (-\infty; 0)$, then $y'' < 0$ - the graph is convex up; no inflection points;

c) the graph of the function $y = \dfrac{x^2+1}{3x}$ is shown in Figure 2.

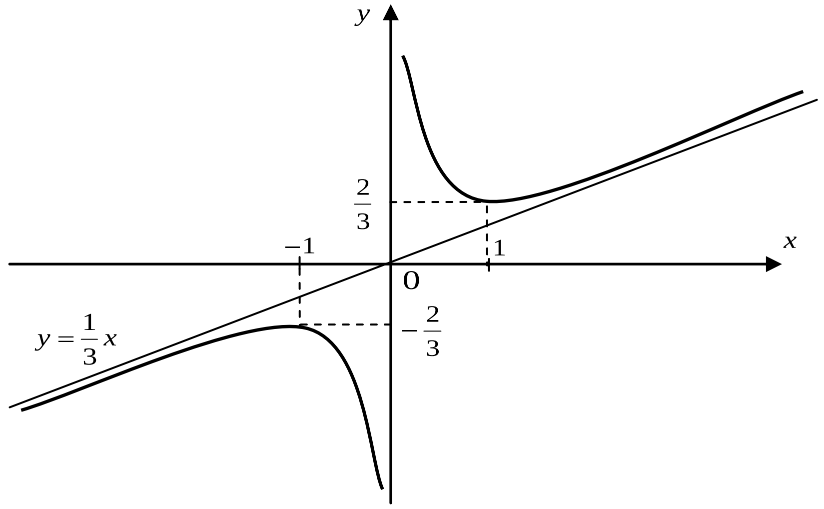

Figure 2

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