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ALMATY UNIVERSITY OF POWER ENGINEERING AND TELECOMMUNICATIONS

The department of
Higher mathematics

MATHEMATICS 1

Methodological Guidelines and Tasks
for carrying out the calculation-graphical work for students of specialties
5B071700 – Heat power engineering,
5B071800 – Electrical power engineering,
5B071900 – Radio engineering, electronics and telecommunications
Part 3

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Methodological Guidelines and Tasks for carrying out the calculation-graphical work contain section of the first semester program of course of mathematics for students of AUPET: integral calculus of one variable function.

The basic theoretical questions of the program and the solution of an exemplary embodiment are given.

Tables 22, figure 1, bibl. 6

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Introduction

Tasks of calculation-graphical work (CGW) №3 are given in the present work. Tasks correspond to the program of section «Integral calculus of one variable function» of discipline «Mathematics 1». The necessary theoretical material is given in Summary of lectures [2].

The basic methodological guidelines are given in the form of formulas for solving the problems. The solution of an exemplary embodiment is given.

CGW is performed in a separate thin notebook. In the number of each task the second digit indicates the variant.

Calculation-graphical work. Integral calculus of one variable function

Purpose: teach students to possess the classical mathematical methods of integration of functions. Give students the skills of finding indefinite and definite integrals, and the ability to use the gained knowledge in the study of further mathematics course and sections of other special disciplines.

1 Theoretical questions

1. Antiderivative. Indefinite integral and its properties.
2. Table of integrals.
3. Decomposition of proper rational function in partial fractions. Integration of proper rational function.
4. Integration of irrational functions.
5. Integration of trigonometric functions. Universal substitution. Partial substitutions and rules for their application.
6. Definite integral as the limit of integral sums. Basic properties. The Newton-Leibniz formula. Application of the definite integral.
7. Substitution method in indefinite and definite integrals.
8. Integration by parts of indefinite and definite integrals.

2 Calculated tasks

Task 1. Take out the function $f(x)$ from the sign of differential.

№	$d(f(x))$	№	$d(f(x))$	№	$d(f(x))$
1.1	$d\left(\frac{1}{\sqrt{x^2-1}}\right)$	1.2	$d\left(\sqrt[4]{2x+5}\right)$	1.3	$d\left(\ln(2-x)\right)$
1.4	$d\left(\cos(3x-1)\right)$	1.5	$d\left(2^{\sin 2x}\right)$	1.6	$d(\sin(4x-5))$
1.7	$d\left(\arcsin(2x-1)\right)$	1.8	$d\left(\frac{2}{\sqrt[3]{x^7}}\right)$	1.9	$d\left(arctg(2x+5)\right)$

1.10	$d\left(e^{\cos 2x}\right)$	1.11	$d\left(\frac{1}{\sqrt{x^2+1}}\right)$	1.12	$d\left(\sqrt[3]{x+35}\right)$
1.13	$d\left(\ln(2x+7)\right)$	1.14	$d\left(2\cos(4x+1)\right)$	1.15	$d\left(3^{\sin 5x}\right)$
1.16	$d(2\sin(x+6))$	1.17	$d\left(e^{tg\,2x}\right)$	1.18	$d\left(\arcsin(x-2)\right)$
1.19	$d\left(arctg(3x+15)\right)$	1.20	$d\left(\arccos(2x-1)\right)$	1.21	$d\left(\frac{25}{\sqrt[3]{x^{10}}}\right)$
1.22	$d\left(\arcsin(2x+5)\right)$	1.23	$d\left(e^{x^2+2}\right)$	1.24	$d\left(\frac{2}{\sqrt{x^2-16}}\right)$
1.25	$d\left(\sqrt[5]{3x-5}\right)$	1.26	$d\left(\ln(2-3x)\right)$	1.27	$d(3\sin(4x+25))$
1.28	$d\left(5^{\sin 2x}\right)$	1.29	$d\left(4\cos(5x-2)\right)$	1.30	$d\left(\arccos(x+3)\right)$

Task 2. Introduce the function $f(x)$ under the sign of differential.

№	$f(x)$	№	$f(x)$	№	$f(x)$
2.1	$\frac{5}{x^3}$	2.2	$\sin(3x+5)$	2.3	3^{2x+1}
2.4	$\cos(3-2x)$	2.5	$\frac{1}{x\ln x}$	2.6	$tg(6x-2)$
2.7	$\frac{1}{\sqrt{x}}$	2.8	$ctg(x+2)$	2.9	$\frac{2}{\sqrt[3]{3-x}}$
2.10	$(2x-1)^7$	2.11	$\frac{15}{(x+2)^3}$	2.12	5^{-2x+1}
2.13	$3\sin(5x+9)$	2.14	$\frac{1}{(x+1)\ln(x+1)}$	2.15	$5\cos(3+4x)$
2.16	$(2+12x)^4$	2.17	$tg(6-2x)$	2.18	$\frac{1}{\sqrt{2x+1}}$
2.19	$-ctg(x-2)$	2.20	$\frac{2}{\sqrt[3]{3+2x}}$	2.21	$\frac{3}{(2x-1)^2}$
2.22	$\frac{1}{\sqrt{4-2x}}$	2.23	4^{2x-15}	2.24	$-\sin(2x-5)$
2.25	$(2x+4)^9$	2.26	$\frac{1}{(2x-1)\ln(2x-1)}$	2.27	$7\cos(5x-1)$
2.28	3^{5-x}	2.29	$-tg(x-2)$	2.30	$ctg(5x+2)$

Task 3. Find the indefinite integral.

№	Task	№	Task	№	Task
3.1	$\int 2^x \cdot 5^x dx$	3.2	$\int 3^x\left(5-7^{-x}\right)dx$	3.3	$\int 7^{-x}\left(\sqrt{7^{2x}}+5^x\right)dx$

3.4	$\int e^{-x}\left(2^x - 3\cdot e^{x+3}\right)dx$	3.5	$\int 5^{-x}\left(4 - 3^{x+1}\right)dx$	3.6	$\int \left(4\cdot e^x \cdot 5^x - \pi^{2x}\right)dx$
3.7	$\int 3^{-x}\left(7 + 2^{x+1}\right)dx$	3.8	$\int \left(e^x + \cdot e^{2x}\right)^2 dx$	3.9	$\int \left(e^x - 2\cdot e^{-x}\right)dx$
3.10	$\int e^x \cdot \sqrt{2^x}\,dx$	3.11	$\int 5^x \cdot 3^{1-x}\,dx$	3.12	$\int \pi^x \cdot e^{x+1}dx$
3.13	$\int \left(5^x - 5^{-x}\right)^2 dx$	3.14	$\int 3^{-x}\left(4 - 5^{x+2}\right)dx$	3.15	$\int \left(5^{x+1} + 5^{-x}\right)^2 dx$
3.16	$\int \frac{2^x + 3^x}{2^{x+1}}dx$	3.17	$\int \frac{3^{-x} + 2^{x+3}}{2^{x+1}}dx$	3.18	$\int \left(3^{\frac{x}{2}} - 3^{\frac{-x}{2}}\right)^2 dx$
3.19	$\int 4^x\left(7 + 2^{-x}\right)dx$	3.20	$\int e^x\left(1 - 5\cdot e^{3-x}\right)dx$	3.21	$\int 2^{-x}\left(3 - 5^{x+2}\right)dx$
3.22	$\int \pi^{2x}\left(\frac{1}{2^x} - 3^{x+1}\right)dx$	3.23	$\int \frac{1}{3^x}\left(e^x + 2\cdot e^{-x}\right)dx$	3.24	$\int \left(e^x - 2\right)^2 dx$
3.25	$\int \left(e^{-x} - e^x\right)^3 dx$	3.26	$\int \left(e^x + 2^{-x}\right)^2 dx$	3.27	$\int 3^x\left(7 + 2^{-x}\right)dx$
3.28	$\int \frac{3^{-x} + 2^{x+3}}{2^{x+1}}dx$	3.29	$\int \left(e^x + \cdot e^{2x}\right)^3 dx$	3.30	$\int \frac{1}{e^x}\left(e^{x+1} + 2\cdot e^{-x}\right)dx$

Task 4. Find the indefinite integral.

№	Task	№	Task	№	Task
4.1	$\int \frac{dx}{3\sqrt{9 - x^2}}$	4.2	$\int \frac{7dx}{\sqrt{x^2 + 9}}$	4.3	$\int \frac{5dx}{\sqrt{x^2 - 9}}$
4.4	$\int \frac{dx}{\sqrt{3x - x^2}}$	4.5	$\int \frac{dx}{\sqrt{5 + x^2}}$	4.6	$\int \frac{dx}{\sqrt{x^2 + 7}}$
4.7	$\int \frac{dx}{3x^2 + 1}$	4.8	$\int \frac{dx}{3x^2 - 1}$	4.9	$\int \frac{dx}{1 - 3x^2}$
4.10	$\int \frac{dx}{4 - 5x^2}$	4.11	$\int \frac{dx}{\sqrt{x^2 - 9}}$	4.12	$\int \frac{dx}{\sqrt{16 - x^2}}$
4.13	$\int \frac{dx}{\sqrt{x^2 - 3}}$	4.14	$\int \frac{dx}{\sqrt{x^2 + 16}}$	4.15	$\int \frac{dx}{\sqrt{1 - 9x^2}}$
4.16	$\int \frac{dx}{\sqrt{3x^2 - 27}}$	4.17	$\int \frac{dx}{\sqrt{2x^2 + 8}}$	4.18	$\int \frac{dx}{\sqrt{4 - x^2}}$
4.19	$\int \frac{dx}{\sqrt{4 + x^2}}$	4.20	$\int \frac{dx}{\sqrt{1 + 9x^2}}$	4.21	$\int \frac{dx}{\sqrt{x^2 + 5}}$
4.22	$\int \frac{dx}{\sqrt{x^2 - 5}}$	4.23	$\int \frac{\sqrt{x^2 - 3}}{x^2 - 3}dx$	4.24	$\int \frac{\sqrt{x^2 + 3}}{2x^2 + 6}dx$

4.25	$\int \frac{x^{2+}\sqrt{x^2-4}}{x^2-4}dx$	4.26	$\int \frac{\sqrt{x^2+9}-x^2}{x^2+9}dx$	4.27	$\int \left(x^2-5\right)^{-\frac{1}{2}}dx$
4.28	$\int \left(x^2+5\right)^{-\frac{1}{2}}dx$	4.29	$\int \left(3-x^2\right)^{-\frac{1}{2}}dx$	4.30	$\int \left(x^2+25\right)^{-\frac{1}{2}}dx$

Task 5. Find the indefinite integral.

№	Task	№	Task	№	Task
5.1	$\int\left(\frac{5}{9+x^2}\right)dx$	5.2	$\int\left(\frac{7}{3-x^2}-\frac{4}{5+x^2}\right)dx$	5.3	$\int \frac{dx}{1-7x^2}$
5.4	$\int \frac{dx}{3x^2+2}$	5.5	$\int \frac{dx}{2x^2-3}$	5.6	$\int \frac{dx}{3+7x^2}$
5.7	$\int \frac{2}{2x^2-1}dx$	5.8	$\int \frac{dx}{5-3x^2}$	5.9	$\int \frac{dx}{3x^2-2}$
5.10	$\int\left(\frac{1}{x^2}-\frac{2}{x^2+2}\right)dx$	5.11	$\int\left(\frac{3}{x^2}-\frac{4}{x^2-2}\right)dx$	5.12	$\int\left(\frac{5}{x^2-3}-\frac{7}{x^2+3}\right)dx$
5.13	$\int \frac{dx}{4x^2+1}$	5.14	$\int\left(\frac{3}{x^2-5}-\frac{2}{x^2+3}\right)dx$	5.15	$\int \frac{dx}{4x^2-1}$
5.16	$\int\left(\frac{3}{4-x^2}-\frac{4}{1+x^2}\right)dx$	5.17	$\int\left(\frac{7}{9-x^2}+\frac{4}{25+x^2}\right)dx$	5.18	$\int \frac{dx}{4x^2+25}$
5.19	$\int \frac{dx}{16-27x^2}$	5.20	$\int \frac{2}{25x^2-1}dx$	5.21	$\int \frac{dx}{63+7x^2}$
5.22	$\int \frac{dx}{25x^2-36}$	5.23	$\int\left(\frac{5}{x^2+36}\right)dx$	5.24	$\int \frac{6dx}{36x^2-25}$
5.25	$\int\left(\frac{1}{x^2}-\frac{2}{x^2+4}\right)dx$	5.26	$\int\left(\frac{2}{9+x^2}+\frac{4}{4-x^2}\right)dx$	5.27	$\int \frac{dx}{49x^2+16}$
5.28	$\int \frac{dx}{4x^2-1}$	5.29	$\int \frac{dx}{64x^2+1}$	5.30	$\int\left(\frac{7}{81-x^2}\right)dx$

Task 6. Find the indefinite integral using the table of integrals and the property of the linear argument of the integrand.

№	Task	№	Task	№	Task
6.1	$\int(\cos 2x-3\sin x)dx$	6.2	$\int \frac{dx}{\cos^2 3x}$	6.3	$\int \frac{dx}{1-\sin^2 5x}$
6.4	$\int tg^2 2xdx$	6.5	$\int\left(e^{5x}-3e^{-x}+4e^{\frac{x}{2}}\right)dx$	6.6	$\int\left(e^{4x}-2e^{-3x}\right)^2 dx$

6.7	$\int \sin(2x+3)dx$	6.8	$\int \frac{2^{3x}-3^x}{3^{2x}}dx$	6.9	$\int \frac{\left(e^{-x}+5\right)^2}{e^x}dx$
6.10	$\int \left(\sqrt[3]{e^x}-x\right)dx$	6.11	$\int \frac{dx}{\sqrt{e^{3x}}}$	6.12	$\int \frac{dx}{\sqrt{6e^{5x-1}}}$
6.13	$\int (\cos x+\sin x)^2 dx$	6.14	$\int (\cos x-\sin x)^2 dx$	6.15	$\int (\cos 3x+\sin 3x)^2 dx$
6.16	$\int (\cos 2x-\sin 2x)^2 dx$	6.17	$\int \frac{dx}{\sin^2 5x}$	6.18	$\int \frac{dx}{\cos^2(x/3)}$
6.19	$\int \cos(1-5x)dx$	6.20	$\int \sin^2 x dx$	6.21	$\int \cos^2 x dx$
6.22	$\int \left(\cos\frac{x}{3}-\sin\frac{x}{2}\right)dx$	6.23	$\int \cos x\cdot(1-5\cdot\cos x)dx$	6.24	$\int \frac{5^{2x}-3^{-x}}{2^x}dx$
6.25	$\int \frac{5^{2x-1}-5^{2-2x}}{4^x}dx$	6.26	$\int \frac{e^x-e^{-x}}{e^{2x}}dx$	6.27	$\int \frac{e^{x+1}+e^{1-x}}{e^x}dx$
6.28	$\int \left(\cos\frac{x}{3}-5\sin\frac{x}{2}\right)^2 dx$	6.29	$\int \cos x(1-25\cos x)dx$	6.30	$\int \sin^2 x dx$

Task 7. Find the indefinite integral.

№	Task	№	Task	№	Task
7.1	$\int \frac{dx}{x-5}$	7.2	$\int \frac{x}{(x+3)^2}dx$	7.3	$\int \frac{dx}{3x+1}$
7.4	$\int \frac{dx}{5-7x}$	7.5	$\int \frac{dx}{\sqrt{x+9}}$	7.6	$\int \frac{dx}{\sqrt[3]{2-x}}$
7.7	$\int (x-a)^4 dx$	7.8	$\int \sqrt[5]{(x+3)^4}dx$	7.9	$\int \frac{dx}{\sqrt[3]{(x+2)^4}}$
7.10	$\int \frac{dx}{(x+1)\cdot\sqrt{2x+2}}$	7.11	$\int \sqrt[3]{(x+2)^5}dx$	7.12	$\int \sqrt{\frac{7}{(x-2)^5}}dx$
7.13	$\int \left(\sqrt[3]{\frac{1}{x+1}}\right)^5 dx$	7.14	$\int \sqrt{\frac{5}{(x-2)^7}}dx$	7.15	$\int \sqrt[4]{(x+a)^5}dx$
7.16	$\int \frac{x}{(x-3)^5}dx$	7.17	$\int \frac{x}{\sqrt{x+2}}dx$	7.18	$\int \frac{1-2x}{\sqrt{x+1}}dx$
7.19	$\int \frac{x}{(x-2)^3}dx$	7.20	$\int \frac{x}{\sqrt{x+2}}dx$	7.21	$\int \frac{x}{(x-3)^5}dx$
7.22	$\int \frac{dx}{x-5}$	7.23	$\int \frac{x}{(x-3)^2}dx$	7.24	$\int \frac{dx}{3x+1}$

7.25	$\int \frac{dx}{(x+25)\cdot\sqrt{x+25}}$	7.26	$\int \sqrt[3]{(x+2)^5}\,dx$	7.27	$\int \sqrt{\frac{1}{(x-21)^5}}\,dx$
7.28	$\int \frac{dx}{(25x+1)\cdot\sqrt{25x+1}}$	7.29	$\int \sqrt[3]{(x+25)^7}\,dx$	7.30	$\int \frac{3dx}{(x-5)^3}$

Task 8. Find the indefinite integral by introducing under the sign of differential.

№	Task	№	Task	№	Task
8.1	$\int e^{x^2}\cdot xdx$	8.2	$\int e^{-x^2}\cdot xdx$	8.3	$\int e^{5x^2}\cdot xdx$
8.4	$\int e^{\frac{x^2}{2}}\cdot xdx$	8.5	$\int e^{x^2+3}\cdot xdx$	8.6	$\int e^{1-3x^2}\cdot xdx$
8.7	$\int x\cdot\sqrt{x^2+1}dx$	8.8	$\int x\cdot\sqrt[3]{(x^2+3)^4}\,dx$	8.9	$\int \frac{xdx}{\sqrt{x^2+9}}$
8.10	$\int \frac{xdx}{x^2+10}$	8.11	$\int \frac{e^x dx}{e^x+5}$	8.12	$\int e^x\cdot\sqrt{3e^x-7}dx$
8.13	$\int \frac{e^x dx}{\sqrt{e^{2x}-5}}$	8.14	$\int \frac{e^x dx}{e^{2x}+10}$	8.15	$\int \frac{\ln^2 x}{x}dx$
8.16	$\int \frac{dx}{x\cdot\ln^3 x}$	8.17	$\int \frac{dx}{x\cdot\ln x}$	8.18	$\int tgxdx$
8.19	$\int ctgxdx$	8.20	$\int \cos x\cdot\sqrt{\sin x}dx$	8.21	$\int e^{-x^3}\cdot x^2 dx$
8.22	$\int \frac{e^x dx}{e^{2x}-4}$	8.23	$\int \frac{dx}{x\cdot(\ln^2 x+1)}$	8.24	$\int x\cdot\sqrt{(x^2+3)^5}\,dx$
8.25	$\int \frac{\sqrt{\ln x}}{x}dx$	8.26	$\int 4^{\cos x}\sin x dx$	8.27	$\int \frac{\cos x}{2+3\sin x}dx$
8.28	$\int \frac{x^2}{1-x^3}dx$	8.29	$\int \frac{6^x}{1-6^x}dx$	8.30	$\int \frac{e^x}{(e^x-1)^2}dx$

Task 9. Find the indefinite integral (integration of quadratic trinomial).

№	Task	№	Task	№	Task
9.1	$\int \frac{dx}{x^2+8x+5}$	9.2	$\int \frac{dx}{x^2-x+1}$	9.3	$\int \frac{dx}{\sqrt{x^2-6x+10}}$
9.4	$\int \frac{dx}{\sqrt{x^2+6x+11}}$	9.5	$\int \frac{dx}{\sqrt{x^2+3x+1}}$	9.6	$\int \frac{dx}{\sqrt{1+2x-x^2}}$

9.7	$\int \frac{dx}{\sqrt{2-2x-x^2}}$	9.8	$\int \frac{xdx}{x^2+2x+5}$	9.9	$\int \frac{(x-1)dx}{x^2-2x+5}$
9.10	$\int \frac{1-x}{x^2-x+1}dx$	9.11	$\int \frac{dx}{x^2+x+2}$	9.12	$\int \frac{dx}{x+(x+1)^2}$
9.13	$\int \frac{3x+1}{(x-2)^2+(x+1)}dx$	9.14	$\int \frac{x^2+7}{x^2+4}dx$	9.15	$\int \frac{dx}{x+x^2+3}$
9.16	$\int \frac{x+1}{x+(x^2+3)}dx$	9.17	$\int \frac{dx}{x^3+1}$	9.18	$\int \frac{dx}{x^3-8}$
9.19	$\int \frac{x}{x^2+6x-1}dx$	9.20	$\int \frac{x}{x^2-4x+1}dx$	9.21	$\int \frac{3x+5}{\sqrt{x^2+6x-5}}dx$
9.22	$\int \frac{1-2x}{x^2-6x+10}dx$	9.23	$\int \frac{dx}{\sqrt{3+2x-x^2}}$	9.24	$\int \frac{dx}{2x^2-3x+1}$
9.25	$\int \frac{dx}{5x-x^2-6}$	9.26	$\int \frac{(2x-3)dx}{4+3x-x^2}$	9.27	$\int \frac{dx}{\sqrt{x^2-4x+5}}$
9.28	$\int \frac{dx}{x^2+x-3}$	9.29	$\int \frac{2x+1}{x^2+x-3}dx$	9.30	$\int \frac{dx}{x^2+8x+7}$

Task 10. Find the indefinite integral using integration by parts.

№	Task	№	Task	№	Task
10.1	$\int x \cdot e^{2x}dx$	10.2	$\int (x-1) \cdot e^{-x}dx$	10.3	$\int (2x+5) \cdot e^{\frac{x}{3}}dx$
10.4	$\int x^2 \cdot e^x dx$	10.5	$\int x \cdot 3^x dx$	10.6	$\int x \cdot \ln x dx$
10.7	$\int x \cdot \ln 5x dx$	10.8	$\int \sqrt{x} \cdot \ln x dx$	10.9	$\int (\ln x - 2\ln^2 x)dx$
10.10	$\int \frac{\ln x}{x^2}dx$	10.11	$\int x \cdot arctgx dx$	10.12	$\int x \cdot \cos x dx$
10.13	$\int x \cdot \sin x dx$	10.14	$\int x \cdot \cos 3x dx$	10.15	$\int x \cdot \sin \frac{x}{2}dx$
10.16	$\int x \cdot \cos \frac{x}{3}dx$	10.17	$\int x \cdot \sin 5x dx$	10.18	$\int x^2 \cdot e^{-x}dx$
10.19	$\int x \cdot 5^x dx$	10.20	$\int \frac{x}{2^x}dx$	10.21	$\int x^2 \sqrt{e^x}dx$
10.22	$\int x \cdot e^{-3x}dx$	10.23	$\int x \cdot \sin \frac{x}{3}dx$	10.24	$\int x \cdot e^{\frac{x}{2}}dx$
10.25	$\int \sqrt[3]{x} \ln x dx$	10.26	$\int (x+5)3^x dx$	10.27	$\int \frac{\ln x}{\sqrt[3]{x}}dx$

10.28	$\int x \cdot \sin 3x dx$	10.29	$\int \frac{x}{7^x} dx$	10.30	$\int x\sqrt{e^x} dx$

Task 11. Find the indefinite integral by selecting the whole part of improper rational function.

№	Task	№	Task	№	Task
11.1	$\int \frac{3x^2+1}{x^2+25} dx$	11.2	$\int \frac{x^2-3+2x+6}{x^2-9} dx$	11.3	$\int \frac{x^3}{x^2+9} dx$
11.4	$\int \frac{x^3}{x^2+9} dx$	11.5	$\int \frac{x^2+1}{x^2+5} dx$	11.6	$\int \frac{2x^2-2}{x^2-16} dx$
11.7	$\int \frac{x^2+1}{x^2-25} dx$	11.8	$\int \frac{x^2+2}{x^2-1} dx$	11.9	$\int \frac{x^2+2x+4}{x^2+2x-1} dx$
11.10	$\int \frac{x^2+2}{x^2-1} dx$	11.11	$\int \frac{3x^2+1}{x^2+6x+25} dx$	11.12	$\int \frac{x^2+x-2}{x^2-8x+20} dx$
11.13	$\int \frac{x^2+2}{x^2-6x+10} dx$	11.14	$\int \frac{x^2+7}{x^2+4} dx$	11.15	$\int \frac{x^4}{x^2+9} dx$
11.16	$\int \frac{x^2+2x}{x^2-8x+25} dx$	11.17	$\int \frac{x^2+1}{x^2+4x+5} dx$	11.18	$\int \frac{x^2+2}{x^2-4x+13} dx$
11.19	$\int \frac{x^3}{x^2-1} dx$	11.20	$\int \frac{x^4}{x^2+1} dx$	11.21	$\int \frac{x^3}{x^3+8} dx$
11.22	$\int \frac{x^2+3}{x^2+9} dx$	11.23	$\int \frac{3x^2+2x+1}{x^2+25} dx$	11.24	$\int \frac{x^2-4x+2}{x^2-1} dx$
11.25	$\int \frac{x^2+2}{x^2+6x-1} dx$	11.26	$\int \frac{x^2+2x+16}{x^2-9} dx$	11.27	$\int \frac{x^2+x+1}{x^2+5} dx$
11.28	$\int \frac{3x^2+1}{x^2-2x+26} dx$	11.29	$\int \frac{x^3+2x}{x^2+49} dx$	11.30	$\int \frac{x^2-3+2x}{x^2-9} dx$

Task 12. Find the indefinite integral of proper rational function.

№	Task	№	Task	№	Task
12.1	$\int \frac{dx}{(x-2)^2(x+3)}$	12.2	$\int \frac{(x+1)}{(x-3)^2(x+4)} dx$	12.3	$\int \frac{(x-1)}{(x+2)^2(x-3)} dx$
12.4	$\int \frac{(x+1)}{x^2(x+3)} dx$	12.5	$\int \frac{x-7}{(x+5)(x-4)^3} dx$	12.6	$\int \frac{(x-14)dx}{(x+2)^2(x+3)}$
12.7	$\int \frac{(2x+1)}{(x+3)^2(x+4)} dx$	12.8	$\int \frac{(2x-1)}{(x+2)^2(2x-3)} dx$	12.9	$\int \frac{(2x+11)}{x^2(x+3)} dx$

12.10	$\int \frac{(x-1)dx}{(x-2)^3(x+3)}$	12.11	$\int \frac{dx}{x^2(x+2)}$	12.12	$\int \frac{dx}{x\cdot(x+1)^2}$
12.13	$\int \frac{3x+1}{(x-2)^2\cdot(x+1)}dx$	12.14	$\int \frac{x^2+7}{(x^2+4)(x-1)}dx$	12.15	$\int \frac{dx}{x\cdot(x^2+3)}$
12.16	$\int \frac{x^2+1}{x\cdot(x^2+3)}dx$	12.17	$\int \frac{dx}{x^3+1}$	12.18	$\int \frac{dx}{x^3-8}$
12.19	$\int \frac{x^3}{x^4-1}dx$	12.20	$\int \frac{x^2}{(x^3+1)(x+1)}dx$	12.21	$\int \frac{(4x+1)}{x^2(x+3)}dx$
12.22	$\int \frac{2x-7}{(x-5)(x-4)^3}dx$	12.23	$\int \frac{(x+12)}{(x-3)^3(x+4)}dx$	12.24	$\int \frac{(2x+1)dx}{(x-2)^3(x+3)}$
12.25	$\int \frac{(3x-1)}{(x+2)^3(x-3)}dx$	12.26	$\int \frac{(x+1)}{x(x+3)^2}dx$	12.27	$\int \frac{3x-7}{(x+5)^2(x-4)}dx$
12.28	$\int \frac{(5x+1)}{x^3(x+3)}dx$	12.29	$\int \frac{(2x-1)}{(x-3)(x+4)^2}dx$	12.30	$\int \frac{(x-1)}{(x+2)(x-3)^2}dx$

Task 13. Find the indefinite integral of proper rational function.

№	Task	№	Task
13.1	$\int \frac{3x+13}{(x-1)(x^2+2x+5)}dx$	13.2	$\int \frac{5x+3}{(x+2)(x^2+2x+4)}dx$
13.3	$\int \frac{12-6x}{(x+1)(x^2-4x+13)}dx$	13.4	$\int \frac{2x^2+2x+20}{(x-1)(x^2+2x+10)}dx$
13.5	$\int \frac{3x-6}{(x+1)(x^2+6x+13)}dx$	13.6	$\int \frac{x^2+3x+20}{(x-1)(x^2+x+1)}dx$
13.7	$\int \frac{36}{(x+2)(x^2-2x+10)}dx$	13.8	$\int \frac{3x+13}{(x+3)(x^2+6x+13)}dx$
13.9	$\int \frac{6}{(x-2)(x^2+4x+13)}dx$	13.10.	$\int \frac{4x^2+2x+17}{(x-1)(x^2+2x+5)}dx$
13.11	$\int \frac{3x+13}{(x-1)(x^2+2x+5)}dx$	13.12	$\int \frac{x^2+2x+40}{(x+2)(x^2-2x+10)}dx$
13.13	$\int \frac{4x-x^2-12}{(x-1)(x^2+2x+10)}dx$	13.14	$\int \frac{x^2-13x+40}{(x+1)(x^2-4x+13)}dx$
13.15	$\int \frac{30}{(x+2)(x^2+6x+25)}dx$	13.16	$\int \frac{7x+3}{(x-4)(x^2+8x+20)}dx$

13.17	$\int \frac{36}{(x+1)(x^2-12x+40)}dx$	13.18	$\int \frac{2x^2+2x+20}{(x-1)(x^2+6x+13)}dx$
13.19	$\int \frac{2x^2+7x+7}{(x+1)(x^2+2x+5)}dx$	13.20	$\int \frac{19ч-x^2-34}{(x+1)(x^2-4x+13)}dx$
13.21	$\int \frac{5x+10}{(x+3)(x^2+2x+10)}dx$	13.22	$\int \frac{4x^2+38}{(x+2)(x^2-2x+10)}dx$
13.23	$\int \frac{8}{(x+1)(x^2+6x+13)}dx$	13.24	$\int \frac{18}{(x-3)(x^2-6x+25)}dx$
13.25	$\int \frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}dx$	13.26	$\int \frac{x+15}{(x+5)(x^2+2x+5)}dx$
13.27	$\int \frac{13x+3}{(x-1)(x^2+12x+40)}dx$	13.28	$\int \frac{72}{(x+2)(x^2-4x+20)}dx$
13.29	$\int \frac{2x^2+2x+20}{(x+1)(x^2+4x+5)}dx$	13.30	$\int \frac{3x+13}{(x-5)(x^2+12x+40)}dx$

Task 14. Find the indefinite integral.

№	Task	№	Task	№	Task
14.1	$\int \frac{1-5x^3}{x^4}dx$	14.2	$\int \frac{\sqrt{x}-3x\cdot\sqrt[3]{x}+x^5}{x^2}dx$	14.3	$\int \frac{\left(\sqrt{x}-2\sqrt[3]{x}\right)^2}{x}dx$
14.4	$\int x\cdot\sqrt[3]{a\cdot x}dx$	14.5	$\int\left(\frac{3}{\sqrt[3]{x^2}}-\frac{x+2}{\sqrt[4]{x}}\right)dx$	14.6	$\int\left(a^{\frac{1}{3}}-x^{\frac{1}{3}}\right)^3 dx$
14.7	$\int\left(5x^3-\frac{5}{\sqrt[3]{x^5}}\right)dx$	14.8	$\int\left(\frac{x^2}{\sqrt{x}}+\frac{a}{x^4}-x\right)dx$	14.9	$\int \frac{\left(\sqrt{x^3}-2x^2+3x^4\right)}{x}dx$
14.10	$\int\left(\sqrt{x}-\frac{1}{\sqrt[3]{x}}\right)^2 dx$	14.11	$\int\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)^3 dx$	14.12	$\int\left(x\cdot\sin a-\frac{4\sqrt{x}}{x^2}\right)dx$
14.13	$\int\left(\sqrt{\frac{x}{a}}-\sqrt[3]{x}\right)^2 dx$	14.14	$\int \frac{x^3-3\sqrt{x^5}}{\sqrt{x}}dx$	14.15	$\int \frac{x^4-2x-\sqrt{x}}{x^2}dx$
14.16	$\int x^2\cdot\sqrt[4]{b\cdot x}dx$	14.17	$\int \frac{\left(\sqrt{b\cdot x}-\frac{x}{b}\right)^2}{x}dx$	14.18	$\int \frac{(x-1)^3}{x\cdot\sqrt{x}}dx$
14.19	$\int \frac{(2x+a)^3}{x^2}dx$	14.20	$\int \frac{\left(x-\frac{a}{x}\right)^2}{\sqrt{x}}dx$	14.21	$\int \frac{\left(b\cdot x+\frac{a}{x}\right)^2}{\sqrt[3]{x}}dx$

14.22	$\int \frac{\left(1-b\sqrt[3]{x}\right)^3}{x^2}dx$	14.23	$\int \frac{(2x+b)^3}{x^3}dx$	14.24	$\int \frac{x^3-2\sqrt{x}+x\sqrt[3]{x}}{x}dx$
14.25	$\int \frac{(2x+5)^3}{x^2}dx$	14.26	$\int \frac{\left(x+\frac{1}{x}\right)^2}{\sqrt[4]{x}}dx$	14.27	$\int \frac{\left(2-a\sqrt{x^3}\right)^2}{x^2}dx$
14.28	$\int \frac{\left(1-b\sqrt{x}\right)^2}{x^3}dx$	14.29	$\int \frac{x^4-2\sqrt{x}+x}{x^2}dx$	14.30	$\int \frac{\left(2-\sqrt{x^3}\right)^3}{x^2}dx$

Task 15. Find the indefinite integral.

№	Task	№	Task
15.1	$\int\left(5\cdot\cos x-\frac{7}{\sin^2 x}\right)dx$	15.2	$\int\left(3\cdot\sin x+\frac{2}{\cos^2 x}\right)dx$
15.3	$\int \frac{\sin^2 x-3\cdot\sin x-\frac{5}{\sin x}}{\sin x}dx$	15.4	$\int \frac{5\cdot\sin 2x-5\cdot tgx}{\sin x\cdot\cos x}dx$
15.5	$\int \frac{\cos 2x-5\cdot\cos^3 x}{\cos^2 x}dx$	15.6	$\int \frac{1+5\cdot\cos^3 x}{\cos^2 2x}dx$
15.7	$\int \frac{1-3\sin^3 x}{\sin^2 x}dx$	15.8	$\int \frac{2+\cos^3 x}{\cos^2 x}dx$
15.9	$\int\left(5\cdot\sin x-\frac{4}{\cos^2 x}\right)dx$	15.10	$\int \frac{\sin 2x}{\cos^3 x}dx$
15.11	$\int \frac{\sin 2x}{\sin^3 x}dx$	15.12	$\int \frac{\cos 2x}{\cos^2 x}dx$
15.13	$\int \frac{\cos 2x}{\sin^2 x}dx$	15.14	$\int(3\cdot\cos x-4\cdot\sin x)dx$
15.15	$\int \frac{\cos^2 x+5\cdot\cos x}{\cos x}dx$	15.16	$\int \frac{\cos^2 2x-\cos^2 x}{\sin^4 x}dx$
15.17	$\int \frac{\cos^2 2x-\cos^2 x}{\sin^4 x}dx$	15.18	$\int\left(\frac{\sin x}{2}+\frac{7}{\cos^2 x}\right)dx$
15.19	$\int \frac{ctgx}{\sin x\cdot\cos x}dx$	15.20	$\int \frac{tgx}{\sin 2x}dx$
15.21	$\int \frac{ctgx}{\sin 2x}dx$	15.22	$\int \frac{\cos^2 2x-\cos^2 x}{\sin^4 x}dx$
15.23	$\int ctg^2 xdx$	15.24	$\int \frac{5\sin 2x}{\cos^3 x}dx$

15.25	$\int\left(1+tg^2x\right)dx$	15.26	$\int\frac{\cos 2x-\cos^2 x}{\sin^4 x}dx$
15.27	$\int\frac{\cos^2 x+5\cdot\cos x}{\cos x}dx$	15.28	$\int\frac{\cos\ x+5\cdot\cos x}{\cos x}dx$
15.29	$\int\frac{\cos 2x-\cos^2 x}{\sin^4 x}dx$	15.30	$\int\frac{5\sin 2x}{\cos^3 x}dx$

Task 16. Find the indefinite integral using the universal trigonometric substitution.

№	Task	№	Task
16.1	$\int\frac{dx}{5+2\sin x+3\cos x}$	16.2	$\int\frac{dx}{5-4\sin x+2\cos x}$
16.3	$\int\frac{dx}{5+3\sin x-5\cos x}$	16.4	$\int\frac{3\sin x-2\cos x}{1+\cos x}dx$
16.5	$\int\frac{dx}{3+2\sin x-\cos x}$	16.6	$\int\frac{dx}{10\sin x+5\cos x}$
16.7	$\int\frac{dx}{5-3\cos x}$	16.8	$\int\frac{dx}{8-4\sin x+7\cos x}$
16.9	$\int\frac{dx}{3+5\cos x}$	16.10	$\int\frac{dx}{3+2\sin x+3\cos x}$
16.11	$\int\frac{dx}{5+4\sin x}$	16.12	$\int\frac{dx}{3\sin x-4\cos x}$
16.13	$\int\frac{dx}{7+3\sin x}$	16.14	$\int\frac{dx}{8+4\cos x}$
16.15	$\int\frac{dx}{2+4\sin x+3\cos x}$	16.16	$\int\frac{dx}{7\sin x-3\cos x}$
16.17	$\int\frac{dx}{3-4\sin x}$	16.18	$\int\frac{dx}{5+\sin x+3\cos x}$
16.19	$\int\frac{dx}{5+4\sin x+3\cos x}$	16.20	$\int\frac{7+6\sin x-5\cos x}{1+\cos x}dx$
16.21	$\int\frac{dx}{3+\sin x+\cos x}$	16.22	$\int\frac{6\sin x+\cos x}{1+\cos x}dx$
16.23	$\int\frac{dx}{5-4\sin x+3\cos x}$	16.24	$\int\frac{dx}{5+3\cos x}$
16.25	$\int\frac{dx}{4\cos x+3\sin x}$	16.26	$\int\frac{dx}{4-4\sin x+3\cos x}$
16.27	$\int\frac{dx}{7-3\sin x+\cos x}$	16.28.	$\int\frac{dx}{3+5\sin x+3\cos x}$

16.29	$\int \frac{dx}{3\sin x - \cos x}$	16.30	$\int \frac{dx}{2 + \sin x - 3\cos x}$

Task 17. Calculate the definite integral.

№	Task	№	Task	№	Task
17.1	$\int_{-1}^{2} x^4 dx$	17.2	$\int_{0}^{1} \left(\sqrt{x-1}\right)^2 dx$	17.3	$\int_{3}^{10} \sqrt[3]{x-2} dx$
17.4	$\int_{1}^{8} \frac{dx}{\sqrt[3]{x^2}}$	17.5	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin 2x dx$	17.6	$\int_{-\frac{\pi}{8}}^{0} \frac{dx}{\cos^2 2x}$
17.7	$\int_{0}^{1} \frac{x+2}{x+1} dx$	17.8	$\int_{0.5}^{1} e^{-2x+1} dx$	17.9	$\int_{1}^{e} \frac{\ln x}{x} dx$
17.10	$\int_{0}^{\frac{\pi}{4}} x \sin x^2 dx$	17.11	$\int_{-1}^{1} \frac{dx}{x^2 + 2x + 5}$	17.12	$\int_{-2}^{-1} \frac{dx}{x^2}$
17.13	$\int_{0.5}^{1} \frac{dx}{(x+1)^2}$	17.14	$\int_{\frac{\pi}{4}}^{0} \cos^2 t dt$	17.15	$\int_{0}^{\frac{\pi}{2}} \frac{dx}{\cos^2\left(\frac{x}{2}\right)}$
17.16	$\int_{0}^{\pi} \left[\sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right)\right]^2 dx$	17.17	$\int_{-1}^{-0.5} \frac{x^2 - 1}{x^4} dx$	17.18	$\int_{-1}^{0} \frac{dx}{\sqrt{x+2}}$
17.19	$\int_{\frac{\pi}{4}}^{0} \sin^2 t dt$	17.20	$\int_{0}^{7} \sqrt[3]{x+1} dx$	17.21	$\int_{0}^{\frac{1}{2}} \frac{dx}{(x-1)^3}$
17.22	$\int_{\frac{\pi}{2}}^{\pi} \frac{dx}{\sin^2\left(\frac{x}{2}\right)}$	17.23	$\int_{\frac{1}{2}}^{1} \frac{x^2 + 1}{x^4} dx$	17.24	$\int_{0}^{16} \sqrt[4]{x^5} dx$
17.25	$\int_{\pi}^{2\pi} \frac{dx}{\cos^2\left(\frac{x}{4}\right)}$	17.26	$\int_{0}^{\frac{\pi}{3}} tg \frac{x}{2} dx$	17.27	$\int_{0.5}^{1} \frac{dx}{(2x+1)^3}$
17.28	$\int_{0.25}^{1} \frac{dx}{(4x+1)^2}$	17.29	$\int_{-2}^{2} \frac{dx}{x^2 + 4x + 5}$	17.30	$\int_{\frac{1}{3}}^{2} \frac{x^2 + 1}{x^4} dx$

Task 18. Calculate the definite integral using the substitution of integrand variable.

№	Task	№	Task
18.1	$\int_0^2 x^3 \cdot e^{x^2} dx$	18.2	$\int_0^{0.5} x^3 \cdot e^{-x^4} dx$
18.3	$\int_1^{\sqrt{2}} x^3 \cdot e^{-x^2} dx$	18.4	$\int_2^3 \frac{dx}{1+\sqrt{2x-1}}$
18.5	$\int_1^e \frac{dx}{x \cdot (\ln^2 x + 1)}$	18.6	$\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\cos x}$
18.7	$\int_1^6 \frac{dx}{x+\sqrt{3x-2}}$	18.8	$\int_4^9 e^{-2\sqrt{x}} dx$
18.9	$\int_0^2 \sqrt{4-x^2} dx$	18.10	$\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x} dx$
18.11	$\int_{\ln 1}^{\frac{\ln 3}{2}} \frac{dx}{e^x + e^{-x}}$	18.12	$\int_{-1}^1 \frac{xdx}{\sqrt{5+4x}}$
18.13	$\int_e^{e^2} \frac{dx}{x \cdot \ln^3 x}$	18.14	$\int_0^3 \sqrt{9-x^2} dx$
18.15	$\int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$	18.16	$\int_1^{\ln^2 2} e^{-\sqrt{x}} dx$
18.17	$\int_0^{\frac{\pi}{2}} \frac{dx}{4+3\cos x}$	18.18	$\int_0^4 \sqrt{x} \cdot e^{-\sqrt{x}} dx$
18.19	$\int_0^1 x^3 \cdot e^{x^2} dx$	18.20	$\int_0^{\sqrt{2}} \frac{xdx}{x^4+2x^2+25}$
18.21	$\int_{\frac{1}{2}}^1 \frac{e^{\frac{1}{x}}}{x^2} dx$	18.22	$\int_e^{e^2} \frac{dx}{x \cdot \sqrt{\ln^3 x}}$
18.23	$\int_{\frac{1}{\sqrt{3}}}^1 \frac{dx}{x \cdot \sqrt{x^2+1}}$	18.24	$\int_0^{\ln 2} \frac{dx}{e^x + 2e^{-x}}$
18.25	$\int_0^1 \sqrt{1-x^2} dx$	18.26	$\int_0^{\sqrt{2}} \frac{xdx}{x^4+12x^2+225}$

18.27	$\int\limits_{\frac{1}{\sqrt{3}}}^{1} \frac{dx}{x\cdot\sqrt{x^2+4}}$	18.28	$\int\limits_{e}^{e^2} \frac{dx}{x\cdot\sqrt{\ln^5 x}}$
18.29	$\int\limits_{e}^{e^2} \frac{dx}{x\cdot\sqrt{\ln^5 x}}$	18.30	$\int\limits_{\frac{1}{2}}^{1} \frac{e^{\frac{1}{x}}}{x^2} dx$

Task 19. Calculate the definite integral using the method of integration by parts and replacing of integrand variable.

№	Task	№	Task	№	Task
19.1	$\int\limits_{2}^{3} x\ln(x-1)dx$	19.2	$\int\limits_{-2}^{0} x^2 e^{-x/2} dx$	19.3	$\int\limits_{0}^{\pi/2} x\cos x dx$
19.4	$\int\limits_{0}^{\pi} x^2 \sin x dx$	19.5	$\int\limits_{-1/2}^{1/2} \arccos 2x dx$	19.6	$\int\limits_{1}^{2} (x-1)\ln(x) dx$
19.7	$\int\limits_{-1/2}^{0} xe^{-2x} dx$	19.8	$\int\limits_{-\pi}^{\pi} x\sin x\cos x dx$	19.9	$\int\limits_{-2/3}^{-1/3} x^2 e^{-3x} dx$
19.10	$\int\limits_{1}^{e} \frac{\ln^2 x}{x^2} dx$	19.11	$\int\limits_{1}^{e^2} \sqrt{x}\ln(x) dx$	19.12	$\int\limits_{0}^{1} arctg\sqrt{x} dx$
19.13	$\int\limits_{0}^{\pi} (x+2)\cos\frac{x}{2} dx$	19.14	$\int\limits_{0}^{\pi/8} x^2 \sin 4x dx$	19.15	$\int\limits_{1}^{3} x^2 \ln(x) dx$
19.16	$\int\limits_{1}^{2} \frac{\ln^2(x+1)}{(x+1)^2} dx$	19.17	$\int\limits_{3/2}^{2} arctg(2x-3) dx$	19.18	$\int\limits_{0}^{\pi/2} (x+3)\sin x dx$
19.19	$\int\limits_{1}^{e} x\ln^2(x) dx$	19.20	$\int\limits_{-3}^{0} (x-2)e^{-x/3} dx$	19.21	$\int\limits_{0}^{\pi/9} \frac{xdx}{\cos^2 3x}$
19.22	$\int\limits_{1/2}^{1} \arcsin(1-x) dx$	19.23	$\int\limits_{1}^{\sqrt{3}} arctg\frac{1}{x} dx$	19.24	$\int\limits_{-1}^{0} x\ln(1-x) dx$
19.25	$\int\limits_{0}^{1} \frac{\arcsin(x/2)}{\sqrt{2-x}} dx$	19.26	$\int\limits_{1}^{2} \ln(3x+2) dx$	19.27	$\int\limits_{0}^{4} x^3\sqrt{x^2+9} dx$
19.28	$\int\limits_{-1}^{0} (x+1)e^{-2x} dx$	19.29	$\int\limits_{0}^{\pi/4} xtg^2 x dx$	19.30	$\int\limits_{0}^{1} xarctgx dx$

Task 20. Calculate the improper integral (or prove its divergence).

№	Task	№	Task	№	Task
20.1	$\int_0^{+\infty} \frac{dx}{9x^2+1}$	20.2	$\int_0^{+\infty} \frac{dx}{\sqrt{3x^2+5}}$	20.3	$\int_2^{+\infty} \frac{dx}{x\ln x}$
20.4	$\int_0^{+\infty} \frac{dx}{\sqrt{4-9x^2}}$	20.5	$\int_1^{+\infty} \frac{e^{-\sqrt{x}}dx}{\sqrt{x}}$	20.6	$\int_1^{+\infty} \frac{dx}{2^{3x-1}}$
20.7	$\int_0^{+\infty} \frac{dx}{(x+2)(2x+1)}$	20.8	$\int_0^{+\infty} \frac{dx}{\sqrt{x^2+25}}$	20.9	$\int_0^{+\infty} \frac{dx}{x^2+9}$
20.10	$\int_2^{+\infty} \frac{dx}{(x+1)\ln(x+1)}$	20.11	$\int_5^{+\infty} \frac{dx}{(x-1)(2x+4)}$	20.12	$\int_0^{+\infty} \frac{dx}{\sqrt{16-x^2}}$
20.13	$\int_1^{+\infty} \frac{dx}{3^{4x-1}}$	20.14	$\int_2^{+\infty} \frac{dx}{(x-1)(2x+6)}$	20.15	$\int_1^{+\infty} \frac{e^{-\sqrt{x+1}}dx}{\sqrt{x+1}}$
20.16	$\int_0^{+\infty} \frac{dx}{\sqrt{25-9x^2}}$	20.17	$\int_0^{+\infty} \frac{dx}{9x^2+16}$	20.18	$\int_0^{+\infty} \frac{dx}{\sqrt{16x^2+1}}$
20.19	$\int_1^{+\infty} \frac{e^{-\sqrt{2x}}dx}{\sqrt{2x}}$	20.20	$\int_2^{+\infty} \frac{dx}{x\ln 2x}$	20.21	$\int_1^{+\infty} \frac{dx}{5^{4x+1}}$
20.22	$\int_0^{+\infty} \frac{dx}{(x+1)(3x+7)}$	20.23	$\int_0^{+\infty} \frac{dx}{(5x+2)(x+3)}$	20.24	$\int_0^{+\infty} \frac{dx}{\sqrt{3x^2+15}}$
20.25	$\int_0^{+\infty} \frac{dx}{4x^2+1}$	20.26	$\int_2^{+\infty} \frac{dx}{\sqrt{64-49x^2}}$	20.27	$\int_1^{+\infty} \frac{e^{-\sqrt{3x+1}}dx}{\sqrt{3x+1}}$
20.28	$\int_1^{+\infty} \frac{dx}{6^{3x+11}}$	20.29	$\int_2^{+\infty} \frac{3dx}{(2x+1)\ln(2x+1)}$	20.30	$\int_0^{+\infty} \frac{dx}{\sqrt{4x^2+5}}$

Task 21. Calculate the improper integral (or prove its divergence).

№	Task	№	Task	№	Task
21.1	$\int_0^1 \frac{dx}{4x^2-1}$	21.2	$\int_0^{64} \frac{dx}{\sqrt[4]{x}}$	21.3	$\int_0^2 \frac{dx}{4x^2\cdot\sqrt[3]{x}}$
21.4	$\int_0^1 \frac{dx}{x^5\sqrt{x}}$	21.5	$\int_1^e \frac{dx}{x\ln x}$	21.6	$\int_{-1}^1 \frac{dx}{(x+1)(3x+7)}$
21.7	$\int_0^5 \frac{dx}{\sqrt{25-x^2}}$	21.8	$\int_0^2 \frac{dx}{4x^2-16}$	21.9	$\int_0^1 \frac{dx}{\sqrt[7]{x}}$

21.10	$\int_1^e \frac{dx}{x\sqrt{\ln x}}$	21.11	$\int_0^3 \frac{dx}{x^2\sqrt{x}}$	21.12	$\int_0^2 \frac{dx}{(x-1)(2x+1)}$
21.13	$\int_0^3 \frac{dx}{x^2\sqrt[3]{x}}$	21.14	$\int_0^{15} \frac{dx}{\sqrt{225-x^2}}$	21.15	$\int_0^1 \frac{dx}{\sqrt[5]{x}}$
21.16	$\int_2^4 \frac{dx}{(x-2)(5x+7)}$	21.17	$\int_1^e \frac{dx}{x\ln^2 x}$	21.18	$\int_0^2 \frac{dx}{x^2-1}$
21.19	$\int_0^3 \frac{dx}{\sqrt[3]{(x-3)^7}}$	21.20	$\int_0^8 \frac{dx}{x\sqrt[3]{x}}$	21.21	$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$
21.22	$\int_0^1 \frac{dx}{\sqrt[3]{x}}$	21.23	$\int_0^3 \frac{dx}{x\sqrt{x}}$	21.24	$\int_{-2}^0 \frac{dx}{(x+1)(3x+7)}$
21.25	$\int_1^9 \frac{dx}{4x^2-32}$	21.26	$\int_1^e \frac{\ln^{-2} x dx}{x}$	21.27	$\int_0^5 \frac{dx}{\sqrt{25-9x^2}}$
21.28	$\int_0^5 \frac{dx}{(x-4)(x+1)}$	21.29	$\int_0^{27} \frac{xdx}{\sqrt[3]{x}}$	21.30	$\int_0^3 \frac{dx}{\sqrt[3]{x-3}}$

Task 22. Calculate the area of the figure *D* bounded by lines.

№	The equations of the domain *D* boundaries	№	The equations of the domain *D* boundaries	№	The equations of the domain *D* boundaries
22.1	$y = x^2,\ y = 4$	22.2	$y = \sqrt{x},\ y = 0,$ $y = 1,\ y = 4$	22.3	$y = \frac{x^2}{2},\ y = 2x + \frac{5}{2}$
22.4	$y^2 = 16x,\ y = x$	22.5	$y^2 = x,\ y = x^2$	22.6	$y = x^2,\ y = 1 - x^2$
22.7	$y = x^2 - 5,\ y = x - 3$	22.8	$y = \cos^2 x,$ $y = 0, x = 0$	22.9	$y = x^2 - 2x,\ y - 3 = 0$
2.10	$y = x^2 - 1,\ y = x + 1$	22.11	$y = tgx,\ y = \sqrt{3},$ $x = 0$	22.12	$y = x^2 - 1,\ y = x + 3$
22.13	$y = ctgx,\ y = -1,$ $x = 0$	22.14	$y = -2x^2 + 3x + 6,$ $y = x + 2.$	22.15	$y = 4x - x^2,\ y = x.$
22.16	$x = y^2, x = 1. y = 0.$	22.17	$y = (x-1)^2,$ $y = x + 1.$	22.18	$y = 2x^2 + 1,\ y = x = 2,$ $y = 1.5$
22.19	$y = x^2 + 2x,$ $y = x + 2$	22.20	$y^2 = 4x,\ x^2 = 4y$	22.21	$y = -x^2,$ $y = -x - 2$
22.22	$y = \frac{x^2}{9},\ y = \frac{x}{3} + 2$	22.23	$y = tgx,\ y = -1,$ $x = 0$	22.24	$y = x^2,\ y = \frac{x^2}{2},$ $y = 5.$

22.25	$y = x^2 - x,\ y = 0,$ $x = 2.$	22.26	$y = x^2 + 4x,$ $y = 2x + 4$	22.27	$y = ctgx,\ y = 1,$ $x = \frac{\pi}{2}$
22.28	$y = tg2x,\ y = 1,$ $x = 0$	22.29	$y = x^2 + x,\ y = x + 1$	22.30	$y = tgx,\ y = 1,$ $x = 0$

3 Methodological guidelines for solving the problems of the section «Integral calculus of one variable function». Solution of an exemplary embodiment

Task 1 is solved by the formula $d(f(x)) = f'(x)dx$.

Task 2 is solved by using the same formula for the reverse action, i.e. $f'(x)dx = d\big(f(x)\big)$ or $f(x)dx = d(F(x))$, where $F(x)$ – the antiderivative function for function $f(x)$.

Tasks 3 – 5 must be solved by the method of direct integration, i.e. using the rules of integration, identity transformations of integrands and table of basic integrals.

We give samples of examples for these tasks. Find the indefinite integral:

1. $\int \frac{\left(a - \sqrt[3]{x}\right)^2}{x^3} dx.$

Solution:

$$\int \frac{\left(a - \sqrt[3]{x}\right)^2}{x^3} dx = \int \frac{a^2 - 2a \cdot \sqrt[3]{x} + \sqrt[3]{x^2}}{x^3} dx = \int \left(\frac{a^2}{x^3} - \frac{2a \cdot \sqrt[3]{x}}{x^3} + \frac{\sqrt[3]{x^2}}{x^3} \right) dx =$$

$$= \int (a^2 x^{-3} dx - 2a x^{-\frac{8}{3}} + x^{-\frac{7}{3}}) dx = a^2 \int x^{-3} dx - 2a \int x^{-\frac{8}{3}} dx + \int x^{-\frac{7}{3}} dx = a^2 \frac{x^{-3+1}}{-3+1} -$$

$$- 2a \frac{x^{-\frac{8}{3}+1}}{-\frac{8}{3}+1} + \frac{x^{-\frac{7}{3}+1}}{-\frac{7}{3}+1} + C = -\frac{a^2 x^{-2}}{2} + \frac{6a x^{-\frac{5}{3}}}{5} - \frac{3x^{-\frac{4}{3}}}{4} + C.$$

2. $\int \frac{5^{2x} - 3^x}{5^x} dx = \int \left(\frac{5^{2x}}{5^x} - \frac{3^x}{5^x} \right) dx = \int 5^x dx - \int \left(\frac{3}{5} \right)^x dx = \frac{5^x}{\ln 5} - \frac{\left(\frac{3}{5} \right)^x}{\ln \frac{3}{5}} + C.$

3. $\int \frac{1 + \cos^2 x}{1 + \cos 2x} dx = \left| \begin{array}{c} \textit{Apply the formula of trigonometry} \\ 1 + \cos 2x = 2\cos^2 x \end{array} \right| =$

$$= \int \frac{1+\cos^2 x}{2\cos^2 x}dx = \frac{1}{2}\int \frac{dx}{\cos^2 x} + \frac{1}{2}\int dx = \frac{1}{2}tg\,x + \frac{1}{2}x + C.$$

4. $$\int\left(\frac{6}{2x^2+9} + \frac{12}{2x^2-9}\right)dx = \int \frac{6dx}{2\left(x^2+\frac{9}{2}\right)} + \int \frac{12dx}{2\left(x^2-\frac{9}{2}\right)} =$$

$$= 3\cdot\frac{1}{\sqrt{\frac{9}{2}}}arctg\frac{x}{\sqrt{\frac{9}{2}}} + 6\cdot\frac{1}{2\cdot\sqrt{\frac{9}{2}}}\ln\left|\frac{\sqrt{\frac{9}{2}}-x}{\sqrt{\frac{9}{2}}+x}\right| + C =$$

$$= \sqrt{2}arctg\frac{\sqrt{2}x}{3} + \sqrt{2}\ln\left|\frac{3-\sqrt{2}x}{3+\sqrt{2}x}\right| + C.$$

5. $$\int \frac{\sqrt{x^2+5}}{2x^2+10}dx = \frac{1}{2}\int \frac{dx}{\sqrt{x^2+5}} = \frac{1}{2}\ln\left(x+\sqrt{x^2+5}\right) + C.$$

Tasks 6 – 7. These tasks are based on using the following special case of changing the variable (formula of integration of functions with a linear argument):

$$\int f(x)dx = F(x) + C \;\Rightarrow \int f(ax+b)dx = \frac{1}{a}F(ax+b) + C \quad a \neq 0.$$

An example of the application of this formula is the integral:

$$\int \cos(3x+8)dx = \frac{1}{3}\int \cos(3x+8)d(3x+8) = \frac{1}{3}\sin(3x+8) + C.$$

Task 8 is solved by using the formula of introducing of function under the sign of the differential, in particular, using the formula $\int \frac{u'(x)dx}{u(x)} = \ln|u(x)| + C$.

For example, $\int \frac{dx}{x+8} = \int \frac{d(x+8)}{x+8} = \ln|x+8| + C$.

In addition, methodological guidelines for the tasks 1 and 2 are used.

For example, we use this method in the following integral:

$$\int e^{x^5}x^4dx = \left|\begin{array}{c} replacement: \;\; x^5 = t, \;\; d\left(x^5\right) = dt, \\ 5x^4dx = dt, \;\; x^4dx = \frac{dt}{5} \end{array}\right| = \int e^t \cdot \frac{dt}{5} = \frac{1}{5}e^t + C = e^{x^5} + C.$$

In Task 9 an example is solved on the theme: integration of functions, containing quadratic trinomial ([2], p.43).

We recommend using one of the known methods of allocating the total square:

$$ax^2+bx+c=a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a} \text{ or } ax^2+bx+c=\left(\sqrt{a}x+\frac{b}{2\sqrt{a}}\right)^2+c-\frac{b^2}{4a}.$$

Using these formulas, we can carry out the integration of functions of the form:

$$\int\frac{dx}{ax^2+bx+c},\ \int\frac{(Ax+B)dx}{ax^2+bx+c},\ \int\frac{dx}{\sqrt{ax^2+bx+c}},\ \int\frac{(Ax+B)dx}{\sqrt{ax^2+bx+c}}.$$

In Task 10 we use the formula of integration by parts ([2], p.42).

It is recommended for integrals: $\int x^n e^{ax}dx$, $\int x^n\sin axdx$, $\int x^n\cos axdx$

take that $u=x^n$, $dv=\left\{e^{ax}dx,\ \sin axdx,\ \cos axdx\right\}$.

And for the integrals:

$\int x^n\ln^m axdx$, $\int x^n\arcsin axdx$, $\int x^n\arccos axdx$, $\int x^n arctgaxdx$, $\int x^n arcctgaxdx$

take that $dv=x^n dx,\ v=\frac{x^{n+1}}{n+1}$,

$$u=\left\{\ln^m ax,\ \arcsin ax,\ \arccos ax,\ arctgax,\ arcctgax\right\}.$$

For example, $\int(2x-7)\sin 5x\,dx$.

We apply the method of integration by parts.

$$\int(2x-7)\sin 5xdx=\left|\begin{array}{c}According\ \ to\ \ the\ formulaof\ integration\ \ by\ parts\\ \int udv=uv-\int vdu,\ \ where\ \ u=2x-7,\\ du=2dx,\ \ dv=\sin 5xdx,\ \ v=-\frac{1}{5}\cos 5x\end{array}\right|=$$

$$=-\frac{1}{5}(2x-7)\cos 5x+\frac{2}{5}\int\cos 5x\,dx=-\frac{1}{5}(2x-7)\cos 5x+\frac{2}{25}\sin 5x+C.$$

Tasks 11, 12, 13. It is necessary to study the theme «integration of rational function» ([2], p.44).

Decomposition of the proper rational function $\frac{Q_m(x)}{P_n(x)} = \frac{b_0x^m + b_1x^{m-1} + ... + b_m}{a_0x^n + a_1x^{n-1} + ... + a_n}$, where $n > m$ into partial fractions has the form:

$$\frac{Q_m(x)}{P_n(x)} = \frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + ... + \frac{A_k}{(x-a)^k} +$$

$$... + \frac{M_1x + N_1}{(x^2 + bx + c)} + ... + \frac{M_rx + N_r}{(x^2 + bx + c)^r}$$

if $a_0x^n + ... + a_n = a_0(x-a)^k ... (x^2 + bx + c)^r ...$

First of all it is necessary to study the method of undetermined coefficients or the method of sampling points for the decomposition of the proper rational function.

Task 14. Learn from lecture notes the theme «integration of irrational functions by rationalizing» ([2], p.46).

For example, in the integrals of the form $\int R\left(x, x^{\frac{l}{k}}, x^{\frac{s}{m}}, ...\right)dx$, it is necessary to take the replacement $x = t^n$, where n – the common denominator of the fractions $\frac{l}{k}, \frac{s}{m}, ...$.

In the integrals of the form $\int R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{l}{k}}, \left(\frac{ax+b}{cx+d}\right)^{\frac{s}{m}}, ...\right)dx$, it is necessary to take the replacement $\frac{ax+b}{cx+d} = t^n$, where n – the common denominator of the fractions $\frac{l}{k}, \frac{s}{m}, ...$.

Task 15. If the given integral has the form $\int R(tgx)dx$, or the integrand has the form $R(-\sin x, -\cos x) = R(\sin x, \cos x)$, then we use the replacement: $tgx = t$. Moreover, on the basis of known formulas, we have:

$$\sin x = \frac{tgx}{\sqrt{1+tg^2x}} = \frac{t}{\sqrt{1+t^2}}; \qquad \cos x = \frac{1}{\sqrt{1+tg^2x}} = \frac{1}{\sqrt{1+t^2}};$$

$$x = arctgt, \quad dx = \frac{dt}{1+t^2}.$$

If the given integral has the form $\int \sin^n x \cdot \cos^m x dx$, and n – odd number,

then we use the replacement: $\cos x = t$, $\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-t^2}$, $x = \arccos t$, $dx = -\dfrac{dt}{\sqrt{1-t^2}}$.

If m – odd number, then we use the replacement:

$$\sin x = t, \quad \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-t^2}, \quad x = \arcsin t, \quad dx = \frac{dt}{\sqrt{1-t^2}}.$$

Example: $\int \dfrac{\sin^3 3x}{\sqrt[5]{\cos 3x}}\,dx.$

$$\int \frac{\sin^3 3x}{\sqrt[5]{\cos 3x}}\,dx = \int \frac{\sin^2 3x \cdot \sin 3x}{\sqrt[5]{\cos 3x}}dx = \int \frac{\left(1-\cos^2 3x\right)\sin 3x}{\sqrt[5]{\cos 3x}}\,dx = \left| \begin{array}{c} replacement \quad \cos 3x = t, \\ dt = -3\sin 3x dx \end{array} \right| =$$

$$= -\frac{1}{3}\int \frac{\left(1-t^2\right)dt}{t^{\frac{1}{5}}} = -\frac{1}{3}\int \left(t^{-\frac{1}{5}} - t^{\frac{9}{5}} \right) dt = -\frac{1}{3}\cdot\frac{t^{\frac{4}{5}}}{\frac{4}{5}} + \frac{1}{3}\cdot\frac{t^{\frac{14}{5}}}{\frac{14}{5}} + C = -\frac{5}{12}\sqrt[5]{\cos^4 3x} +$$

$$+ \frac{5}{42}\sqrt[5]{\cos^{14} 3x} + C.$$

Task 16. We use the universal trigonometric substitution to the integral of the form $\int R(\sin x, \cos x)dx$. The universal substitution consists of the following:

$$tg\frac{x}{2} = t; \sin x = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2t}{1+t^2}; \cos x = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}; x = 2arctgt,\ dx = \frac{2dt}{1+t^2}.$$

Task 17 is solved by Newton-Leibniz formula ([2], p.51) and using the properties of integration of functions with a linear argument (see methodological guidelines for the tasks 6 and 7).

For example, calculate the definite integral $\int\limits_{-1}^{2} \dfrac{dx}{(x+3)^3}$.

Solution:

$$\int\limits_{-1}^{2} \frac{dx}{(x+3)^3} = \int\limits_{-1}^{2} \frac{d(x+3)}{(x+3)^3} = \int\limits_{-1}^{2} (x+3)^{-3} d(x+3) = \left.\frac{(x+3)^{-3+1}}{-2}\right|_{-1}^{2} = -\frac{1}{2}\left[(2+3)^{-2} - (-1+3)^{-2}\right] =$$

$$= -\frac{1}{2}\left(5^{-2} - 2^{-2}\right) = -\frac{1}{2}\left(\frac{1}{25} - \frac{1}{4}\right) = \frac{21}{200}.$$

In the Task 18 we apply the method of replacing the variable for the definite integral and Newton-Leibniz formula ([2], p.51).

Calculate the definite integral $\int_0^1 \sqrt{4-x^2}\,dx$, using the replacement of variable $x = 2\cos t$.

$$\int_0^1 \sqrt{4-x^2}\,dx = \left| \begin{array}{l} x = 2\cos t, \quad dx = -2\sin t\,dt, \\ x = 0 \ \Rightarrow t = \dfrac{\pi}{2}, \\ x = 1 \ \Rightarrow t = \dfrac{\pi}{3} \end{array} \right| = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \sqrt{4-4\cos^2 t}\cdot(-2\sin t)dt =$$

$$= -4\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \cdot (\sin t)^2\,dt = -4\cdot\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \frac{1-\cos 2t}{2}\cdot dt = -2\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} (1-\cos 2t)dt = -2\left(t - \frac{1}{2}\sin 2t\right)\Bigg|_{\frac{\pi}{2}}^{\frac{\pi}{3}} =$$

$$= -2\left(\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3} - \left(\frac{\pi}{2} - \frac{1}{2}\sin\pi\right)\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2}.$$

Task 19 is solved by using the formula for integration by parts for definite integral ([2], p.51).

For example, calculate the definite integral $\int_0^1 x\cos 2x\,dx$.

$$\int_0^1 x\cos 2x dx = \left| \begin{array}{l} by \quad parts: \quad x = u, \quad dx = du, \\ \cos 2x dx = dv, \quad v = \dfrac{1}{2}\sin 2x \end{array} \right| = \frac{x\sin 2x}{2}\Bigg|_0^1 - \int_0^1 \frac{1}{2}\sin 2x dx =$$

$$= \frac{\sin 2}{2} + \frac{1}{4}\cos 2x\Big|_0^1 = \frac{\sin 2}{2} + \frac{1}{4}(\cos 2 - \cos 0) = \frac{\sin 2}{2} + \frac{1}{4}(\cos 2 - 1).$$

Task 20. It is recommended to solve the task on the model of example:

Calculate the improper integral (with infinite limits)

$\int_{-\infty}^{\infty} \dfrac{dx}{x^2+8x+17}$ (or prove its divergence).

$$\int_{-\infty}^{\infty} \frac{dx}{x^2+8x+17} = \int_{-\infty}^{\infty} \frac{dx}{(x+4)^2+1} = \int_{-\infty}^{\infty} \frac{d(x+4)}{(x+4)^2+1} = arctg(x+4)\Big|_{-\infty}^{\infty} =$$

$$= arctg(\infty) - arctg(-\infty) = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi .$$

Conclusion: the improper integral converges and is equal to the number π .

Tasks 21. It is recommended to solve the task on the model of example:

Calculate the improper integral (the integrand contains the function having a break point of the second kind in the area of integration):

$\int_{-1}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}}$ (or prove its divergence).

$$\int_{-1}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}} = \int_{-1}^{1} \frac{dx}{\sqrt[3]{(x-1)^2}} +$$

$$\int_{1}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}} = \lim_{\alpha \to 1-0} \int_{-1}^{\alpha} (x-1)^{-\frac{2}{3}} d(x-1) + \lim_{\beta \to 1+0} \int_{\beta}^{2} (x-1)^{-\frac{2}{3}} d(x-1) =$$

$$= 3 \lim_{\alpha \to 1-0} (x-1)^{\frac{1}{3}} \Big|_{-1}^{\alpha} + 3 \lim_{\beta \to 1+0} (x-1)^{\frac{1}{3}} \Big|_{\beta}^{2} = 3 \lim_{\alpha \to 1-0} \left[(\alpha-1)^{\frac{1}{3}} + 2^{\frac{1}{3}} \right] + 3 \lim_{\beta \to 1+0} \left[1 - (\beta-1)^{\frac{1}{3}} \right] =$$

$$= 3 \cdot \sqrt[3]{2} + 3 = 3\left(\sqrt[3]{2} + 1\right).$$

Conclusion: the improper integral converges and is equal to the specified number.

Task 22. Calculate the area of the figure D bounded by lines.

$$D: \quad y = \cos x, \quad y = -2\cos x,$$
$$-\frac{\pi}{2} \le x \le \frac{\pi}{2}.$$

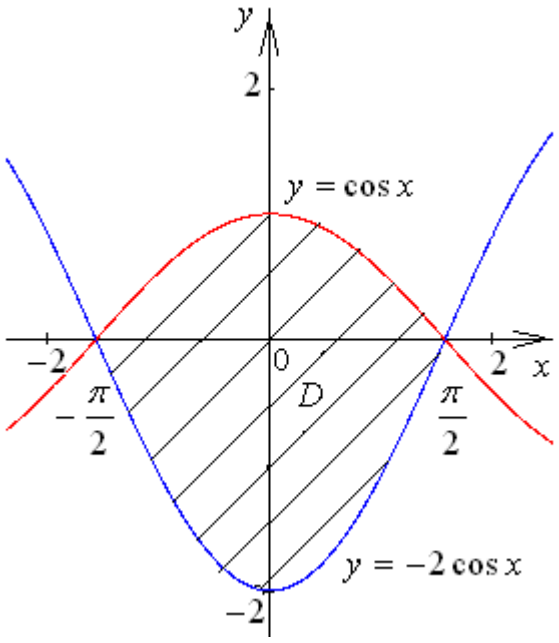

Figure 1

Solution:

$$S = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\cos x - (-2\cos x)] dx =$$

$$= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 3 \sin x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$$

$$= 3\left(\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right) \right) = 3\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2} \right) = 6\sin\frac{\pi}{2} = 6.$$

4 Reference material

Table of integrals

1. $\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \quad (\alpha \neq -1)$

2. $\int \frac{dx}{x} = \ln|x| + C$

3. $\int \sin x dx = -\cos x + C$

4 $\int \cos x dx = \sin x + C$

5. $\int \frac{dx}{\cos^2 x} = tgx + C$

6. $\int \frac{dx}{\sin^2 x} = -ctgx + C$

7. $\int tgx dx = -\ln|\cos x| + C$

8. $\int ctgx dx = \ln|\sin x| + C$

9. $\int e^x dx = e^x + C$

10. $\int a^x dx = \frac{a^x}{\ln a} + C$

11. $\int \frac{dx}{1+x^2} = arctgx + C$

11′. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} arctg \frac{x}{a} + C$

12. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C$

13. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$

13′. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$

14. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

15. $\int shx dx = chx + C$

16. $\int chx dx = shx + C$

17. $\int \frac{dx}{ch^2 x} = thx + C$

18. $\int \frac{dx}{sh^2 x} = -cthx + C$

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5 Индивидуальные задания по высшей математике: Комплексные числа. Неопределенные и определенные интегралы. Функции нескольких переменных. Обыкновенные дифференциальные уравнения ч. 2: Учеб. пособие /под ред. А.П. Рябушко – Мн.: Выш.шк., 2000.-396 с.

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Kim Regina Evgenievna

MATHEMATICS 1

Methodological Guidelines and Tasks
for carrying out the calculation-graphical work for students of specialties
5B071700 «Heat power engineering»,
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