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ALMATY UNIVERSITY OF POWER ENGINEERING AND TELECOMMUNICATIONS

Department of
Higher mathematics

COMPUTER SOLUTION OF PROBLEMS IN OPERATIONAL CALCULUS AND PROBABILITY THEORY

Methodological Guidelines and Tasks
for carrying out calculation-graphical work for students of specialty
5B071900 «Radio engineering, electronics and telecommunications»

Part 1 – 2

Almaty 2016

AUTHORS: Kim R.E., Masanova A.Zh. Computer solution of problems in operational calculus and probability theory. Methodological Guidelines and Tasks for carrying out calculation-graphical work for students of specialty 5B071900 – Radio engineering, electronics and telecommunications. Part 1 – 2. – Almaty: AUPET, 2016. – 38 p.

The methodological Guidelines and Tasks for carrying out the calculation-graphical works №1 and №2 on discipline CSPOCPT are worked out for students of specialty 5B071900 – Radio engineering, electronics and telecommunications.

The presented material corresponds to the section "Probability theory" provided in the curriculum for students of this specialty.

There are basic theoretical questions, solutions of a standard version as well as examples of tasks performance in MathCad system in the guidelines.

Tables 17, figures 1, bibl. 9

Reviewer: senior lecturer of the Department of foreign languages, S.B. Bukhina

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Calculation-graphical work (CGW) №1. Random events and random variables

Theoretical questions

1. Subject of probability theory. Random events, frequency. Statistical and geometrical definition of probability.

2. Space of elementary events. Algebra of events. Classical definition of probability.

3. Theorems of addition and multiplication of probabilities. Conditional probability. Dependent and independent events.

4. Formula of total probability. Bayes' formula.

5. Repeating the tests. Bernoulli's formula.

6. Laplace's local and integral theorems. Laplace's function. Poisson's formula.

7. Discrete and continuous random variables. Law of distribution of a discrete random variable. Binomial distribution, Poisson's distribution.

8. Integral distribution function (distribution function), properties and graph.

9. Differential distribution function (distribution density), properties and graph.

10. Numerical characteristics of random variables. Mathematical expectation of discrete and continuous random variables.

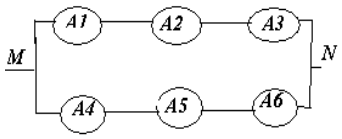
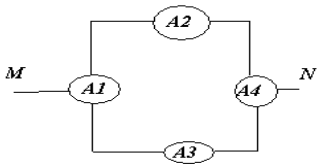
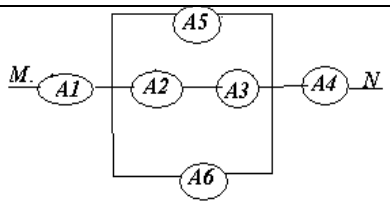
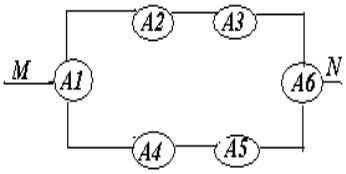
11. Dispersion and standard deviation of discrete and continuous random variables.

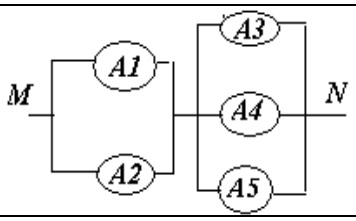
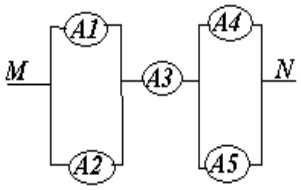
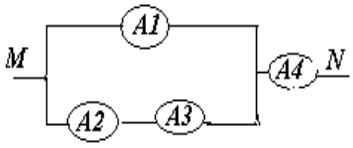
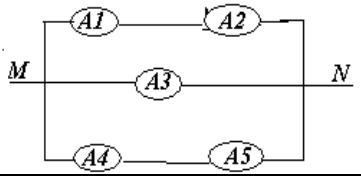
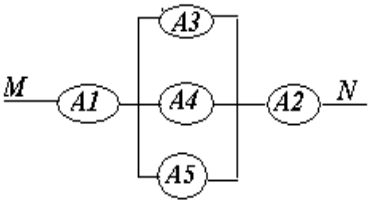
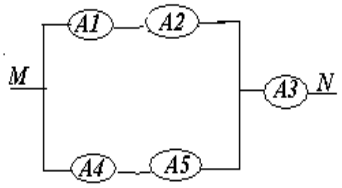
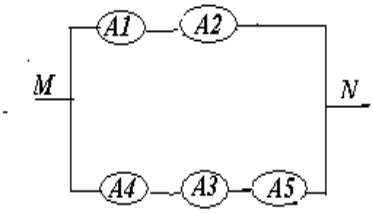
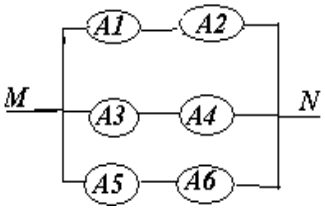
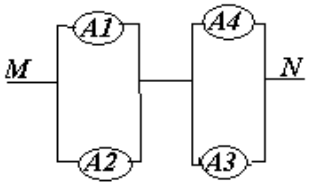
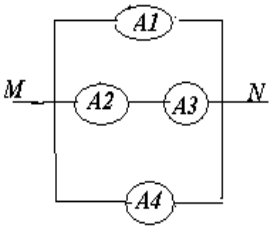
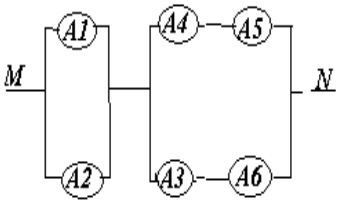
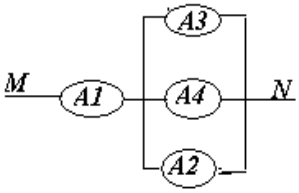
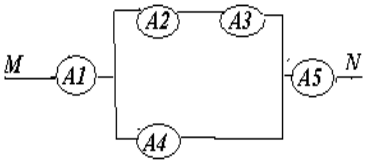
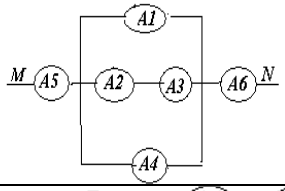
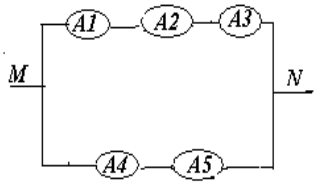
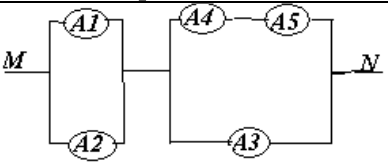
12. Initial and central moments of distribution, asymmetry, excess, mode and median.

Tasks for calculation

Task 1. Piece *MN* of electrical circuit consists of elements connected according to the scheme given in Table 1. The failure of various elements of the system during time *T* is independent events $\overline{A_i}$, with probabilities p_i, given in Table 2. It is required to calculate the probability of breaking the circuit for the given time interval.

Table 1

1.1		1.2	
1.3		1.4	

1.5		1.6	
1.7		1.8	
1.9		1.10	
1.11		1.12	
1.13		1.14	
1.15		1.16	
1.17		1.18	
1.19		1.20	

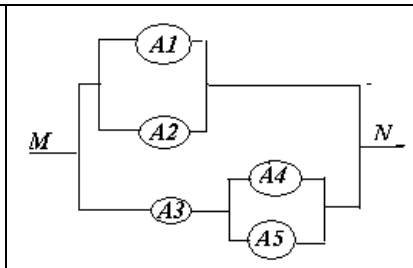
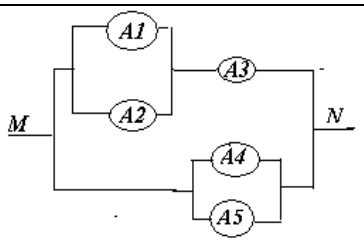
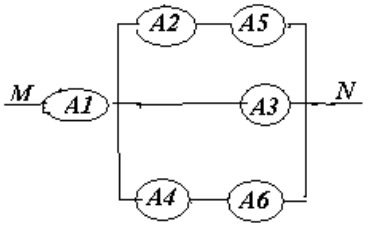
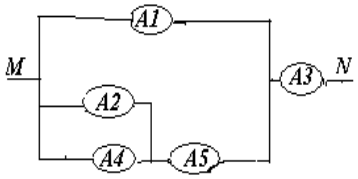
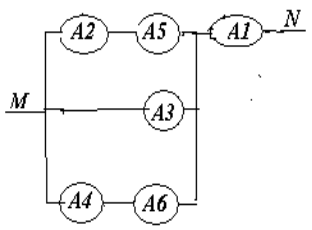
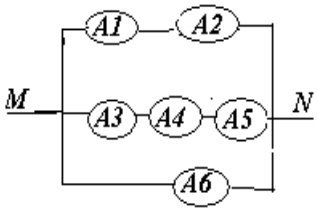
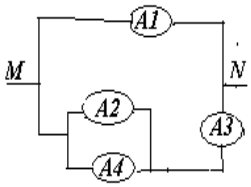
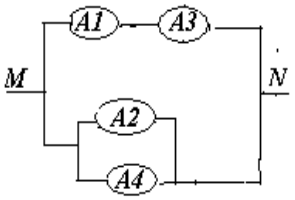
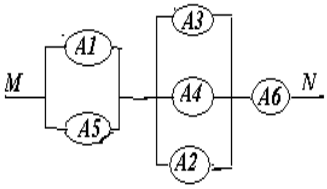
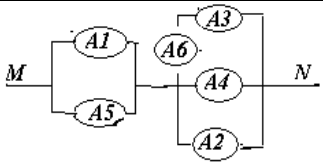
1.21		1.22	
1.23		1.24	
1.25		1.26	
1.27		1.28	
1.29		1.30	

Table 2

№	A_1	A_2	A_3	A_4	A_5	A_6
	P_1	P_2	P_3	P_4	P_5	P_6
1.1	0,2	0,3	0,6	0,4	0,1	0,5
1.2	0,3	0,2	0,1	0,4	-	-
1.3	0,1	0,2	0,5	0,3	0,6	-
1.4	0,3	0,6	0,1	0,4	0,5	0,2
1.5	0,3	0,6	0,5	0,4	0,2	-
1.6	0,3	0,2	0,5	0,1	-	-
1.7	0,4	0,5	0,1	0,2	0,3	-
1.8	0,1	0,3	0,5	0,4	0,2	-
1.9	0,4	0,5	0,6	0,3	0,1	-
1.10	0,4	0,2	0,3	0,6	0,5	-
1.11	0,1	0,5	0,2	0,3	0,4	-
1.12	0,3	0,5	0,1	0,4	0,6	0,2

1.13	0,6	0,5	0,1	0,3	-	-
1.14	0,4	0,6	0,1	0,5	-	-
1.15	0,2	0,4	0,1	0,3	0,6	0,5
1.16	0,3	0,2	0,1	0,5	-	-
1.17	0,2	0,5	0,1	0,6	0,4	-
1.18	0,5	0,6	0,1	0,3	0,4	0,2
1.19	0,2	0,5	0,1	0,6	0,4	-
1.20	0,5	0,2	0,3	0,4	0,1	-
1.21	0,2	0,5	0,1	06	0,4	-
1.22	0,4	0,5	0,6	0,3	0,1	-
1.23	0,3	0,5	0,1	0,4	0,6	0,2
1.24	0,1	0,3	0,5	0,4	0,2	-
1.25	0,3	0,6	0,1	0,4	0,5	0,2
1.26	0,2	0,3	0,6	0,4	0,1	0,5
1.27	0,3	0,2	0,1	0,4	-	-
1.28	0,3	0,2	0,5	0,1	-	-
1.29	0,5	0,6	0,1	0,3	0,4	0,2
1.30	0,2	0,3	0,6	0,4	0,1	0,5

Task 2. Components from four automatic machines come for the assembly. The first machine produces n_1%, the second – n_2%, the third – n_3% from the total quantity of all components. The first machine gives m_1% of defects, the second – m_2%, the third – m_3%, and the fourth – m_4%. It is required to find the probability of the event when:

a) a standard component will go to the assembly;

b) a component, gone to the assembly, is made by i – th machine (i = 1, 2, 3, 4).

№	n_1	n_2	n_3	m_1	m_2	m_3	m_4	i
2.1	10	25	45	7	8	5	5	4
2.2	43	18	25	5	4	7	8	2
2.3	17	54	10	6	5	8	4	4
2.4	65	12	25	10	9	8	7	2
2.5	40	18	35	7	10	5	4	1
2.6	12	38	40	10	6	9	5	2
2.7	27	34	50	9	5	4	6	3
2.8	43	12	23	10	7	6	1	4
2.9	36	12	12	5	10	8	3	1
2.10	42	21	21	8	7	6	4	4
2.11	37	13	15	10	6	5	9	2
2.12	41	20	18	5	10	8	2	3
2.13	28	51	17	10	6	5	5	3
2.14	71	12	10	2	10	4	4	4
2.15	46	24	14	5	9	7	7	4

2.16	52	22	11	5	8	7	3	1
2.17	27	41	13	10	5	9	9	4
2.18	25	14	35	8	7	4	8	2
2.19	19	38	30	5	9	30	5	4
2.20	29	61	20	6	3	3	4	2
2.21	27	43	20	10	6	4	6	4
2.22	28	36	22	7	10	9	4	1
2.23	52	11	17	5	7	10	9	1
2.24	24	29	35	9	8	4	4	4
2.25	31	41	19	7	2	5	4	3
2.26	52	11	15	3	6	7	5	4
2.27	28	31	18	9	8	4	4	2
2.28	40	32	10	4	5	8	5	1
2.29	35	24	20	9	8	7	9	4
2.30	19	52	18	5	2	4	6	3

Task 3. In message transmission the probability of error of one sign is equal to 1–p. Find the probability of the event when in the message of n signs:

a) exactly k_1 signs will not be distorted;

b) no less than k_2 signs will not be distorted;

c) no more than k_3 signs will not be distorted.

№	n	k_1	k_2	k_3	p	№	n	k_1	k_2	k_3	p
3.1	4	2	3	2	0.9	3.16	5	3	4	2	0.8
3.2	4	3	3	2	0.8	3.17	4	3	3	1	0.7
3.3	5	4	4	2	0.7	3.18	4	2	3	2	0.6
3.4	5	3	3	2	0.6	3.19	5	3	4	1	0.5
3.5	6	5	5	1	0.5	3.20	6	4	5	2	0.4
3.6	6	4	4	1	0.4	3.21	7	5	6	2	0.3
3.7	7	5	5	2	0.3	3.22	8	3	7	2	0.2
3.8	7	4	4	1	0.2	3.23	8	4	7	1	0.3
3.9	8	4	7	2	0.3	3.24	7	5	6	2	0.4
3.10	8	3	6	1	0.4	3.25	6	3	5	2	0.5
3.11	7	4	6	2	0.5	3.26	5	2	4	1	0.6
3.12	7	5	6	1	0.6	3.27	4	2	3	2	0.7
3.13	6	3	4	2	0.7	3.28	5	3	3	3	0.8
3.14	6	2	4	2	0.8	3.29	6	4	4	2	0.9
3.15	5	4	4	1	0.9	3.30	7	5	6	1	0.9

Task 4. An industrial television system contains n transistors. The probability of each of the transistors failure is equal to p. It is required to find the probability of failure of k transistors.

№	p	n	k	№	p	n	k	№	p	n	k
4.1	0.002	1000	7	4.11	0.01	200	8	4.21	0.004	500	9
4.2	0.003	1000	7	4.12	0.01	300	8	4.22	0.005	600	9
4.3	0.004	1000	7	4.13	0.02	200	8	4.23	0.01	400	9
4.4	0.005	1000	7	4.14	0.01	500	8	4.24	0.01	500	9
4.5	0.006	1000	7	4.15	0.02	300	8	4.25	0.01	600	9
4.6	0.007	1000	7	4.16	0.01	700	8	4.26	0.007	1000	9
4.7	0.008	1000	7	4.17	0.02	400	8	4.27	0.008	1000	9
4.8	0.009	1000	7	4.18	0.01	900	8	4.28	0.009	1000	9
4.9	0.01	1000	7	4.19	0.02	500	8	4.29	0.01	1000	9
4.10	0.011	1000	7	4.20	0.011	1000	8	4.30	0.012	1000	9

Task 5. The probability of event A in each test is equal to p. Find the probability of the event that in n tests event A occurs

a) exactly k_2 times;

b) from k_1 to k_2 times for the variants № 1 – 15;

c) more than k_2 times for the variants № 16 – 30.

№	n	k_1	k_2	p	№	n	k_1	k_2	p
5.1	100	80	90	0.8	5.16	100	90	95	0.6
5.2	100	85	95	0.8	5.17	100	62	82	0.6
5.3	100	70	95	0.8	5.18	100	50	70	0.8
5.4	100	83	93	0.7	5.19	100	55	75	0.8
5.5	100	50	60	0.7	5.20	100	45	80	0.8
5.6	100	65	75	0.7	5.21	100	40	60	0.8
5.7	100	70	80	0.7	5.22	100	35	70	0.3
5.8	100	40	50	0.6	5.23	100	50	80	0.3
5.9	100	65	80	0.75	5.24	100	40	65	0.3
5.10	100	70	85	0.75	5.25	200	45	75	0.4
5.11	100	78	92	0.75	5.26	200	100	150	0.4
5.12	100	20	60	0.7	5.27	200	80	170	0.4
5.13	100	30	85	0.7	5.28	300	150	180	0.8
5.14	100	40	79	0.7	5.29	400	100	190	0.6
5.15	100	80	95	0.6	5.30	400	200	295	0.7

Task 6. Discrete random variable X is given by distribution series. Find:

a) its distribution function $F(x)$, make a graph of function $F(x)$;

b) mathematical expectation, dispersion, standard deviation and mode;

c) probability of getting the variable X to the interval (a, b).

№	X	x_1	x_2	x_3	x_4	x_5	x_6	a	b
	P	p_1	p_2	p_3	p_4	p_5	p_6		
6.1	X	0	1	2	4	6	9	-2	7
	P	0.05	0.15	0.3	0.25	0.15	0.1		

6.2	X	-3	-2	-1	0	2	4	-1	3
	P	0.15	0.3	0.02	0.14	0.18	0.31		
6.3	X	1	2	3	5	7	8	-3	6
	P	0.3	0.14	0.16	0.1	0.2	0.1		
6.4	X	-4	-3	-2	0	1	2	0	1
	P	0.2	0.08	0.23	0.27	0.12	0.1		
6.5	X	1	2	4	5	7	9	3	8
	P	0.19	0.21	0.06	0.14	0.12	0.28		
6.6	X	-1	0	2	3	5	7	-4	4
	P	0.26	0.14	0.07	0.2	0.03	0.3		
6.7	X	-2	-1	0	3	5	7	1	6
	P	0.18	0.09	0.01	0.2	0.22	0.3		
6.8	X	1	2	4	5	6	8	0	6
	P	0.3	0.17	0.13	0.1	0.2	0.1		
6.9	X	1	2	3	4	7	9	5	8
	P	0.11	0.29	0.06	0.14	0.17	0.23		
6.10	X	0	1	2	3	7	9	4	8
	P	0.06	0.14	0.3	0.25	0.15	0.1		
6.11	X	-3	-2	0	1	2	4	-1	3
	P	0.15	0.3	0.01	0.14	0.19	0.31		
6.12	X	-1	0	3	5	7	8	1	6
	P	0.25	0.14	0.16	0.1	0.2	0.15		
6.13	X	-4	-3	-2	0	2	4	-1	3
	P	0.2	0.07	0.24	0.26	0.13	0.1		
6.14	X	-3	-1	0	3	4	7	-2	6
	P	0.12	0.09	0.01	0.2	0.28	0.3		
6.15	X	-1	0	1	3	7	8	2	6
	P	0.26	0.14	0.15	0.2	0.3	0.15		
6.16	X	-2	-1	0	1	2	7	-3	5
	P	0.17	0.09	0.01	0.3	0.23	0.2		
6.17	X	1	2	3	5	6	7	0	4
	P	0.1	0.14	0.16	0.1	0.2	0.3		
6.18	X	-3	-1	0	3	5	6	-2	4
	P	0.16	0.09	0.01	0.3	0.24	0.2		
6.19	X	1	2	5	6	7	8	3	6
	P	0.2	0.15	0.15	0.1	0.3	0.1		
6.20	X	-1	0	2	4	7	8	1	5
	P	0.23	0.18	0.12	0.2	0.1	0.17		
6.21	X	1	2	4	5	6	8	0	7
	P	0.3	0.14	0.16	0.03	0.2	0.17		
6.22	X	-4	-3	-1	0	1	3	-2	2
	P	0.2	0.03	0.24	0.26	0.17	0.1		
6.23	X	1	2	3	4	7	9	0	8

	P	0.17	0.23	0.09	0.11	0.12	0.28		
6.24	*X*	0	1	3	5	7	8	2	6
	P	0.2	0.14	0.16	0.12	0.3	0.08		
6.25	*X*	-5	-3	-2	0	1	3	-4	2
	P	0.2	0.06	0.21	0.29	0.14	0.1		
6.26	*X*	1	2	3	5	8	9	4	7
	P	0.18	0.22	0.05	0.15	0.12	0.28		
6.27	*X*	1	3	4	5	7	8	2	6
	P	0.3	0.16	0.14	0.01	0.2	0.19		
6.28	*X*	-5	-3	-1	0	1	3	-4	2
	P	0.1	0.03	0.14	0.36	0.17	0.2		
6.29	*X*	0	2	3	4	6	8	1	7
	P	0.26	0.14	0.05	0.15	0.12	0.28		
6.30	*X*	-1	0	2	3	7	8	1	6
	P	0.21	0.16	0.14	0.1	0.2	0.19		

Task 7. Continuous random variable X is given by distribution function $F(x)$. Find:

a) its probability density function $f(x)$;

b) mathematical expectation, dispersion, standard deviation, mode and median;

c) probability of getting the variable X to the interval (a, b).

Make graphs of functions $F(x)$ and $f(x)$.

№	*F(x)*	*a*	*b*	№	*F(x)*	*a*	*b*
7.1	$\begin{cases} 0, if\ x\langle 0 \\ \frac{1}{8}x^3, if\ 0\le x\le 2 \\ 1, if\ x\rangle 2 \end{cases}$	0	4	7.16	$\begin{cases} 0, if\ x<0 \\ \frac{1}{5}(x+1), if\ -1\le x\le 4 \\ 1, if\ x>4 \end{cases}$	0	3
7.2	$\begin{cases} 0, if\ x<0 \\ \frac{1}{33}(2x^2+5x), if\ 0\le x\le 3 \\ 1, if\ x>3 \end{cases}$	1	2	7.17	$\begin{cases} 0, if\ x<0 \\ \sin x, if\ 0\le x\le \pi/2 \\ 1, if\ x>/2\pi \end{cases}$	0	$\frac{\pi}{6}$
7.3	$\begin{cases} 0, if\ x<0 \\ \frac{1}{9}x^2, if\ 0\le x\le 3 \\ 1, if\ x>3 \end{cases}$	0	1	7.18	$\begin{cases} 0, if\ x<0 \\ (x^3+3x), if\ 0\le x\le 2 \\ 1, if\ x>2 \end{cases}$	0,2	0,9
7.4	$\begin{cases} 0, if\ x<0 \\ \frac{1}{24}(2x^2+5x), if\ 0\le x\le 4 \\ 1, if\ x>4 \end{cases}$	0	1	7.19	$\begin{cases} 0, if\ x<0 \\ (x^2-x)/2, if\ 1\le x\le 2 \\ 1, if\ x>2 \end{cases}$	1,5	2

7.5	$\begin{cases} 0, \text{ if } x<0 \\ \frac{1}{10}(x^3+x), \text{ if } 0\le x\le 2 \\ 1, \text{ if } x>2 \end{cases}$	0	1	7.20	$\begin{cases} 0, \text{ if } x<0 \\ (x^2+x)/6, \text{ if } 0\le x\le 2 \\ 1, \text{ if } x>2 \end{cases}$	0	1,5
7.6	$\begin{cases} 0, \text{ if } x<0 \\ \frac{1}{20}(x^3+x), \text{ if } 0\le x\le 4 \\ 1, \text{ if } x>4 \end{cases}$	0	3	7.21	$\begin{cases} 0, \text{ if } x<0 \\ \frac{1}{10}(x^2+3x), \text{ if } 0\le x\le 2 \\ 1, \text{ if } x>2 \end{cases}$	1	1,5
7.7	$\begin{cases} 0, \text{ if } x<0 \\ \cos 2x, \text{ if } 3\pi/4\le x\le \pi \\ 1, \text{ if } x>\pi \end{cases}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	7.22	$\begin{cases} 0, \text{ if } x<0 \\ \frac{1}{4}(x^2-2x), \text{ if } 1\le x\le 2 \\ 1, \text{ if } x>2 \end{cases}$	1,2	1,5
7.8	$\begin{cases} 0, \text{ if } x<0 \\ 1-\cos x, \text{ if } 0\le x\le \pi/2 \\ 1, \text{ if } x>\pi/2 \end{cases}$	0	$\frac{\pi}{3}$	7.23	$\begin{cases} 0, \text{ if } x<2 \\ \frac{1}{2}x-1, \text{ if } 2\le x\le 4 \\ 1, \text{ if } x>4 \end{cases}$	1	3
7.9	$\begin{cases} 0, \text{ if } x<0 \\ \frac{1}{96}(x^3+8x), \text{ if } 0\le x\le 4 \\ 1, \text{ if } x>4 \end{cases}$	0	2	7.24	$\begin{cases} 0, \text{ if } x<0 \\ \frac{1}{6}x, \text{ if } 0\le x\le 6 \\ 1, \text{ if } x>6 \end{cases}$	2	5
7.10	$\begin{cases} 0, \text{ if } x<-1 \\ \frac{1}{9}(x+1)^2, \text{ if } -1\le x\le 2 \\ 1, \text{ if } x>2 \end{cases}$	1	2	7.25	$\begin{cases} 0, \text{ if } x<-1 \\ \frac{1}{2}x+\frac{1}{2}, \text{ if } -1\le x\le 1 \\ 1, \text{ if } x>1 \end{cases}$	-0,5	0,2
7.11	$\begin{cases} 0, \text{ if } x<\pi/2 \\ 1-\sin x, \text{ if } \pi/2\le x\le \pi \\ 1, \text{ if } x>\pi \end{cases}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	7.26	$\begin{cases} 0, \text{ if } x<2 \\ (x-2)^2, \text{ if } 2\le x\le 3 \\ 1, \text{ if } x>3 \end{cases}$	2,5	2,8
7.12	$\begin{cases} 0, \text{ if } x<-1 \\ \frac{1}{9}(x^3+1), \text{ if } -1\le x\le 2 \\ 1, \text{ if } x>2 \end{cases}$	1	2	7.27	$\begin{cases} 0, \text{ if } x<1 \\ \frac{1}{2}(x^2-x), \text{ if } 1\le x\le 2 \\ 1, \text{ if } x>2 \end{cases}$	1,5	1,9
7.13	$\begin{cases} 0, \text{ if } x<0 \\ \frac{1}{33}(3x^2+2x), \text{ if } 0\le x\le 3 \\ 1, \text{ if } x>3 \end{cases}$	0	2	7.28	$\begin{cases} 0, \text{ if } x<\pi/2 \\ -\cos x, \text{ if } \pi/2\le x\le \pi \\ 1, \text{ if } x>\pi \end{cases}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$

7.14	$\begin{cases} 0, \text{ if } x<3\pi/2 \\ \cos x, \text{ if } 3\pi/2 \le x \le 2\pi \\ 1, \text{ if } x>2\pi \end{cases}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	7.29	$\begin{cases} 0, \text{ if } x<1 \\ \frac{1}{4}(x-1), \text{ if } 1 \le x \le 5 \\ 1, \text{ if } x>5 \end{cases}$	2	4
7.15	$\begin{cases} 0, \text{ if } x<0 \\ \frac{1}{15}(x^2+2x), \text{ if } 0 \le x \le 3 \\ 1, \text{ if } x>3 \end{cases}$	0	2	7.30	$\begin{cases} 0, \text{ if } x<0 \\ \frac{1}{2}(1-\cos x), \text{ if } 0 \le x \le \pi \\ 1, \text{ if } x>\pi \end{cases}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

Solution of a standard version

Task 1. Piece *MN* of electrical circuit consists of elements connected according to the scheme given in Figure 1. All circuit elements 1-5 operate independently from each other and the probabilities of failure over a given time interval are, respectively: $P_1 = 0{,}1;\ P_2 = 0{,}2;\ P_3 = 0{,}5;\ P_4 = 0{,}3;\ P_5 = 0{,}1.$

It is required to calculate the probability of breaking the circuit over a given time interval.

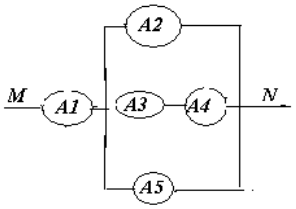


Figure 1

Solution:

Let's suppose that $\overline{A_i}$ (i = 1, 2, 3, 4, 5) are events meaning the failure of the i-th element in the circuit. The circuit is inseparable (event A) over a given period of time if events A_1 and $A_2+A_3A_4+A_5$ occur together (the product of events), i.e.

$$P(A) = P(A_1) \cdot P(A_2 + A_3 \cdot A_4 + A_5),$$

where the sum of the events $A_2+A_3A_4+A_5$ indicates normal operation of at least one of the three branches of the circuit, it is more convenient to calculate this probability using an opposite event, when all these three branches of the circuit fail simultaneously:

$$P(A_2 + A_3 \cdot A_4 + A_5) = 1 - P(\overline{A}_2) \cdot P(\overline{A}_5) \cdot P(\overline{B}),$$

$$P(\overline{B}) = 1 - P(A_3) \cdot P(A_4).$$

According to the problem we have $P(A_i)=1-p_i$, then

$$P(A) = P(A_1) \cdot (1 - P(\overline{A}_2) \cdot P(\overline{A}_5) \cdot (1 - P(A_3) \cdot P(A_4)) = 0{,}9(1 - 0{,}1 \cdot 0{,}2 \cdot (1 - 0{,}5 \cdot 0{,}7)) = 0{,}8883.$$

But the probability of circuit breaking over a given period of time is an opposite event:

$$P(\overline{A}) = 1 - P(A) = 1 - 0{,}8883 = 0{,}1117.$$

Task 2. Components from four automatic machines come for the assembly. The first machine produces 10%, the second – 30%, the third – 40% from the total quantity of all components. The first machine gives 5% of defects, the second – 4%, the third – 6%, and the fourth – 3%. It is required to find the probability of event when:

a) a standard component will go to the assembly;

b) a component, gone to the assembly, is made by the 2–nd machine.

Solution:

a) let's suppose that event *A* means that a standard item is received, and events B_1, B_2, B_3, B_4 mean that the item is made by the first, the second, the third, the fourth machine respectively (these events are called the hypotheses).

The probability of event *A* is calculated according to the formula of total probability:

$P(A) = P(B_1)P(A/B_1)+P(B_2)P(A/B_2)+P(B_3)P(A/B_2)+P(B_4)P(A/B_4)$,

where $P(A/B_i)$ is conditional probability of the event that a received standard component is made by the *i*–th automatic machine (*i*=1, 2, 3, 4).

According to the problem we have:

$P(B_1) = 10/100 = 0,1;\ P(B_2) = 30/100 = 0,3;\ P(B_3) = 40/100 = 0,4;$

$P(B_4) = (100-10-30-40)/100 = 0,2;$

$P(A/B_1)=1-0,05=0,95;\ P(A/B_2)=1-0,04=0,96;$

$P(A/B_3)=1-0,06=0,94;\ P(A/B_4)=1-0,03=0,97.$

Therefore $P(A) = 0,1\cdot 0.95 + 0,3\cdot 0,96 + 0,4\cdot 0,94 + 0,2\cdot 0,97 = 0,961$;

b) conditional probability that a received standard item is made by the 2nd automatic machine $P(B_2/A)$ is calculated according to the Bayes's formula:

$$P(B_i / A) = \frac{P(B_i)P(A/B_i)}{P(A)},\ i = 2,$$

where $P(A)$ is total probability.

Then $P(B_2 / A) = \frac{P(B_2)P(A/B_2)}{P(A)} = \frac{0,3\cdot 0,96}{0,961} = 0,299.$

Task 3. In message transmission the probability of error of one sign is equal to 0,2. Find the probability of the event when in the message of 10 signs:

a) exactly 8 signs will not be distorted (event *A*);

b) no less than 9 signs will not be distorted (event *B*);

c) no more than 2 signs will not be distorted (event *C*).

Solution:

a) to determine the probability of event *A* we use Bernoulli's formula:

$P_n(k) = C_n^k p^k q^{n-k}$,

where $P_n(k)$ is the probability that an event will occur *k* ($k = 0,1,2,...n$) times in series of *n* independent tests, where $q = 1 - p = 0,2;\ p = 0.8$,

$P(A) = P_{10}(8) = C_{10}^{8} \cdot 0{,}8^{8} \cdot 0{,}2^{2} = 0{,}302;$

b) the probability of event *B* is defined as the sum of probabilities: $P_n(k) + P_n(k+1) + ... + P_n(n)$ is the probability that the event will occur no less than *k* times in *n* independent tests;

$P(B) = P_{10}(9) + P_{10}(10) = 0{,}376;$

c) the probability of event *C* is defined as the sum of probabilities: $P_n(0) + P_n(1) + ... + P_n(k)$ is the probability that the event will occur no more than *k* times in *n* independent tests, i.e.

$P(C) = P_{10}(0) + P_{10}(1) + P_{10}(2) = 0{,}000078.$

The copy of file with Mathcad calculations is given below:

$C(Q, R) := \text{combin}(Q, R)$

$C(10, 8) \cdot 0.8^{8} \cdot 0.2^{2} = 0.302$

$C(10, 9) \cdot 0.8^{9} \cdot 0.2^{1} + C(10, 10) \cdot 0.8^{10} \cdot 0.2^{0} = 0.376$

$C(10, 0) \cdot 0.8^{0} \cdot 0.2^{10} + C(10, 1) \cdot 0.8^{1} \cdot 0.2^{9} + C(10, 2) \cdot 0.8^{2} \cdot 0.2^{8} = 7.793 \times 10^{-5}$

Task 4. An industrial television system contains 1000 transistors. The probability of each of the transistors failure is equal to 0,003. It is required to find the probability of failure of 6 transistors.

Solution: since n is very large, p is very small, and product $\lambda = n \cdot p$ is a small number, to determine the probability that an event occurs k times in series of *n* independent tests it is easier to use the Poisson's formula instead of the Bernoulli's one:

$P_n(k) \approx \lambda^{k} \cdot e^{-\lambda} / k!.$

In our problem $n = 1000$, $k = 6$, $p = 0{,}003$, $\lambda = 1000 \cdot 0{,}003 = 3$, therefore $P_{1000}(6) = 3^{6} \cdot e^{-3} / 6! = 0{,}05.$

While calculating we can use the table of values for function $p(k, \lambda) = \lambda^{k} \cdot e^{-\lambda} / k!$, given in some textbooks, or function *dpois* in MathCad. The copy of file with MathCad calculations is given below:

$p(k, \lambda) := \text{dpois}(k, \lambda)$

$p(6, 3) = 0.05$

Task 5. The probability of event in each test is equal to 0,8. Find out the probability of the event that in 100 tests the event occurs

a) exactly 80 times (event *A*);

b) from 70 to 80 times (event *B*);

c) more than 80 times (event *C*);

d) less than 70 times (event *D*).

Solution:

As the number of independent tests *n* is large, and $p \approx 0{,}5$ is not small, the probability $P_n(k)$ that an event will occur *k* times in series of *n* independent tests is determined by de Moivre-Laplace's local theorem, and approximately is equal to

$$P_n(k) \cong \frac{1}{\sqrt{npq}}\varphi(x), \text{ where } x = \frac{k-np}{\sqrt{npq}}, \quad 0 < p < 1, \quad \varphi(x) = \frac{1}{\sqrt{2\pi}}\exp(-x^2/2)$$

(values of this function can be found from the tables in Appendix 1 or by using the built-in function *dnorm* in MathCad system).

To determine the probability of events B, C and D we use de Moivre-Laplace's integral theorem: the probability $P_n(k_1,k_2)$ that the number *k* of occurrence of an event will be in the interval from k_1 to k_2, equals approximately:

$$P_n(k_1,k_2) \approx \Phi(x_2) - \Phi(x_1), \qquad \text{where} \qquad x_2 = \frac{k_2-np}{\sqrt{npq}}, \qquad x_1 = \frac{k_1-np}{\sqrt{npq}},$$

$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_0^x \exp(-t^2/2)dt$ is Laplace's function, the values of which can be found in special tables in Appendix 2 or using built-in function *pnorm* in MathCad system.

a) $x = \frac{80-100\cdot 0{,}8}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}} = 0,\ P_{100}(80) \cong \frac{1}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}}\varphi(0) = 0{,}399;$

b) $P_{100}(70{,}80) \approx \Phi\left(\frac{80-100\cdot 0{,}8}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}}\right) - \Phi\left(\frac{70-100\cdot 0{,}8}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}}\right) = \Phi(0) - \Phi(-2{,}5) = 0{,}494;$

c) $P_{100}(k>80) = P_{100}(80{,}100) \approx \Phi\left(\frac{100-100\cdot 0{,}8}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}}\right) - \Phi\left(\frac{80-100\cdot 0{,}8}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}}\right) = \Phi(5) - \Phi(0) = 0{,}5;$

d)

$$P_{100}(k<70) = P_{100}(0{,}70) \approx \Phi\left(\frac{70-100\cdot 0{,}8}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}}\right) - \Phi\left(\frac{-100\cdot 0{,}8}{\sqrt{100\cdot 0{,}8\cdot 0{,}2}}\right) = \Phi(-2{,}5) - \Phi(-20) = 0{,}006.$$

The copy of file with MathCad calculations is presented below:

$n := 100$, $k1 := 70$, $k2 := 80$,

$p := 0.8$, $q := 1 - p$,

$$x1 := \frac{k1 - n\cdot p}{\sqrt{n\cdot p\cdot q}}, \quad x2 := \frac{k2 - n\cdot p}{\sqrt{n\cdot p\cdot q}}, \quad x3 := \frac{n - n\cdot p}{\sqrt{n\cdot p\cdot q}}, \quad x4 := \frac{0 - n\cdot p}{\sqrt{n\cdot p\cdot q}},$$

$x1 = -2.5$, $x2 = 0$. $dnorm(x2,0,1) = 0.399$,

$pnorm(x1,0,1) = 6.21 \times 10^{-3}$

$pnorm(x2,0,1) = 0.5$

$pnorm(x2,0,1) - pnorm(x1,0,1) = 0.494$

or another variant:

$\Phi(x) := pnorm(x,0,1) - 0.5$,

$P(k1, k2) := \Phi(x2) - \Phi(x1)$

$\Phi(x1) = -0.494$, $\Phi(x2) = 0$,

$P(k1, k2) = 0.494$, $x3 = 5$, $x4 = -20$,

$\Phi(x3) = 0.5$, $\Phi(x4) = -0.5$,

$\Phi(x3) - \Phi(x2) = 0.5$, $\Phi(x1) - \Phi(x4) = 6.21 \times 10^{-3}$.

Task 6. Discrete random variable X is given by distribution series.

X	0	10	20	30	40	50
P	0,05	0,15	0,3	0,25	0,2	0,05

Find:

a) its distribution function $F(x)$, make a graph of function $F(x)$;

b) mathematical expectation, dispersion, standard deviation and mode;

c) the probability of getting the variable X to the interval (15; 45).

Solution:

a) integral distribution function $F(x)$ of random variable X determines the probability of the event $X<x$. For discrete random variable it is determined by the formula $F(x) = P(X < x) = \sum_{x_i < x} p_i = \sum_{x_i < x} P(X = x_i)$, where the summation is performed for all i, for which $x_i < x$.

So,

1) if $x \le 0$, then $F(x) = P(X < 0) = 0$;

2) if $0 < x \le 10$, then $F(x) = P(X = 0) = 0{,}05$;

3) if $10 < x \le 20$, then $F(x) = P(X = 0) + P(X = 10) = 0{,}05 + 0{,}15 = 0{,}2$;

4) if $20 < x \le 30$, then $F(x) = P(X = 0) + P(X = 10) + P(X = 20) = 0{,}2 + 0{,}3 = 0{,}5$;

5) if $30 < x \le 40$, then $F(x) = P(X = 0) + P(X = 10) + P(X = 20) + P(X = 30) =$
$= 0{,}5 + 0{,}25 = 0{,}75$;

6) if $40 < x \le 50$, then $F(x) = P(X = 0) + P(X = 10) + P(X = 20) + P(X = 30) +$
$+ P(X = 40) = 0{,}75 + 0{,}2 = 0{,}95$;

7) if $x > 50$, then $F(x) = P(X = 0) + P(X = 10) + P(X = 20) + P(X = 30) +$
$+ P(X = 40) + P(X = 50) = 0{,}95 + 0{,}05 = 1$.

Thus,
$$F(x) = \begin{cases} 0, \ if \ x \le 0 \\ 0{,}05, \ if \ 0 < x \le 10 \\ 0{,}2, \ if \ 10 < x \le 20 \\ 0{,}5, \ if \ 20 < x \le 30 \\ 0{,}75, \ if \ 30 < x \le 40 \\ 0{,}95, \ if \ 40 < x \le 50 \\ 1, \ if \ x > 50 \end{cases};$$

b) numerical characteristics of discrete random variable X are:

$M(X) = \sum_i x_i p_i$; - mathematical expectation;

$D(X) = \sum_i (x_i - M(X))^2 \cdot p_i = \sum_i x_i^2 p_i - (M(X))^2$ - dispersion;

$\sigma(x) = \sqrt{D(x)}$ - standard deviation;

M_0 - mode and value X_i taken with the greatest probability P_i;

$P(a \le X \le b) = F(b) - F(a)$ - the probability of getting X to the interval (a;b).

In our problem, these values are:

$M(X) = 0 \cdot 0{,}15 + 10 \cdot 0{,}15 + 20 \cdot 0{,}3 + 30 \cdot 0{,}25 + 40 \cdot 0{,}2 + 50 \cdot 0{,}05 = 25{,}5;$

$D(x)=154{,}75;\ \sigma(x) = \sqrt{154{,}75} = 12{,}44;\ \ M_0=20;$

$P(15;45) = F(45) - F(15)=0{,}75.$

The copy of file with MathCad calculations is presented below (dispersion calculations are carried out using both formulas):

ORIGIN := 1 s := (0 10 20 30 40 50),

p := (0.05 0.15 0.3 0.25 0.2 0.05),

M := $s^T \cdot p^T$, M = 25.5 , s0 := s^T − M,

$$s0 = \begin{pmatrix} -25.5 \\ -15.5 \\ -5.5 \\ 4.5 \\ 14.5 \\ 24.5 \end{pmatrix},$$

D := $\left[\overrightarrow{(s0 \cdot s0)}\right]^T \cdot p^T$,

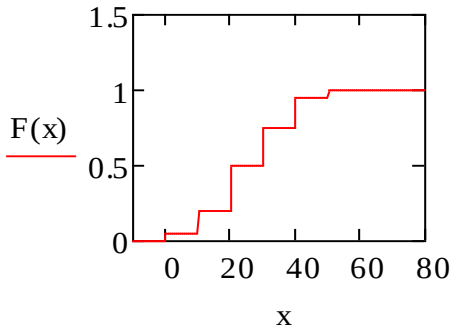
D = 154.75 , σ := $\sqrt{D}$ = 12.44 ,

s2 := $\left(0^2 \ \ 10^2 \ \ 20^2 \ \ 30^2 \ \ 40^2 \ \ 50^2\right)$,

D1 := $s2^T \cdot p^T$ − M^2, D1 = 154.75 ,

$$F(x) := \begin{cases} 0 & \text{if } x \le 0 \\ 0.05 & \text{if } 0 < x \le 10 \\ 0.2 & \text{if } 10 < x \le 20 \\ 0.5 & \text{if } 20 < x \le 30 \\ 0.75 & \text{if } 30 < x \le 40 \\ 0.95 & \text{if } 40 < x \le 50 \\ 1 & \text{if } x > 50 \end{cases}$$

$$F(45) - F(15) = 0.75$$



Task 7. Continuous random variable X is given by distribution function

$$F(x) = \begin{cases} 0, \ if \ \ x < 2 \\ (x-2)^2, \ if \ \ 2 \le x \le 3 \\ 1, \ if \ \ x > 3 \end{cases}.$$

Find:

a) its probability density function $f(x)$;

b) mathematical expectation, dispersion, standard deviation, mode and median;

c) the probability of getting the variable X to the interval (1; 2,5).

Construct graphs of functions $F(x)$ and $f(x)$.

Solution:

a) the probability density distribution of a continuous random variable equals the derivative of its distribution function: $f(x) = F'(x)$, therefore

$$f(x) = F'(x) = \begin{cases} 0, \ if \ \ x < 2 \\ 2(x-2), \ if \ 2 \le x \le 3 \\ 0, \ if \ \ x > 3 \end{cases};$$

b) numerical characteristics of continuous random variables are: $M(x) = \int_{-\infty(a)}^{\infty(b)} xf(x)dx$ - mathematical expectation;

$D(x) = \int_{-\infty(a)}^{\infty(b)} (x - M(x))^2 f(x)dx$ $D(x) = \int_{-\infty(a)}^{\infty(b)} x^2 f(x)dx - [M(x)]^2$ - dispersion;

$\sigma(x) = \sqrt{D(x)}$ - standard deviation;

M_o - mode of continuous random variable X is its value at which the distribution density function has the maximum;

M_e - median of continuous random variable X is its value at which $P(X < M_e) = P(X > M_e) = 0{,}5$.

Thus, in our problem

$$M(x) = \int_2^3 2x(x-2) = 8/3; \qquad D(x) = \int_2^3 x^2 \cdot 2(x-2)dx - (8/3)^2 = 1/18;$$

$\sigma(X) = \sqrt{1/18} = 0{,}236$. To determine the mode it is necessary to find the maximum of the function in the interval [2; 3]: $f_{\max} = f(M_0) = \max(2(x-2)) = f(3) = 2$.

So, $M_o = 3$.

Midian can be found from the condition:

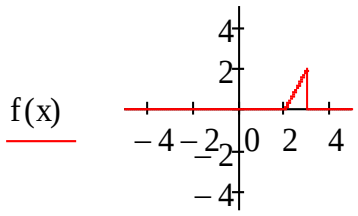
$$P(X < M_e) = P(-\infty < X < M_e) = P(2 < X < M_e) = \int_2^{M_e} 2(x-2)dx = (M_e - 2)^2 = 0{,}5,$$

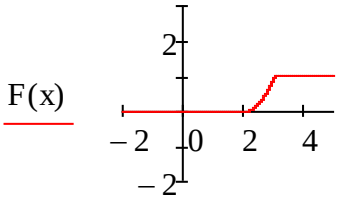
solving the equation, we obtain two roots 1,29 and 2,71, one of which is suitable: $M_e = 2{,}71$;

c) the probability of getting X to the interval (1; 2,5) we find from the distribution function $P(1 < X < 2{,}5) = F(2{,}5) - F(1) = (2{,}5-2)^2 - 0 = 0{,}25$.

We make the graph for functions $F(x)$ and $f(x)$ and other calculations in Mathcad system:

$$f(x) := \begin{cases} 0 & \text{if } x < 2 \\ [2\cdot(x-2)] & \text{if } 2 \le x \le 3 \\ 0 & \text{if } x > 3 \end{cases}, \qquad F(x) := \begin{cases} 0 & \text{if } x < 2 \\ (x-2)^2 & \text{if } 2 \le x \le 3 \\ 1 & \text{if } x > 3 \end{cases},$$





$$\int_2^3 2\cdot x\cdot(x-2)\,dx \to \frac{8}{3} = 2.667,$$

$$\int_2^3 2\cdot x^2\cdot(x-2)\,dx - \left(\frac{8}{3}\right)^2 \to \frac{1}{18} = 0.056, \qquad \sqrt{\frac{1}{18}} = 0.236,$$

$$\int_2^y 2\cdot(x-2)\,dx \rightarrow (y-2)^2$$

$$(y-2)^2 - 0.5 \text{ solve} \rightarrow \begin{pmatrix} 1.2928932188134524756 \\ 2.7071067811865475244 \end{pmatrix}$$

Calculation-graphical work (CGW) №2. Basic distribution laws of random variables used in radio engineering

Theoretical questions

1. Binomial distribution.
2. Poisson's distribution.
3. Uniform distribution.
4. Exponential distribution, reliability function.
5. Normal distribution (Laplace's function, distribution function).
6. Normal distribution (probability of getting to the interval, the probability of given deviation, «three sigma rule»).
7. Systems of random variables and their numerical characteristics.
8. Linear mean square regression.
9. Concept of limit theorems. The law of large numbers, central limit theorem.
10. Chebyshev's inequality. Markov's inequality.

Tasks for calculation

Uniform distribution.

Task 1. Random variable X is uniformly distributed in the interval [0; a]. Find out:

a) its probability density function $f(x)$;

b) distribution function $F(x)$;

c) mathematical expectation, dispersion;

d) the probability that $X < m$.

Make graphs of $F(x)$ and $f(x)$.

№	a	m	№	a	m	№	a	m
1.1	0,2	0,04	1.11	0,3	0,08	1.21	19	8
1.2	0,3	0,02	1.12	0,6	0,01	1.22	20	5
1.3	0,1	0,06	1.13	0,9	0,06	1.23	25	5
1.4	0,5	0,01	1.14	0,5	0,05	1.24	9	3
1.5	0,6	0,05	1.15	0,8	0,07	1.25	14	7
1.6	0,9	0,02	1.16	5	3	1.26	18	9
1.7	0,1	0,08	1.17	10	4	1.27	24	8
1.8	0,7	0,01	1.18	15	5	1.28	6	3
1.9	0,4	0,06	1.19	6	2	1.29	12	6
1.10	0,5	0,07	1.20	20	10	1.30	16	8

Exponential distribution.

Task 2. Uptime of a radio tube t has an exponential distribution, where T_0 - the mean operating time of the radio tube. It is required to find:

a) probability density function $f(t)$;

b) distribution function $F(t)$, specify its probabilistic meaning;

c) reliability function $R(t)$, specify its probabilistic meaning;

d) mathematical expectation, dispersion;

e) the probability that during the time t the radio tube will stop working and the probability that during the time t the radio tube will not stop working;

f) make graphs of F(t), R(t) and f(t).

№	T_0	t	№	T_0	t	№	T_0	t
2.1	1	5	2.11	2	5	2.21	3	8
2.2	2	10	2.12	3	10	2.22	4	4
2.3	3	6	2.13	4	6	2.23	6	3
2.4	4	8	2.14	6	8	2.24	7	2
2.5	6	4	2.15	7	4	2.25	8	1
2.6	7	3	2.16	8	3	2.26	9	10
2.7	8	2	2.17	9	2	2.27	10	6
2.8	9	1	2.18	10	1	2.28	1	7
2.9	10	7	2.19	1	10	2.29	2	8
2.10	1	9	2.20	2	6	2.30	3	2

Normal distribution.

Task 3. Random variable X is subordinated to a normal distribution law with mathematical expectation a, and dispersion σ^2. Find:

a) probability density function *f(x);*

b) distribution function *F(x);*

c) the probability of getting to the interval $(\alpha;\beta)$;

d) the probability that deviation of random variable X from a will be got with an error not exceeding in absolute value the value of δ;

e) make graphs of *F(x)* and *f(x).*

№	a	σ	α	β	δ	№	a	σ	α	β	δ
3.1	10	1	8	14	2	3.16	10	2	9	14	2
3.2	12	2	7	14	3	3.17	12	4	5	14	3
3.3	14	3	10	15	5	3.18	14	1	9	15	5
3.4	11	5	9	12	3	3.19	11	6	8	12	3
3.5	13	2	6	13	2	3.20	13	4	6	17	2
3.6	12	3	7	15	4	3.21	12	9	8	15	4
3.7	10	2	8	17	2	3.22	10	3	6	17	2
3.8	12	4	6	14	6	3.23	12	5	6	13	6
3.9	14	6	11	19	5	3.24	14	2	12	19	5
3.10	15	5	8	12	3	3.25	15	3	4	12	3
3.11	17	4	6	14	2	3.26	17	1	5	14	2
3.12	12	5	7	18	4	3.27	12	4	9	18	4
3.13	18	5	6	12	3	3.28	11	3	4	12	3
3.14	10	4	6	15	2	3.29	17	2	5	19	5
3.15	12	3	5	18	4	3.30	13	5	6	18	3

Binomial distribution

Task 4. The probability of an event in each test, from the series of N tests is p. It is required to find:

a) mathematical expectation and dispersion;

b) using Chebyshev's inequality, estimate the probability that the frequency of this event deviates from its probability no more than $\varepsilon = 0.01$.

№	N	p	№	N	p	№	N	p
4.1	300	1/3	4.11	400	1/6	4.21	300	1/5
4.2	200	1/2	4.12	500	1/3	4.22	200	1/6
4.3	400	2/3	4.13	300	1/4	4.23	400	2/9
4.4	500	2/5	4.14	200	2/3	4.24	500	1/8
4.5	300	3/4	4.15	400	2/7	4.25	300	1/7
4.6	200	1/5	4.16	500	2/4	4.26	200	2/5
4.7	400	1/5	4.17	300	1/9	4.27	400	2/3
4.8	500	1/7	4.18	200	2/3	4.28	500	2/4
4.9	300	1/2	4.19	400	1/4	4.29	300	1/8
4.10	200	1/5	4.20	500	2/8	4.30	200	2/6

Task 5. The distribution law of two-dimensional random value (X,Y) is given. It is required to find:

a) mathematical expectations $M(x)$, $M(y)$;

b) dispersions $D(x)$, $D(y)$ and their standard deviations;

c) correlation moment $K(x,y)$, correlation coefficient r_{xy};

d) linear mean square regression of random variable Y on random variable X.

5.1

X \ Y	1	3	4
2	0,16	0,10	0,28
3	0,14	0,20	0,12

5.2

X \ Y	2	3	5
1	0,06	0,18	0,24
4	0,12	0,13	0,27

5.16

X \ Y	5	7	9
4	0,14	0,15	0,21
7	0,16	0,20	0,14

5.17

X \ Y	1	4	6
3	0,14	0,12	0,13
4	0,13	0,20	0,18

5.3

Y / X	1	2	4
3	0,12	0,24	0,22
4	0,20	0,15	0,07

5.4

X / Y	2	3	4
1	0,16	0,10	0,28
3	0,14	0,20	0,12

5.5

X / Y	2	3	5
4	0,06	0,18	0,24
6	0,12	0,13	0,27

5.6

X / Y	1	3	7
5	0,16	0,10	0,28
7	0,14	0,20	0,12

5.7

X / Y	3	5	7
1	0,12	0,13	0,24
4	0,18	0,06	0,27

5.8

Y / X	4	5	6
2	0,06	0,18	0,24
3	0,12	0,13	0,27

5.18

X / Y	5	8	10
2	0,11	0,13	0,26
6	0,21	0,06	0,23

5.19

X / Y	4	7	9
4	0,22	0,09	0,32
7	0,14	0,17	0,06

5.20

X / Y	8	9	12
1	0,14	0,11	0,18
6	0,23	0,04	0,30

5.21

Y / X	3	6	8
2	0,21	0,07	0,23
8	0,11	0,20	0,18

5.22

X / Y	3	4	7
4	0,15	0,23	0,15
8	0,21	0,09	0,17

5.23

Y / X	4	5	8
3	0,13	0,14	0,19
5	0,24	0,08	0,22

5.9

X \ Y	2	4	5
1	0,12	0,13	0,24
3	0,18	0,06	0,27

5.10

X \ Y	1	3	6
3	0,14	0,24	0,12
6	0,18	0,06	0,26

5.11

Y \ X	1	3	4
3	0,13	0,24	0,12
5	0,18	0,06	0,27

5.12

Y \ X	3	5	6
1	0,12	0,24	0,22
3	0,20	0,15	0,07

5.13

Y \ X	4	6	8
3	0,13	0,08	0,12
5	0,20	0,16	0,31

5.14

Y \ X	3	4	7
3	0,30	0,20	0,10
6	0,05	0,12	0,23

5.24

Y \ X	6	9	12
5	0,23	0,07	0,15
9	0,17	0,20	0,18

5.25

Y \ X	5	8	10
2	0,11	0,21	0,14
7	0,20	0,09	0,25

5.26

Y \ X	4	7	9
4	0,30	0,12	0,10
10	0,08	0,12	0,28

5.27

Y \ X	6	9	12
5	0,21	0,18	0,14
9	0,08	0,14	0,25

5.28

Y \ X	4	7	9
2	0,09	0,15	0,16
7	0,17	0,23	0,20

5.29

Y \ X	1	4	8
4	0,11	0,24	0,17
8	0,21	0,08	0,19

5.15

Y \ X	4	6	8
2	0,24	0,30	0,05
5	0,10	0,11	0,19

5.30

Y \ X	4	8	14
3	0,12	0,13	0,20
5	0,23	0,12	0,20

Solution of a standard version

Task 1. Random variable X is uniformly distributed in the interval [0; 0,2]. Find:

a) its probability density function $f(x)$;

b) distribution function $F(x)$;

c) mathematical expectation, dispersion;

d) the probability that $X < 0.04$;

f) make graphs of $F(x)$ and $f(x)$.

Solution:

a) random variable X is uniformly distributed in the interval $[a;b]=[0;0,2]$, where $b-a=0,2$ – length of the interval in which the possible values of X are included. The density of uniform distribution can be found by formula:

$$f(x)=\begin{cases}\dfrac{1}{b-a}, \ if \ \ x\in(a,b),\\ 0, \ \ if \ \ x\notin(a,b),\end{cases}$$

therefore we obtain

$$f(x)=\begin{cases}\dfrac{1}{0,2}=5, if \ \ x\in(0;0,2),\\ 0, \ \ if \ \ x\notin(0;0,2);\end{cases}$$

b) function of uniform distribution – $F(x)=\begin{cases}0, \ \ if\ x\le a,\\ \dfrac{x-a}{b-a}, if\ a<x\le b,\\ 1, \ if\ x>b.\end{cases}$

Therefore in our problem:

$$F(x)=\begin{cases}0, \ \ if\ x\le 0,\\ \dfrac{x}{0,2}=5x, if\ 0<x\le 0,2,;\\ 1, \ if\ x>0,2;\end{cases}$$

c) mathematical expectation and dispersion are calculated according to formulas:

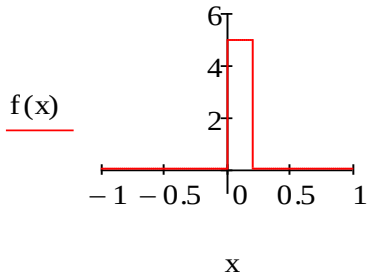
$$M(X)=\frac{a+b}{2},\ D(X)=\frac{(b-a)^2}{12};$$

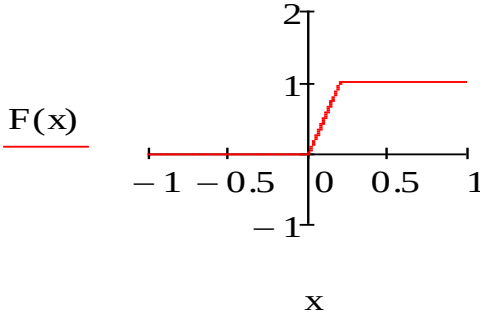
$$M(X)=\frac{0,2+0}{2}=0,1;\quad D(X)=\frac{(0,2-0)^2}{12}=0,003;$$

d) the probability of getting to the interval (α,β) is defined by formula: $P(a\le\alpha<X<\beta\le b)=\frac{\beta-\alpha}{b-a}=F(\beta)-F(\alpha)$. The probability of the event that $X<0,04$, equals $P(X<0,04)=P(-\infty<X<0,04)=F(0,04)-F(0)=\frac{0,04}{0,2}-\frac{0}{0,2}=0,2;$

f) it is convenient to make graphs of $F(x)$ and $f(x)$ in Mathcad software system:

$$f(x):=\begin{cases}0 & \text{if } x\le 0\\ 5 & \text{if } 0<x\le 0.2\\ 0 & \text{if } x>0.2\end{cases}\qquad F(x):=\begin{cases}0 & \text{if } x\le 0\\ (5\cdot x) & \text{if } 0<x\le 0.2\\ 1 & \text{if } x>0.2\end{cases}$$





In Mathcad, special functions with the root word *unif* correspond to the uniform distribution law: *dunif(x,a,b)* displays the values of probability density function; *punif(x,a,b)* displays the values of the distribution function; *runif(n,a,b)* displays an array of *n* values of independent random numbers distributed uniformly over the interval *(a,b).*

Task 2. Random value *t* is uptime of a radio tube, has an exponential distribution; the mean operating time of the radio tube is T_0 = 2 months. It is required to find:

a) probability density function $f(t)$;

b) distribution function $F(t)$, specify its probabilistic meaning;

c) reliability function $R(t)$, specify its probabilistic meaning;

d) mathematical expectation and dispersion;

e) the probability that during the time t=5 months the radio tube will stop working and the probability that during the time t=5 months the radio tube will not stop working;

f) make graphs of $F(t)$, $R(t)$ and $f(t)$.

Solution:

a) distribution law of continuous random variable X with probability density function $f(x)=\begin{cases} 0, \text{ if } x<0 \\ \lambda e^{-\lambda x}, \text{ if } x\geq 0 \end{cases}$ and distribution function $F(x)=\begin{cases} 0, \text{ if } x<0 \\ 1-e^{-\lambda x}, \text{ if } x\geq 0 \end{cases}$ is called exponential; so, function $F(t)=P(T<t)=1-e^{-\lambda t}$ determines the probability of failure of the element during time t; $R(t)=P(T>t)=e^{-\lambda t}$ is reliability function, which determines the probability of no-failure operation of the element during time t. Basic numerical characteristics of the exponential distribution are expressed using a single parameter λ:

$$M(X)=\frac{1}{\lambda},\ D(X)=\frac{1}{\lambda^2},\ \sigma(X)=\frac{1}{\lambda};\ P(\alpha<X<\beta)=e^{-\lambda\alpha}-e^{-\lambda\beta}.$$

a) unknown parameter λ is determined by the mean operating time of the radio tube (its mathematical expectation):

$M(X)=\frac{1}{\lambda}=T_0=2\rightarrow\lambda=\frac{1}{M(x)}=\frac{1}{2}=0{,}5$; and $f(t)=0{,}5e^{-0{,}5t}$ is respectively probability density function;

b) $F(t)=P(T<t)=1-e^{-0{,}5t}$ determines the probability of failure of the element during time t;

c) $R(t)=e^{-0{,}5t}$ determines the probability of no-failure operation of the element during time t;

d) $M(t)=\frac{1}{0{,}5}=2$, $D(t)=\frac{1}{0{,}5^2}=4$;

e) the probability of failure during time t=5 is determined by the distribution function:

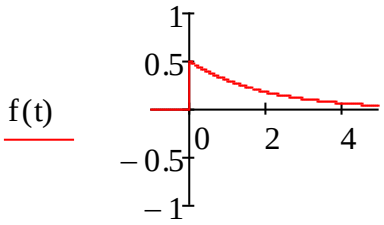
$F(5)=P(T<5)=P(0<X<5)=e^{-0{,}5\cdot 0}-e^{-0{,}5\cdot 5}=\ =1-e^{-0{,}5\cdot 5}=1-e^{-2{,}5}=0{,}918$; and the probability of an opposite event "vacuum tube will not refuse to operate" during time t = 5 is determined by reliability function:

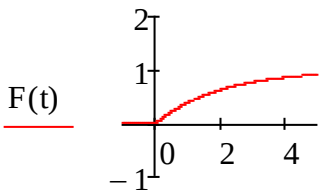
$R(5)=e^{-0{,}5\cdot 5}=e^{-2{,}5}=0{,}082$ or $1-0{,}918=0{,}082$;

f) it is convenient to make the graphs of $F(t)$, $R(t)$ and $f(t)$ and to carry out some calculations using Mathcad software system:

$$f(t):=\begin{cases} 0 & \text{if } t<0 \\ 0.5\cdot e^{-0.5\cdot t} & \text{if } t\geq 0 \end{cases}$$

$$F(t):=\begin{cases} 0 & \text{if } t<0 \\ 1-e^{-0.5\cdot t} & \text{if } t\geq 0 \end{cases}$$

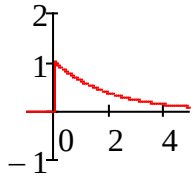




$e^{-2.5} = 0.082$ $\quad 1 - e^{-2.5} = 0.918$

$$R(t) := \begin{cases} 0 & \text{if } t < 0 \\ e^{-0.5 \cdot t} & \text{if } t \geq 0 \end{cases}$$

R(t)



t

In Mathcad system, special functions with the root word *exp* correspond to the exponential distribution law: *dexp(x, λ)* displays the values of the probability density function; *pexp(x, λ)* displays the values of the distribution function; *rexp(n, λ)* displays an array of *n* values of independent random numbers distributed exponentially with parameter *λ*.

Task 3. Random variable *X* is subordinated to a normal distribution law with parameters *a*=10 and σ=2. It is required to find:

a) probability density function *f(x);*

b) distribution function *F(x);*

c) mathematical expectation and dispersion;

d) the probability of getting to the interval (12;14);

e) the probability that the deviation of random variable *X* from *a* will be received with an error not exceeding in absolute value the value of δ=3;

f) make graphs of *F(x)* and *f(x).*

Solution:

a) *N(a,σ)* - distribution law of continuous random variable *X* is called normal if it has the following probability density function and distribution function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}};$$

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[\frac{-(t-a)^2}{2\sigma^2}\right] dt \text{ or } F(x) = \Phi\left(\frac{x-a}{\sigma}\right) + 0{,}5,$$

where $a = M(X)$ – mathematical expectation;

$\sigma = \sigma(X)$ – standard deviation,

$\Phi(x) = \frac{1}{\sqrt{2\sigma}} \int_{0}^{x} e^{-\frac{t^2}{2}} dt$ – Laplace's function, its values are tabulated (see Appendix 2) or you can find them in Mathcad system. Substituting parameters *a*=10 and σ=2, we obtain:

$$f(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-10)^2}{8}};$$

b) $F(x)=\Phi\left(\frac{x-10}{2}\right)+0{,}5$;

c) $M(X)=a=10$, $\sigma(X)=\sigma=2$, $D(X)=\sigma^2=4$;

d) the probability of getting of random variable X to the interval $(\alpha;\ \beta)$ for normal distribution is determined by Laplace's function:

$$P(\alpha<X<\beta)=F(\beta)-F(\alpha)=\Phi\left(\frac{\beta-a}{\sigma}\right)-\Phi\left(\frac{\alpha-a}{\sigma}\right);$$

$$P(12<X<14)=\Phi\left(\frac{14-10}{2}\right)-\Phi\left(\frac{12-10}{2}\right)=\Phi(2)-\Phi(1)=0{,}4772-$$

0,3413=0,1359;

e) the probability that random variable X will deviate from its mathematical expectation a no more than δ, is determined by formula:

$$P(|X-a|\le\delta)=2\Phi\left(\frac{\delta}{\sigma}\right).$$

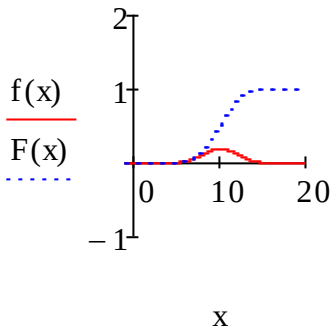
Therefore the probability that the deviation of the random variable X from a will be received with an error not exceeding in absolute value the value of δ=3, is equal to

$$P(|X-10|\le 3)=2\Phi\left(\frac{3}{2}\right)=2\Phi(1{,}5)=0{,}4332,$$

where the values of the Laplace's function are taken from the tables given in Appendix 2;

f) in Mathcad system, special functions with the root word *norm* and starting with letters *d, p, q, r* correspond to the normal distribution law. To graph $F(t)$ and $f(t)$ we use, for example, *dnorm(x,a,* σ*)* that displays the values of the probability density function *f(x); pnorm(x,a,*σ*)* displays the values of the distribution function *F(x).* Mathcad copy is given below.

$f(x) := \text{dnorm}(x,10,2)$ $\quad$ $F(x) := \text{pnorm}(x,10,2)$



Task 4. The probability of an event in each test, from series of 900 tests, is 1/3. It is required to estimate the probability that the frequency of this event deviates from its probability no more than ε = 0,01 according to:

a) Chebyshev's inequality;

b) Moivre-Laplace's integral formula.

Solution:

a) Chebyshev's inequality for random variable *X* has the form

$$P(|X - M(X)| < \varepsilon) \geq 1 - \frac{D(X)}{\varepsilon^2};$$

the same Chebyshev's inequality for frequency deviation *m/n* of event from its probability *p* is expressed:

$$P(|m/n - p| < \varepsilon) = P(|m - np| < \varepsilon \cdot n) \geq 1 - \frac{D(X)}{(\varepsilon \cdot n)^2}.$$

Taking into account the fact that Bernoulli's distribution law is given, mathematical expectation and dispersion are expressed:

*M(X)=np=9000*1/3=3000; D(X)=npq=9000*1/3*(1-1 /3) =2000.*

Then

$$P(|m/n - p| < 0{,}01) = P(|m - np| < 0{,}01 \cdot 9000) \geq 1 - \frac{2000}{(0{,}01 \cdot 9000)^2}. = 1 - \frac{20}{81} =$$

$$= \frac{61}{81} \approx 0{,}7531;$$

b) let's evaluate the deviation by Moivre-Laplace's integral formula

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right) \quad \text{or} \quad P(|X - a| \leq \delta) = 2\Phi\left(\frac{\delta}{\sigma}\right),$$

that in case of frequency deviation *m/n* from probability *p* takes the form

$$P(|m/n - p| \leq \varepsilon) = P(|m - np| \leq n\varepsilon) = 2\Phi\left(\frac{n \cdot \varepsilon}{\sqrt{D(X)}}\right),$$

$$P(|m/n - p| \leq 0{,}01) = 2\Phi\left(\frac{0{,}01 \cdot 9000}{\sqrt{2000}}\right) = 2\Phi(2{,}01) = 0{,}9545.$$

Task 5. The distribution law of two-dimensional random value (*X,Y*) is given by the table:

X \ *Y*	-1	0	1
0	0,15	0,40	0,05
1	0,20	0,10	0,10

It is required to find:

a) mathematical expectations *M(x), M(y);*

b) dispersions *D(x), D(y)* and their standard deviations;

c) correlation moment *K(x,y)* and correlation coefficient r_{xy};

d) linear mean square regression of random variable *Y* on random variable *X*.

Solution:

a) let's find the probability of values X for all possible values of Y by summing the probabilities of the rows. We obtain the distribution law of the random variable X:

X	0	1
p	0,6	0,4

Similarly, having summed the probabilities of columns, we obtain the distribution law of random variable Y:

Y	-1	0	1
p	0,35	0,50	0,15

Then we find mathematical expectations of the components:

$$M(x) = 0 \cdot 0{,}6 + 1 \cdot 0{,}4 = 0{,}4, \quad M(y) = -1 \cdot 0{,}35 + 0 \cdot 0{,}50 + 1 \cdot 0{,}15 = -0{,}2;$$

b) we find the dispersions of components:

$$D(x) = M(x^2) - M^2(x) = \left(0^2 \cdot 0{,}6 + 1^2 \cdot 0{,}4\right) - 0{,}4^2 = 0{,}24,$$

$$D(y) = M(y^2) - M^2(y) = \left((-1)^2 \cdot 0{,}35 + 0^2 \cdot 0{,}50 + 1^2 \cdot 0{,}15\right) - \left(-0{,}2\right)^2 = 0{,}46.$$

So: $\sigma(x) = \sqrt{0{,}24} \approx 0{,}49, \quad \sigma(y) = \sqrt{0{,}46} \approx 0{,}68.$

c) we find the correlation moment *K(x,y)*, using formula:

$$K(x, y) = M(xy) - M(x) \cdot M(y).$$

As

$$M(xy) = \sum_i \sum_j p_{ij} x_i y_j = 0 \cdot (-1) \cdot 1{,}15 + 0 \cdot 0 \cdot 0{,}4 + 0 \cdot 1 \cdot 0{,}05 + 1 \cdot (-1) \cdot 0{,}2 +$$

$$1 \cdot 0 \cdot 0{,}1 + 1 \cdot 1 \cdot 0{,}1 = -1 \cdot 0{,}20 + 0 \cdot 0{,}70 + 1 \cdot 0{,}10 = -0{,}1,$$

$$K(x, y) = -0{,}1 - 0{,}4 \cdot (-0{,}20) = -0{,}02.$$

We find the correlation coefficient:

$$r_{xy} = \frac{K(x, y)}{\sigma(x) \cdot \sigma(y)} = \frac{-0{,}02}{0{,}49 \cdot 0{,}68} \approx -0{,}06;$$

d) linear mean square regression of random variable Y on random variable X has the form:

$$y(x) = M(y) + r_{xy} \frac{\sigma_y}{\sigma_x}\left(x - M(x)\right).$$

Using the data, we obtain

$$y(x) = -0{,}2 - 0{,}06 \cdot \frac{0{,}68}{0{,}49}\left(x - 0{,}4\right), \quad \text{т.е.} \quad y(x) = -0{,}083x + 0{,}167.$$

It is also convenient to make the graph of regression line of random variable Y on X and to carry out some calculations using Mathcad software system:

ORIGIN := 1

$$A := \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0.15 & 0.40 & 0.05 \\ 1 & 0.20 & 0.10 & 0.10 \end{pmatrix} \qquad n := \sum_{i=2}^{3} \sum_{j=2}^{4} A_{i,j} \qquad n = 1$$

$$x := \sum_{i=2}^{3} \sum_{j=2}^{4} \left(A_{i,1} A_{i,j} \right) \qquad y := \sum_{i=2}^{3} \sum_{j=2}^{4} \left(A_{1,j} A_{i,j} \right) \qquad y = -0.2$$

$$x = 0.4$$

$$Dx := \sum_{i=2}^{3} \sum_{j=2}^{4} \left[\left(A_{i,1} \right)^2 A_{i,j} \right] \qquad Dy := \sum_{i=2}^{3} \sum_{j=2}^{4} \left[\left(A_{1,j} \right)^2 A_{i,j} \right]$$

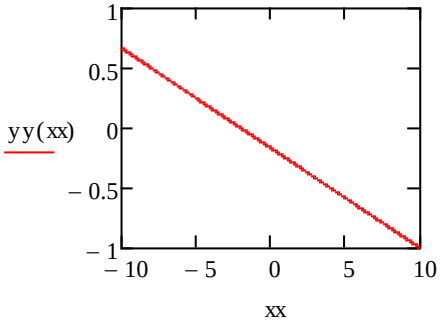
$$dx1 := Dx - x^2 \qquad dy1 := Dy - y^2 \qquad dx1 = 0.24 \qquad dy1 = 0.46$$

$$\sigma x := \sqrt{dx1} \qquad \sigma y := \sqrt{dy1} \qquad \sigma x = 0.49 \qquad \sigma y = 0.678$$

$$MXY := \sum_{i=2}^{3} \sum_{j=2}^{4} \left(A_{i,1} A_{1,j} A_{i,j} \right) \qquad Kxy := MXY - x \cdot y$$

$$rxy := \frac{Kxy}{\sigma x \cdot \sigma y} \qquad Kxy = -0.02 \qquad MXY = -0.1 \qquad rxy = -0.06$$

$$yy(xx) := y + rxy \cdot \frac{\sigma y}{\sigma x} \cdot (xx - x)$$



Appendix 1

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

Table of function values

	0	1	2	3	4	5	6	7	8	9
0,0	0,3989	3989	3989	3988	3986	3984	3982	3980	3977	3973
0,1	3970	3965	3961	3956	3951	3945	3939	3932	3925	3918
0,2	3910	3902	0894	3885	3876	3867	3857	3847	3836	3825
0,3	3814	3802	3790	3778	3765	3752	3739	3726	3712	3697
0,4	3683	3668	3652	3637	3621	3605	3589	3572	3555	3538
0,5	3521	3503	3485	3467	34448	3429	3410	3391	3372	3352
0,6	3332	3312	3292	3271	3251	3230	3209	3187	3166	3144
0,7	3123	3101	3079	3056	3034	3011	2989	2966	2943	2920
0,8	2897	2874	2827	2803	2780	2756	2756	2732	2709	2685
0,9	2661	2637	2613	2589	2565	2541	2516	2492	2468	2444
1,0	0,2420	2396	2371	2347	2323	2299	2275	2251	2227	2203
1,1	2179	2155	2131	2107	2083	2059	2036	2012	1989	1965
1,2	1942	1919	1895	1872	1849	1826	1804	1781	1758	1736
1,3	1714	1691	1609	1647	1626	1604	1582	1561	1539	1518
1,4	1497	1476	1456	1435	1415	1394	1374	1354	1334	1315
1,5	1295	1276	1257	1238	1219	1200	1182	1163	1145	1127
1,6	1109	1092	1074	1057	1040	1023	1006	0989	0973	0957
1,7	0940	0925	0909	0893	0878	0863	0848	0833	0818	0804
1,8	0709	0775	0761	0748	0734	0721	0707	0694	0681	0669
1,9	0656	0644	0632	0620	0608	0596	0584	0573	0562	0551
2,0	0,0540	0529	0519	0508	0498	0488	0478	0468	0459	0449
2,1	0440	0431	0422	0413	0404	0396	0387	0379	0371	0363
2,2	0355	0347	0339	0332	0325	0317	0310	0303	0297	0290
2,3	0283	0277	0270	0264	0258	0252	0246	0241	0235	0229
2,4	0224	0219	0213	0208	0203	0198	0194	0189	0184	0180
2,5	0175	0171	0167	0163	0158	0154	0151	0147	0143	0139
2,6	0136	0132	0129	0126	0122	0119	0116	0113	0110	0107
2,7	0104	0101	0099	0096	0093	0091	0088	0086	0084	0081
2,8	0079	0077	0075	0073	0071	0069	0067	0065	0063	0061
2,9	0060	0058	0056	0055	0053	0051	0050	0048	0047	0046
3,0	0,0044	0043	0042	0040	0039	0038	0037	0036	0035	0034
3,1	0033	0032	0031	0030	0029	0028	0027	0026	0025	0025
3,2	0024	0023	0022	0022	0021	0020	0020	0019	0018	0018
3,3	0017	0017	0016	0016	0015	0015	0014	0014	0013	0013
3,4	0012	0012	0012	0011	0011	0010	0010	0010	0009	0009
3,5	0009	0008	0008	0008	0008	0007	0007	0007	0007	0006
3,6	0006	0006	0006	0005	0005	0005	0005	0005	0005	0001
3,7	0004	0004	0004	0004	0004	0004	0003	0003	0003	0003
3,8	0003	0003	0003	0003	0003	0002	0002	0002	0002	0002
3,9	0002	0002	0002	0002	0002	0002	0002	0002	0001	0001

Appendix 2

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp\left(-\frac{t^2}{2}\right) dt$$

Table of function values

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0,00	0,0000								
0,01	0,0040	0,51	0,1950	1,01	0,3438	1,51	0,4345	2,02	04783
0,02	0,0080	0,52	0,1985	1,02	0,3461	1,52	0,4357	2,04	0,4793
0,03	0,0120	0,53	0,2019	1,03	0,3485	1,53	0,4370	2,06	0,4803
0,04	0,0160	0,54	0,2054	1,04	0,3508	1,54	0,4382	2,08	0,4812
0,05	0,0199	0,55	0,2088	1,05	0,3531	1,55	0,4394	2,10	0,4821
0,06	0,0239	0,56	0,2123	1,06	0,3554	1,56	0,4406	2,12	0,4830
0,07	0,0279	0,57	0,2157	1,07	0,3577	1,57	0,4418	2,14	0,4838
0,08	0,0319	0,58	0,2190	1,08	0,3599	1,58	0,4429	2,16	0,4846
0,09	0,0359	0,59	0,2224	1,09	0,3621	1,59	0,4441	2,18	0,4854
0,10	0,0398	0,60	0,2257	1,10	0,3643	1,60	0,4452	2,20	0,4861
0,11	0,0438	0,61	0,2291	1,11	0,3665	1,61	0,4463	2,22	0,4868
0,12	0,0478	0,62	0,2324	1,12	0,3686	1,62	0,4474	2,24	0,4875
0,13	0,0517	0,63	0,2357	1,13	0,3708	1,63	0,4484	2,26	0,4881
0,14	0,0557	0,64	0,2389	1,14	0,3729	1,64	0,4495	2,28	0,4887
0,15	0,0596	0,65	0,2422	1,15	0,3749	1,65	0,4205	2,30	0,4893
0,16	0,0636	0,66	0,2454	1,16	0,3770	1,66	0,4515	2,32	0,4898
0,17	0,0675	0,67	0,2486	1,17	0,3790	1,67	0,4525	2,34	0,4904
0,18	0,0714	0,68	0,2517	1,18	0,3810	1,68	0,4535	2,36	0,4909
0,19	0,0753	0,69	0,2549	1,19	0,3830	1,69	0,4545	2,38	0,4913
0,20	0,0793	0,70	0,2580	1,20	0,3849	1,70	0,4554	2,40	0,4918
0,21	0,0832	0,71	0,2611	1,21	0,3869	1,71	0,4564	2,42	0,4922
0,22	0,0871	0,72	0,2642	1,22	0,3883	1,72	0,4573	2,44	0,4927
0,23	0,0910	0,73	0,2673	1,23	0,3907	1,73	0,4582	2,46	0,4931
0,24	0,0948	0,74	0,2703	1,24	0,3925	1,74	0,4591	2,48	0,4934
0,25	0,0987	0,75	0,2734	1,25	0,3944	1,75	0,4599	2,50	0,4938
0,26	0,126	0,76	0,2764	1,26	0,3962	1,76	0,4608	2,52	0,4941
0,27	0,1064	0,77	0,2794	1,27	0,3980	1,77	0,4616	2,54	0,4945
0,28	0,1103	0,78	0,2823	1,28	0,3997	1,78	0,4625	2,56	0,4948
0,29	0,1141	0,79	0,2852	1,29	0,4015	1,79	0,4633	2,58	0,4951
0,30	0,1179	0,80	0,2881	1,30	0,4032	1,80	0,4641	2,60	0,4953
0,31	0,1217	0,81	0,2910	1,31	0,4049	1,81	0,4649	2,62	0,4956
0,32	0,1255	0,82	0,2939	1,32	0,4066	1,82	0,4656	2,64	0,4959
0,33	0,1293	0,83	0,2967	1,33	0,4082	1,83	0,4664	2,66	0,4961
0,34	0,1331	0,84	0,2995	1,34	0,4099	1,84	0,4671	2,68	0,4963
0,35	0,1368	0,85	0,3023	1,35	0,4115	1,85	0,4678	2,70	0,4965
0,36	0,1406	0,86	0,3051	1,35	0,4131	1,86	0,4686	2,72	0,4967
0,37	0,1443	0,87	0,3078	1,37	0,4147	1,87	0,4693	2,74	0,4969
0,38	0,1480	0,88	0,3106	1,38	0,4162	1,88	0,4699	2,76	0,4971
0,39	0,1517	0,89	0,3133	1,39	0,4177	1,89	0,4706	2,78	0,4973

0,40	0,1554	0,90	0,3159	1,40	0,4192	1,90	0,4713	2,80	0,4974
0,41	0,1591	0,91	0,3186	1,41	0,4207	1,91	0,4719	2,82	0,4976
0,42	0,1628	0,92	0,3212	1,42	0,4222	1,92	0,4726	2,84	0,4977
0,43	0,1664	0,93	0,3238	1,43	0,4236	1,93	0,1732	2,86	0,4979
0,44	0,1700	0,94	0,3264	1,44	0,4251	1,94	0,4738	2,88	0,4980
0,45	0,1736	0,95	0,3289	1,45	0,4265	1,95	0,4744	2,90	0,4981
0,46	0,1772	0,96	0,3315	1,46	0,4279	1,96	0,4750	2,92	0,4982
0,47	0,1808	0,97	0,3340	1,47	0,4292	1,97	0,4756	2,94	0,4984
0,48	0,1844	0,98	0,3365	1,48	0,4306	1,98	0,4761	2,96	0,4985
0,49	0,1879	0,99	0,3389	1,49	0,4319	1,99	0,4767	2,98	0,4986
0,50	0,1915	1,00	0,3413	1,50	0,4332	2,00	0,4772	3,00	0,49865
						3,2	0,49931	3,8	0,499928
						3,4	0,49966	4,00	0,499968
						3,6	0,499841	4,50	0,499997
								5,00	0,499997

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Kim Regina Evgenievna
Masanova Aida Zhaylauovna

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Methodological Guidelines and Tasks
for carrying out calculation-graphical work for students of specialty
5B071900 – Radio engineering, electronics and telecommunications

Part 1 – 2

Editor V.S. Kozlov
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