Non-Profit Joint Stock Company

ALMATY UNIVERSITY OF POWER ENGINEERING AND TELECOMMUNICATIONS

Department of
Higher mathematics

COMPUTER SOLUTION OF PROBLEMS IN OPERATIONAL CALCULUS AND PROBABILITY THEORY

Methodological Guidelines and Tasks
for carrying out calculation-graphical work for students of specialty
5B071900 «Radio engineering, electronics and telecommunications»
Part 3 – 4

Almaty 2016

AUTHORS: Kim R.E., Masanova A.Zh. Computer solution of problems in operational calculus and probability theory. Methodological Guidelines and Tasks for carrying out calculation-graphical work for students of specialty 5B071900 – Radio engineering, electronics and telecommunications. Part 3 – 4. – Almaty: AUPET, 2016. – 40 p.

The methodological Guidelines and Tasks for carrying out the calculation-graphical works №3, №4 on discipline CSPOCPT are for students of specialty 5B071900 – Radio engineering, electronics and telecommunications are given.

The presented material corresponds to the sections "Mathematical Statistics", "Operational calculus" provided in the curriculum for students of this specialty.

There are basic theoretical questions, solutions of a standard version as well as examples of tasks performance in MathCad system in the guidelines.

Tables 3, bibl. 9

Reviewer: senior lecturer of the Department of foreign languages, S.B. Bukhina

Printed according to the Publishing plan of Non-Profit Joint Stock Company "Almaty University of Power Engineering and Telecommunications" in 2016

© Non-Profit Joint Stock Company
"Almaty University of Power Engineering and Telecommunications", 2016

Calculation-graphical work (CGW) №3. Mathematical Statistics

Theoretical questions.

1. Subject of mathematical statistics and its basic objectives. Basic concepts (sample, sample size, variants, statistical series, interval series).
2. Empirical distribution function, polygon, histogram.
3. Determination of unknown parameters of distribution (sample mean, sample dispersion and corrected sample dispersion).
4. Point and interval estimations of distribution parameters.
5. Accuracy and reliability of estimation. Confidence interval. Confidence interval to estimate mathematical expectation for normally distributed random variable with known σ.
6. Concept of correlation dependence. Correlation coefficient, positive and negative correlations. Functions and regression lines.

Tasks for calculation

Task 1. Determine for a given sample:

a) variational series (sample in increasing order);

b) interval statistical series (minimum and maximum variants, range of the sample, number of intervals, empirical frequencies of intervals);

c) construct a polygon of frequencies, histogram of relative frequencies and a graph of empirical distribution function;

d) find numerical characteristics of the sample: sample mean, sample dispersion and corrected sample dispersion;

e) corrected sample standard deviation;

f) sample mode and median;

g) sample central moments of the third and the fourth order;

h) sample excess;

i) asymmetry coefficient.

1.1.

17,1	21,4	15,9	19,1	22,4	20,7	17,9	18,6	21,8	16,1
19,1	20,5	14,2	16,9	17,8	18,1	19,1	15,8	18,8	17,2
16,2	17,3	22,5	19,9	21,1	15,1	17,7	19,8	14,9	20,5
17,5	19,2	18,5	15,7	14,0	18,6	21,2	16,8	19,3	17,8
18,8	14,3	17,1	19,5	16,3	20,3	17,9	23,0	17,2	15,2
15,6	17,4	21,3	22,1	20,1	14,5	19,3	18,4	16,7	18,2
16,4	18,7	14,3	18,2	19,1	15,3	21,5	17,2	22,6	20,4
22,8	17,5	20,2	15,5	21,6	18,1	20,5	14,0	18,9	16,5
20,8	16,6	18,3	21,7	17,4	23,0	21,1	19,8	15,4	18,1
18,9	14,7	19,5	20,9	15,8	20,2	21,8	18,2	21,2	20,1

1.2.

16,8	17,9	21,4	14,1	19,1	18,1	15,1	18,2	20,3	16,7
19,5	18,5	22,5	18,4	16,2	18,3	19,1	21,4	14,5	16,1
21,5	14,9	18,6	20,4	15,2	18,5	17,1	22,4	20,8	19,8
17,2	19,7	16,3	18,7	14,4	18,8	19,5	21,6	15,3	17,3
22,8	17,4	22,3	16,5	21,7	15,4	21,3	14,3	20,5	16,4
20,6	15,5	19,4	17,5	20,9	23,0	18,9	15,9	18,2	20,7
17,9	21,8	14,2	21,2	16,1	18,4	17,5	19,3	22,7	19,6
22,1	17,6	16,7	20,4	15,7	18,1	16,6	18,3	15,5	17,7
19,2	14,8	19,7	17,7	16,5	17,8	18,5	14,0	21,9	16,9
15,8	20,8	17,1	20,1	22,6	18,9	15,6	21,2	20,2	15,1

1.3.

189	207	213	208	186	210	198	219	231	227
202	211	220	236	227	220	210	183	213	190
197	227	187	226	213	191	209	196	202	235
211	214	220	195	182	228	202	207	192	226
193	203	232	202	215	195	220	233	214	185
234	215	196	220	203	236	225	221	193	215
204	184	2174	193	216	205	197	203	229	204
225	216	233	223	208	204	207	182	216	191
210	190	207	205	232	222	198	217	211	201
185	217	225	201	208	211	189	205	207	199

1.4.

9,4	7,9	0,3	6,8	4,2	11,9	7,8	1,7	5,1	8,8
8,7	11,1	7,7	1,8	5,5	10,5	4,3	3,8	1,4	11,2
1,1	7,3	3,7	4,4	11,8	8,6	1,9	5,6	10,1	8,4
10,0	11,6	5,2	2,1	5,7	4,8	7,4	0,8	4,7	3,6
8,3	7,6	0,7	7,3	3,41	11,4	5,7	9,9	2,2	7,2
2,3	4,7	9,7	11,3	5,8	4,9	3,3	0,5	7,5	4,6
5,0	0,4	8,9	7,1	9,6	11,5	5,9	9,0	5,3	2,4
9,25	5,9	1,0	9,1	2,5	6,0	8,2	3,2	10,9	6,1
10,2	2,6	4,25	3,1	6,2	11,7	6,3	0,2	7,0	9,2
1,2	6,4	11,9	6,9	8,1	6,5	2,9	6,2	4,4	10,3

1.5.

1,6	4,4	10,9	6,4	4,0	2,8	5,2	1,2	7,6	3,4
2,9	5,3	1,7	7,7	6,9	10,1	5,4	4,1	8,8	6,5
6,6	4,2	5,5	0,5	8,9	4,5	1,8	5,6	7,8	3,0
1,9	10,2	7,9	2,5	5,7	3,1	6,7	4,3	0,6	9,0
6,8	3,2	4,4	9,1	10,3	6,0	7,9	6,9	8,0	2,0
7,0	10,7	8,1	2,1	5,8	6,4	0,3	4,5	9,2	3,3
7,6	9,3	3,4	4,6	5,0	3,8	5,9	8,2	2,2	7,1

2,3	0,8	7,2	8,3	11,1	6,5	3,5	9,4	10,8	4,7
4,8	6,1	3,6	9,5	8,4	2,4	6,2	7,3	5,7	0,9
7,4	8,5	5,8	1,1	5,9	4,9	3,7	9,6	2,6	6,1

1.6.

20	26	32	34	26	28	22	30	17	24
30	28	18	22	24	26	34	28	22	20
34	24	28	20	32	17	22	24	26	30
30	22	26	35	28	24	30	32	28	18
20	30	17	24	32	28	22	26	24	30
34	26	24	28	22	30	35	32	20	17
28	22	36	30	20	26	28	23	24	32
20	26	30	24	32	17	22	28	35	26
28	35	32	22	26	24	26	24	30	24
18	24	26	28	35	30	26	22	26	28

1.7.

57	46	33	49	29	50	38	41	27	34
37	49	51	26	55	42	59	43	46	30
31	43	58	41	35	47	33	45	49	37
47	34	54	39	60	49	25	50	31	53
38	41	30	51	37	55	47	43	35	42
35	46	27	45	41	34	50	29	51	39
42	59	43	31	38	58	54	37	26	43
29	42	33	41	24	39	53	45	3,3	51
45	25	54	50	37	30	41	60	42	46
38	53	34	47	35	49	57	39	55	31

1.8.

37	49	43	31	44	38	40	31	28	43
32	44	47	29	51	25	43	38	41	32
38	24	49	40	32	34	31	28	37	46
41	35	43	25	37	46	38	24	41	50
38	29	41	32	34	49	44	37	31	47
50	34	25	37	40	32	35	28	44	43
46	37	41	35	29	43	38	31	26	34
49	32	46	26	38	35	40	51	37	46
37	25	40	34	24	44	32	28	34	38
44	34	29	47	37	49	43	35	47	50

1.9.

70	95	75	85	60	77	55	63	80	67
90	78	57	76	84	82	75	68	73	62
62	81	77	72	97	68	85	56	92	71

73	78	98	63	83	85	70	90	66	91
86	68	55	93	71	96	77	81	86	72
82	62	70	78	67	87	91	99	78	87
91	58	81	97	75	83	71	66	61	76
73	85	65	90	86	61	54	75	78	93
87	58	72	92	66	98	65	81	76	63
95	83	65	57	80	87	61	92	56	71

1.10.

57,3	75,1	78,1	69,3	60,1	77,3	66,1	69,5	72,1	68,7
81,1	69,4	63,1	67,4	77,1	82,6	64,8	72,5	62,5	80,7
77,6	65,8	78,3	57,7	80,7	64,4	82,8	67,3	83,1	70,6
75,3	58,0	60,7	81,3	67,1	69,6	82,4	62,3	66,9	80,6
62,7	73,8	68,9	83,8	57,0	72,6	65,6	78,7	59,5	70,0
73,5	58,1	64,0	83,9	84,0	63,5	74,1	77,7	68,5	80,5
66,3	73,0	79,1	71,1	80,4	62,1	66,7	83,7	76,8	59,3
71,3	63,7	71,2	78,96	65,2	77,9	74,9	69,1	70,8	74,8
71,6	72,9	61,9	71,5	75,4	71,7	59,9	74,3	76,1	70,9
61,3	71,4	71,8	65,0	67,8	75,5	71,9	64,9	74,7	62,9

1.11.

181	141	162	103	136	124	41	117	69	153
101	24	67	154	172	110	62	59	197	121
135	58	199	159	81	39	142	87	179	85
171	107	125	192	163	200	133	150	178	98
148	56	113	169	73	138	104	31	90	109
127	116	190	20	111	94	57	119	53	76
66	132	166	9	44	115	72	26	128	149
46	75	105	137	82	64	186	96	176	97
156	33	188	58	112	139	86	174	106	77
152	130	43	108	119	129	37	71	96	114

1.12.

32	105	48	80	144	128	64	112	18	81
66	129	113	17	94	78	90	51	104	34
110	149	36	103	82	53	93	130	68	150
114	84	55	131	70	38	102	77	16	135
41	19	142	61	85	159	115	57	72	101
56	100	86	146	73	40	141	25	87	126
151	71	94	15	125	76	54	99	39	140
17	124	52	98	13	37	147	88	69	109
35	158	67	30	93	123	50	138	21	97
96	121	49	137	89	145	91	65	92	33

1.13.

0,053	0,026	0,037	0,056	0,041	0,035	0,031	0,046	0,021	0,054
0,035	0,039	0,043	0,031	0,038	0,023	0,045	0,026	0,037	0,042
0,030	0,041	0,021	0,047	0,026	0,046	0,033	0,038	0,053	0,035
0,049	0,054	0,039	0,034	0,051	0,029	0,046	0,023	0,038	0,043
0,026	0,039	0,033	0,020	0,042	0,050	0,025	0,037	0,041	0,029
0,029	0,038	0,027	0,043	0,035	0,030	0,049	0,055	0,039	0,034
0,022	0,045	0,034	0,055	0,037	0,025	0,033	0,051	0,027	0,045
0,041	0,051	0,027	0,046	0,029	0,038	0,042	0,020	0,039	0,031
0,025	0,047	0,030	0,050	0,023	0,039	0,035	0,049	0,030	0,047
0,034	0,022	0,042	0,031	0,049	0,033	0,056	0,037	0,050	0,025

1.14.

0,026	0,034	0,028	0,036	0,030	0,038	0,041	0,038	0,030	0,028
0,028	0,030	0,034	0,038	0,040	0,036	0,034	0,023	0,032	0,026
0,034	0,032	0,024	0,036	0,032	0,026	0,030	0,028	0,038	0,034
0,038	0,041	0,028	0,026	0,030	0,034	0,032	0,040	0,036	0,032
0,030	0,036	0,034	0,032	0,023	0,032	0,028	0,032	0,026	0,038
0,026	0,032	0,028	0,040	0,038	0,030	0,032	0,024	0,036	0,030
0,024	0,032	0,030	0,036	0,028	0,041	0,032	0,038	0,034	0,026
0,041	0,034	0,023	0,038	0,026	0,030	0,028	0,036	0,040	0,028
0,030	0,026	0,034	0,028	0,024	0,036	0,032	0,030	0,038	0,034
0,028	0,034	0,040	0,036	0,030	0,038	0,023	0,034	0,032	0,026

1.15.

0,86	1,04	1,45	1,31	1,22	1,09	0,73	1,11	0,95	0,84
0,96	0,78	1,23	1,13	1,04	1,44	1,32	1,29	0,68	0,86
1,33	1,08	0,87	0,67	1,28	0,97	1,14	0,83	1,33	1,40
1,24	1,43	0,98	1,34	0,81	0,88	1,10	0,70	1,15	1,23
1,34	1,09	0,80	1,16	1,24	0,75	0,99	1,41	0,88	0,79
1,36	1,25	0,89	1,26	1,42	1,35	0,80	1,17	0,90	1,00
1,11	0,69	1,18	0,82	1,01	0,90	1,36	1,25	0,67	0,91
1,37	1,02	0,92	1,27	1,19	1,38	1,46	0,93	1,27	0,83
1,04	1,11	1,47	1,07	0,72	0,93	1,26	0,77	1,20	1,28
0,77	1,10	0,95	1,05	1,08	1,11	1,10	1,48	1,07	0,92

1.16.

0,76	0,82	0,70	0,86	0,78	0,96	0,68	0,83	0,92	0,86
0,86	0,84	0,66	0,92	0,76	0,95	0,84	1,91	0,78	0,70
0,78	0,70	0,82	0,99	0,83	0,86	0,67	0,91	0,75	0,86
0,83	0,75	0,95	0,79	0,65	0,84	0,78	0,88	0,70	0,95
0,87	0,71	0,92	1,00	0,75	0,87	0,80	0,79	0,66	0,90
0,79	0,82	0,65	0,83	0,88	0,96	0,75	0,91	0,71	0,87
0,76	0,90	0,71	0,87	0,74	0,94	0,80	1,00	0,95	0,79

0,96	0,98	0,84	0,79	0,91	0,71	0,65	0,90	0,88	0,74
0,74	0,67	0,94	0,72	1,01	0,82	0,80	0,83	0,99	0,83
0,88	0,80	0,72	0,91	0,84	0,74	0,94	0,72	0,83	0,87

1.17.

1,66	2,21	1,21	1,46	1,16	1,81	0,86	1,74	2,08	1,38
2,27	0,81	2,39	2,19	2,25	1,67	1,84	1,37	2,12	2,37
1,15	2,17	1,45	1,75	1,14	1,94	1,53	0,83	1,68	1,35
2,39	1,63	1,86	1,24	1,73	1,07	2,10	1,13	1,91	1,31
1,78	2,09	1,54	1,79	1,08	1,42	0,80	1,96	1,19	0,85
1,88	1,27	0,84	2,60	1,44	1,77	2,45	1,10	2,16	1,59
1,56	2,30	2,48	0,99	1,18	2,11	1,64	2,28	1,29	1,93
2,15	1,72	1,83	1,47	1,87	1,17	2,29	1,90	1,71	2,55
2,31	1,39	1,85	2,38	1,65	2,51	1,48	1,28	2,18	1,49
2,14	1,76	1,51	1,82	0,91	2,51	2,34	2,59	1,69	2,13

1.18.

2,1	2,3	1,5	3,1	2,7	1,9	2,4	0,9	2,5	1,1
1,3	2,3	2,3	3,9	2,4	3,6	1,6	3,2	2,9	2,0
2,1	3,3	0,8	3,5	1,7	2,6	4,1	2,8	1,2	2,5
1,1	2,4	1,5	3,2	2,7	1,5	3,7	1,9	3,1	4,0
4,1	2,9	2,0	2,0	1,1	0,7	3,3	2,5	1,6	2,4
2,1	3,2	0,9	2,8	4,2	2,8	1,9	1,2	1,7	3,5
2,1	3,9	2,4	1,7	3,6	2,5	0,8	3,1	2,1	1,3
3,2	1,6	0,7	2,6	1,3	2,0	3,7	2,9	4,0	3,1
2,8	4,1	1,9	3,6	3,3	2,9	0,6	1,5	1,2	2,4
1,1	3,5	1,6	2,4	3,9	2,7	2,5	1,9	2,6	3,2

1.19.

19,3	44,5	49,9	26,9	50,2	51,1	18,6	72,7	35,4	25,4
42,7	17,5	51,7	49,3	26,2	47,1	71,4	27,1	75,7	43,2
25,5	27,2	80,4	50,4	70,2	14,9	52,4	62,3	41,7	49,5
40,6	14,5	62,8	34,5	53,4	26,1	69,3	52,5	27,3	80,3
25,3	43,1	27,4	80,1	68,4	63,3	13,4	55,4	39,5	33,1
38,4	19,7	63,8	40,4	80,8	56,4	66,1	27,5	79,1	24,6
28,6	47,9	78,4	57,4	66,5	37,3	23,4	67,6	11,1	64,3
22,7	64,8	36,2	58,7	10,8	47,7	58,4	29,2	46,7	77,2
51,9	31,3	44,7	66,3	20,1	65,3	45,5	76,3	67,8	35,1
66,9	18,9	42,9	50,7	34,9	43,5	32,5	48,4	53,1	65,8

1.20.

56,5	47,3	23,1	38,6	92,5	50,9	74,9	67,5	47,5	83,9
11,8	70,1	57,1	39,9	54,7	70,9	47,4	28,1	39,1	76,2
32,3	92,1	20,7	48,6	87,1	66,3	45,8	41,4	56,9	22,6

45,8	58,4	53,4	51,4	11,6	30,9	31,4	37,4	65,8	19,3
45,3	74,4	21,2	25,7	56,7	20,3	48,3	60,1	46,2	64,1
15,1	47,7	12,7	92,6	29,5	52,0	60,2	32,1	74,5	54,2
36,1	47,2	26,1	65,3	42,0	50,1	72,1	56,4	25,1	75,1
83,8	38,7	81,2	65,1	87,4	35,3	92,4	85,6	83,5	20,5
76,3	69,4	41,6	35,9	29,7	80,9	49,9	59,5	83,4	76,5
24,4	55,9	74,2	27,3	76,7	29,9	69,1	30,1	65,4	18,4

1.21.

15,2	23,1	27,1	18,6	25,1	27,5	16,0	28,8	22,7	18,8
24,9	26,3	21,2	28,0	25,5	27,7	20,9	31,9	16,8	29,1
26,8	17,4	31,5	21,4	24,8	17,2	30,8	23,7	29,7	21,1
20,4	24,5	26,0	28,7	20,0	33,0	27,9	24,5	20,6	32,1
26,9	19,7	21,5	19,8	16,8	21,7	26,4	23,2	22,9	26,6
25,3	25,8	16,6	23,6	15,0	22,3	24,0	22,4	32,5	19,1
24,7	29,8	18,2	29,6	23,4	18,1	16,9	24,2	24,1	32,2
24,4	18,4	22,1	30,1	22,0	17,8	28,0	25,7	30,9	22,5
30,7	22,5	30,0	27,3	25,4	26,2	20,7	28,1	19,3	28,9
20,3	30,4	24,3	31,6	30,0	22,6	29,2	32,7	26,7	15,8

1.22.

19,1	23,5	19,6	27,5	33,3	31,2	27,7	21,4	27,3	20,5
21,9	20,7	15,2	27,3	23,0	31,7	18,9	23,7	33,1	27,9
23,9	18,5	24,1	28,1	22,0	16,4	30,8	27,1	19,9	30,4
20,5	30,9	31,9	26,9	19,8	28,3	22,7	15,6	22,4	18,3
28,5	16,2	22,5	18,1	28,4	33,9	30,8	19,6	26,7	32,5
21,1	24,3	26,5	15,4	24,5	26,4	28,7	17,9	30,6	23,1
32,1	23,2	17,7	28,9	22,9	20,1	30,4	26,3	16,0	25,4
26,1	15,8	30,2	19,4	25,1	25,3	17,5	24,7	21,7	29,1
21,2	21,8	17,3	33,5	29,3	24,9	30,0	15,0	25,2	25,8
33,7	24,5	25,6	23,3	29,8	17,2	25,1	22,4	29,6	19,3

1.23.

81	106	135	170	206	60	181	178	154	103
78	176	31	204	145	85	229	47	108	234
110	207	241	168	133	68	174	143	89	182
203	153	172	93	48	228	255	134	112	58
144	235	114	77	208	183	59	170	95	154
104	202	39	164	247	226	110	67	121	193
123	91	164	57	209	30	185	162	250	225
201	160	239	211	131	142	101	153	76	125
137	54	127	87	66	190	158	241	33	221
100	195	156	146	231	220	129	83	151	56

1.24.

76	28	151	91	60	204	177	102	128	217
120	66	207	126	124	152	27	221	131	51
241	77	250	134	123	147	184	195	47	160
159	74	169	178	79	129	250	223	182	96
135	199	56	25	82	116	44	229	145	203
88	209	146	224	239	103	201	245	130	163
71	165	176	194	78	154	99	78	127	69
171	173	31	181	117	84	73	161	240	149
247	107	140	53	205	155	29	132	185	179
180	128	42	114	93	191	174	210	133	226

1.25.

157,2	137,1	136,0	131,1	142,1	152,0	150,2	125,74	146,6	141,6
138,3	143,4	143,7	144,2	158,3	146,0	140,8	135,8	150,9	156,4
145,1	122,4	139,1	155,5	150,2	146,2	159,6	146,2	164,1	140,5
156,4	141,6	134,4	149,2	145,3	128,4	150,6	133,7	142,1	136,9
127,2	138,2	160,8	155,2	121,8	150,5	144,5	150,5	141,4	128,0
136,2	145,9	162,5	136,9	142,9	146,4	153,2	161,4	150,8	141,6
149,8	154,1	148,4	144,8	150,8	129,3	145,3	141,2	146,4	135,5
134,8	147,1	137,5	159,7	142,7	145,7	150,3	123,5	139,6	153,6
138,4	166,8	148,8	152,5	151,6	133,4	145,6	144,5	144,4	140,8
152,1	137,4	132,1	149,7	166,2	151,1	145,1	139,5	130,1	145,6

1.26.

2,85	5,92	3,06	2,47	6,28	3,86	2,19	5,81	3,88	3,01
3,91	3,11	1,46	4,67	3,95	5,76	3,08	3,99	6,38	1,51
2,34	4,19	5,72	4,14	3,03	4,08	6,47	4,05	5,96	4,01
4,23	2,16	6,55	3,14	4,26	4,31	1,48	4,45	2,71	5,69
6,60	4,69	2,93	7,68	0,65	6,68	3,18	5,64	4,56	3,36
2,64	3,23	6,75	4,57	5,61	3,29	7,08	2,91	4,59	2,59
4,61	1,98	6,21	3,39	4,62	2,28	4,64	3,45	5,56	4,07
3,58	4,73	3,61	2,24	4,31	3,81	5,52	4,26	4,17	7,49
1,29	4,45	4,78	5,01	7,85	5,49	2,01	4,89	0,98	4,84
2,26	5,47	4,63	4,98	5,42	4,60	5,10	4,96	4,63	5,05

1.27.

76,23	45,29	92,41	35,48	56,81	45,67	54,01	45,88	25,56	65,91
48,11	6,32	26,31	74,27	27,82	88,04	36,12	56,97	4,97	46,31
55,78	46,85	57,31	37,28	66,41	28,53	72,48	29,34	38,34	62,35
46,82	39,47	81,04	54,06	48,64	61,22	40,56	30,11	78,45	48,53
86,24	47,51	66,92	42,74	4,83	47,83	64,02	57,84	41,63	53,75
65,21	43,82	58,31	33,71	44,95	68,91	32,84	45,21	84,47	31,27
49,29	83,09	55,11	94,75	49,85	58,86	55,30	69,44	50,41	35,07
67,24	41,78	50,56	34,05	37,91	71,25	17,84	14,51	18,23	51,93

50,89	9,41	16,31	51,33	70,58	15,91	51,84	59,31	25,01	60,31
85,52	59,77	75,26	52,22	95,73	19,04	60,85	22,91	53,84	15,02

1.28.

1,58	1,95	0,89	1,76	1,54	2,18	1,13	2,59	1,91	1,60
1,19	1,70	2,58	1,31	2,54	1,90	2,20	1,49	2,69	1,51
1,77	1,93	1,48	2,21	1,64	2,92	1,25	1,97	0,90	1,78
1,12	2,48	1,38	1,79	1,75	0,67	2,22	1,62	1,82	1,09
1,61	1,71	0,95	2,23	1,46	1,99	2,24	1,72	2,03	1,25
1,28	2,04	1,83	1,69	1,81	1,22	2,05	1,07	1,74	1,88
1,80	0,69	2,07	1,29	2,27	2,75	1,41	2,08	2,30	2,18
1,34	1,84	1,73	2,31	1,86	1,40	2,46	0,73	2,33	1,85
1,02	2,13	1,66	2,84	1,16	2,34	1,44	2,89	2,09	2,90
1,87	1,43	2,11	0,84	1,91	2,44	2,10	1,75	2,60	1,68

1.29.

30,2	51,9	43,1	58,9	34,1	55,2	47,9	43,7	53,2	34,9
47,8	65,7	37,8	68,6	48,4	67,5	27,3	66,1	52,0	55,6
54,1	26,9	53,6	42,5	59,3	44,8	52,8	42,3	55,9	48,1
44,5	69,8	47,3	35,6	70,1	39,5	70,3	33,7	51,8	56,1
28,4	48,7	41,9	58,1	20,4	56,3	46,5	41,8	59,5	38,1
41,4	70,4	31,4	52,5	45,2	52,3	40,2	60,4	27,6	57,4
29,3	53,8	46,3	40,1	50,3	48,9	35,8	61,7	49,2	45,8
45,3	71,5	35,1	57,8	28,1	57,6	49,6	45,5	36,2	63,2
61,9	25,1	65,1	49,7	62,1	46,1	39,9	62,4	50,1	33,1
33,3	49,8	39,8	45,9	37,3	78,0	64,9	28,8	62,5	58,7

1.30.

88	72	100	60	116	74	36	143	114	70
56	75	30	76	89	53	117	90	135	103
35	128	71	86	43	76	61	113	34	83
62	84	50	69	120	91	102	47	119	99
33	76	91	37	85	17	85	63	121	74
46	85	63	104	77	92	54	78	42	105
85	79	49	80	93	32	106	81	64	79
73	19	80	65	107	123	51	94	80	108
52	83	124	81	96	82	109	20	95	68
66	41	82	98	111	67	125	97	112	58

Task 2. For the interval series from task №1 it is required:

a) to calculate theoretical frequencies, by assuming the null hypothesis that the general set has a normal distribution;

b) using the Pearson's criterion at the significance level α = 0,05 to compare theoretical and empirical frequencies. Calculate χ^2_{obs} and compare with $\chi^2_{\alpha,\kappa}$.

c) find confidence intervals for mathematical expectation and standard deviation with reliability $\gamma = 0{,}95$.

Solution of a standard version

Task 1. For a given sample

20	15	17	19	23	18	21	15	16	13
20	16	19	20	14	20	16	14	20	19
15	19	17	16	15	22	21	12	10	21
18	14	14	18	18	13	19	18	20	23
16	20	19	17	19	17	21	17	19	17
13	17	11	18	19					

determine:

a) variational series (sample in increasing order);

b) interval statistical series (minimum and maximum variants, range of the sample, number of intervals, empirical frequencies of intervals);

c) construct a polygon of frequencies, histogram of relative frequencies and a graph of empirical distribution function;

d) find numerical characteristics of the sample: sample mean, sample dispersion and corrected sample dispersion;

e) corrected sample standard deviation;

f) sample mode and median;

g) sample central moments of the third and the fourth order;

h) sample excess;

i) asymmetry coefficient.

Solution: let's note that calculation and making graphs is carried out in Mathcad system. A copy of Mathcad file is given below. The results obtained in Mathcad should be prepared and explained;

a) sample size *n = 55*. Data of variational series are introduced by specifying matrix X with size (55×1);

Y - sample in increasing order, *sort*(X) command:

$Y^T =$

	0	1	2	3	4	5	6	7	8	9
0	10	11	12	13	13	14	14	14	14	...

b) for construction of interval statistical series let's first define the following: the highest and the lowest variants: $a = x_{\min} = 10$, $b = x_{\max} = 23$; range of the sample: $R = b - a = 13$; the value of intervals is rounded up to whole number, and is found by Sturgess's formula:

$$h = \frac{x_{\max} - x_{\min}}{1 + \log_2 55} \approx 2 \to m = \frac{13}{2} = 6{,}781 \approx 7;$$

It is recommended to take value

$$x_{initial} = x_{\min} - \frac{h}{2} = 10 - \frac{h}{2} = 9.041 \approx 9$$

for the beginning of the first interval.
Intervals are expressed by matrix t in Mathcad; middle of intervals – by matrix x:

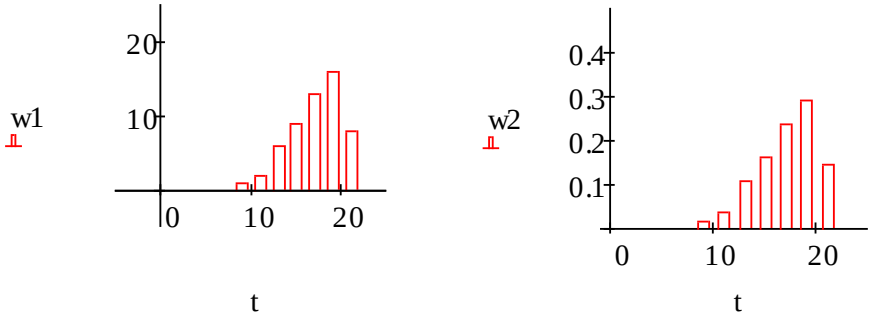
$$t^T = (9 \quad 11 \quad 13 \quad 15 \quad 17 \quad 19 \quad 21 \quad 23) \quad ;$$

calculation of empirical frequencies - n_i (the number of variants being plased in each interval) and relative frequencies (i.e. $p_i = \frac{n_i}{n}$) are obtained in Mathcad using the command w1=*hist*(*t*,*X*), (see $w1^T$ and $w2^T$).

Thus, the required interval series has the form:

intervals	[9,11)	[11,13)	[13,15)	[15,17)	[17,19)	[19,21)	[21,23]
n_i	1	2	6	9	13	16	8
$p_i = \frac{n_i}{n}$	0,018	0,036	0,109	0,164	0,236	0,291	0,145

c) using statistical interval series construct a histogram of frequencies and relative frequencies (in Mathcad):



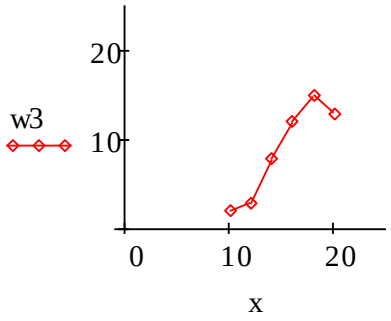
To construct an appropriate discrete statistical series we find the middles of intervals $X_i = \frac{x_i + x_{i+1}}{2}$:

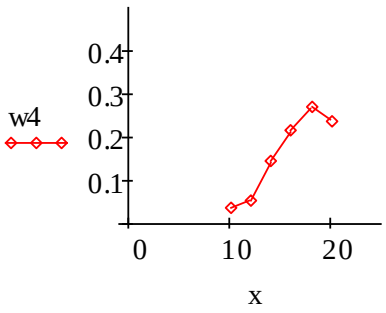
$$x^T = (10 \quad 12 \quad 14 \quad 16 \quad 18 \quad 20 \quad 22) \ ,$$

and corresponding frequencies - w1 as well as relative frequencies - w2 from interval series. The required discrete statistical series has the form:

$\frac{x_i + x_{i+1}}{2}$	10	12	14	16	18	20	22
n_i	1	2	6	9	13	16	8
p_i	0,018	0,036	0,109	0,164	0,236	0,291	0,145

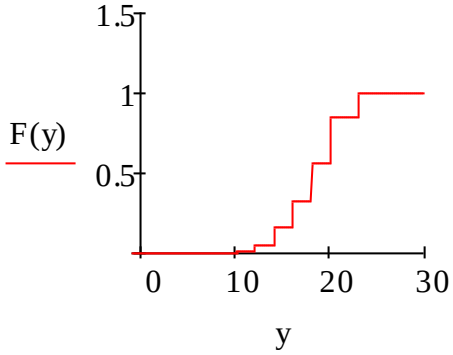
According to the given discrete statistical series we construct a polygon of frequencies and relative frequencies - commands w3=*hist*(x,X); w4=w3/n;





and also the empirical distribution function:

$$F(x) = \begin{cases} 0, \text{ if } x \le 10 \\ 0{,}018, \text{ if } 10 < x \le 12 \\ 0{,}054, \text{ if } 12 < x \le 14 \\ 0{,}163, \text{ if } 14 < x \le 16 \\ 0{,}327, \text{ if } 16 < x \le 18 \\ 0{,}563, \text{ if } 18 < x \le 20 \\ 0{,}854, \text{ if } 20 < x \le 23 \\ 1, \text{ if } x > 23 \end{cases}$$



;

d) sample mean is calculated by the formula (or by using command *mean*(X))

$$\bar{x}_в = \frac{\sum_i x_i n_i}{n} = 17.564;$$

sample dispersion (command *var*(X)):

$$D_в = \frac{\sum_i n_i \cdot x_i^2}{n} - \left(\bar{x}_в\right)^2 = 8{,}428;$$

corrected sample dispersion (*stdev*(X)):

$$S^2 = \frac{n}{n-1}\left(D_в\right) = 8{,}584;$$

e) $\sigma = \sqrt{s} = 2{,}93$ - corrected sample standard deviation;

f) mode $M_0 = 19$ defines the variant having the highest frequency, median $M_e = 18$ defines the middle of variational series and depends on the parity of the sample size (commands *mode*(X) and *median*(X));

g) sample central moments of the third and the fourth order are calculated according to formulas:

$$\mu_3 = \frac{\sum_i n_i \cdot (x_i - x_в)^3}{n} = -14{,}73; \quad \mu_4 = \frac{\sum_i n_i \cdot (x_i - x_в)^4}{n} = 196{,}112;$$

h) sample excess

$$E = \frac{\mu_4}{\sigma_в^4} - 3 = \frac{196{,}112}{2{,}93^4} - 3 = -0{,}19552;$$

i) asymmetry coefficient

$$A = \frac{\mu_3}{\sigma_в^3} = \frac{-14{,}73}{2{,}93^3} = -0{,}6091.$$

A copy of Mathcad file:

n := 55

X := (20 15 17 19 23 18 21 15 16 13 20 16 19 20 14 20 16 14 20 19 15 19 17 16 15 22 21 12 10 21 18

Y := sort(X^T) a := min(X) b := max(X) a = 10 b = 23

X^T =

	0
0	20
1	15
2	17
3	19
4	23
5	18
6	21
7	15
8	16
9	13
10	20
11	16
12	19
13	20
14	14
15	...

Y^T =

	0	1	2	3	4	5	6	7	8	9
0	10	11	12	13	13	14	14	14	14	...

R := b – a R = 13

$$h := \frac{b - a}{1 + \frac{\ln(55)}{\ln(2)}} \qquad h = 1.917 \qquad h1 := 2$$

$$m := \frac{R}{h} \qquad m = 6.781$$

$$x0 := a - \frac{h}{2} \qquad x0 = 9.041$$

$a1 := 9 \qquad m1 := 7$

$j := 0 .. m1$

$i := 0 .. m1 - 1$

$$t_j := a1 + h1 \cdot j \qquad x_i := t_i + \frac{h1}{2}$$

$x^T = (10 \quad 12 \quad 14 \quad 16 \quad 18 \quad 20 \quad 22)$

$w1 := hist(t, X) \qquad w1^T = (1 \quad 2 \quad 6 \quad 9 \quad 13 \quad 16 \quad 8)$

$$w2 := \frac{w1}{n} \qquad w2^T = (0.018 \quad 0.036 \quad 0.109 \quad 0.164 \quad 0.236 \quad 0.291 \quad 0.145)$$

$0.018 + 0.036 + 0.109 + 0.164 + 0.236 + 0.291 + 0.145 = 0.999$

$mode(X) = 19 \qquad x1 := mean(X) \qquad x1 = 17.564$

$M := median(X) \qquad M = 18$

$x2 := var(X) \qquad x2 = 8.428$

$$s := \frac{n}{n - 1} \cdot x2 \qquad s = 8.584$$

$\sigma := \sqrt{s} \qquad \sigma = 2.93$

$stdev(X) = 2.903$

Accumulated frequencies function:

$i := 0 .. 6$

$w := (0.018 \quad 0.036 \quad 0.109 \quad 0.164 \quad 0.236 \quad 0.291 \quad 0.145)^T$

$$F_i := \sum_{j=0}^{i} w_j \qquad F^T = (0.018 \quad 0.054 \quad 0.163 \quad 0.327 \quad 0.563 \quad 0.854 \quad 0.999)$$

$$F(y) := \begin{cases} 0 & \text{if } y \le 10 \\ 0.018 & \text{if } 10 < y \le 12 \\ 0.054 & \text{if } 12 < y \le 14 \\ 0.163 & \text{if } 14 < y \le 16 \\ 0.327 & \text{if } 16 < y \le 18 \\ 0.563 & \text{if } 18 < y \le 20 \\ 0.854 & \text{if } 20 < y \le 23 \\ 1 & \text{if } y > 23 \end{cases}$$

Task 2. Interval series with sample size *n=300* is given:

Number i	1	2	3	4	5	6	7
Interval	[-20;-10)	[-10;0)	[0;10)	[10;20)	[20;30)	[30;40)	[40;50]
Frequency	20	47	80	89	40	16	8

It is required:

a) to calculate theoretical frequencies, by assuming null hypothesis that the general set has a normal distribution;

b) using Pearson's criterion at significance level α = 0,05 to compare theoretical and empirical frequencies. To calculate χ^2_{obs} and compare with $\chi^2_{crit} = \chi^2_{\alpha,k}$;

c) find confidence intervals for mathematical expectation and standard deviation with reliability $\gamma = 0{,}95$.

Solution:

a) for the given sample we find the sample mean, dispersion and standard deviation:

$$\bar{x}_в = \frac{1}{300}\sum_i x_i \cdot n_i = 10{,}4;$$

$$D_в = \frac{\sum_i n_i \cdot x_i^2}{n} - \left(\bar{x}_в\right)^2 = 186{,}84; \qquad \sigma = \sqrt{D_в} = 13{,}67.$$

We calculate the theoretical frequencies taking into account *n=300, h=10,* $\sigma = 13{,}67$ by formula:

$$nt_i = \frac{nh}{\sigma}\varphi(z_i) = \frac{300 \cdot 10}{13.67}\varphi(z_i);$$

$$z_i = \frac{xx_i - x_в}{\sigma};$$

$$xx_i = \frac{x_{i+1} + x_i}{2}.$$

The values of Gauss function *φ(x)* are given in Appendix 1 [9]. The sequence of calculation of theoretical frequencies (*nt*) in Excel is given below

i - interval	xi	xi+1	xxi	zi=(xxi-xv)/sigma	φ(zi)	nt	ne	(nt-ne)^2/nt
1	-20	-10	-15	-1,86	0,07	15,58	20,00	1,26
2	-10	0	-5	-1,13	0,21	46,42	47,00	0,01
3	0	10	5	-0,40	0,37	80,99	80,00	0,01
4	10	20	15	0,34	0,38	82,74	89,00	0,47
5	20	30	25	1,07	0,23	49,49	40,00	1,82
6	30	40	35	1,80	0,08	17,34	16,00	0,10
7	40	50	45	2,53	0,02	3,56	8,00	5,55
sum						296,12	300	9,23
								χ(obs)
10,4	xv							
186,84	Dv							
13,6689429	sigma							

b) to compare empirical (*ne*) and theoretical (*nt*) frequencies using Pearson's criterion it is necessary to calculate an observed value of the criterion:

$$\chi^2_{obs} = \sum_i \frac{(ne_i - nt_i)^2}{nt_i} \quad or \quad \chi^2_{obs} = \sum_i \frac{(ne_i)^2}{nt_i} - n = 9{,}23.$$

For given level of significance α=0,05 we find the number of freedom degrees *k=m-3=7-3=4*, where *m=7* – number of sample groups, and determine critical value $\chi^2_{crit} = \chi^2_{\alpha,\kappa} = \chi^2_{0.05,4} = 9{,}5$.

Pearson's criterion. If $\chi^2_{obs} < \chi^2_{crit}$, there is no reason to reject the null hypothesis H_0 about normal distribution, i.e. theoretical and empirical frequencies differ insignificantly. If $\chi^2_{obs} > \chi^2_{cr}$, the null hypothesis H_0 is rejected, i.e. the difference of empirical and theoretical frequencies is significant.

Let us compare the obtained results:

$\chi^2_{obs} = 9{,}23 < \chi^2_{crit} = 9{,}5$ - hypothesis H_0 about normal distribution is accepted;

c) if a general set is normally distributed, we can claim with reliability $\gamma = 0{,}95$, that mathematical expectation *a* of random variable *X* is covered by the confidence interval

$$x_в - \frac{\sigma_в}{\sqrt{n}} t_\gamma < a < x_в + \frac{\sigma_в}{\sqrt{n}} t_\gamma,$$

where t_γ is determined by Laplace function (Appendix [9])

$$\Phi(t_\gamma) = \frac{\gamma}{2} = \frac{0{,}95}{2} \Rightarrow t_\gamma = 1{,}984.$$

In our case the confidence interval has the form

$$10{,}4 - \frac{13{,}67}{\sqrt{300}} 1{,}984 < a < 10{,}4 + \frac{13{,}67}{\sqrt{300}} 1{,}984 \quad \text{or} \quad 8{,}8341 < a < 11{,}9658.$$

Confidence interval covering standard deviation with reliability $\gamma = 0{,}95$ is expressed:

$$(\sigma_в(1-q) < \sigma < \sigma_в(1+q)),$$

where *q=q(γ;n)* is determined by data in Appendix 2. If $\gamma = 0{,}95$ and *n=300* , then *q= 0.079*. In our case the confidence interval for σ has the form:

$$13{,}67 \cdot (1 - 0{,}079) < \sigma < 13{,}67 \cdot (1 + 0{,}079) \quad or \quad 12{,}59 < \sigma < 14{,}75.$$

Calculation-graphical work (CGW) №4. Operational calculus

Theoretical questions.

1. Complex variable functions. Cauchy-Riemann's condition.
2. Laplace's Transformation. Original and image.
3. Main properties of originals and images.
4. Convolution of two functions. Borel's theorem. Duhamel's formulas.
5. Finding an original by a known image.
6. Applications of operational calculus.

Tasks for calculation

Task 1. It is required for a given function of complex variable $w=f(z)=f(x+iy)$:

a) to separate real $u(x, y)$ and imaginary $v(x, y)$ parts (i.e. to present the function in the form $w(x+iy)=u(x,y)+iv(x,y)$);

b) to check the Cauchy-Riemann's condition (whether function $w = f(z)$ is an analytic function). If so, find its derivative.

№	$w=f(z)$	№	$w=f(z)$
1.1	$sh5z$	1.2	$z^2\cos 2i$
1.3	e^{z-1}	1.4	z^2-3i
1.5	e^{4z^2}	1.6	$sh(3z+2)$
1.7	$\cos 4z$	1.8	$2z^2(z+2)$
1.9	e^{7z}	1.10	e^{5z-5}
1.11	$2z-z^2$	1.12	$3z(1-2i)$
1.13	$z-2i$	1.14	$2(z-2i)^2$
1.15	$ch3z$	1.16	$\sin 9z-1$
1.17	e^{iz^2}	1.18	$e^{3z}+\sin 3i$
1.19	$3z^3-3z-1$	1.20	$sh2z-5z$
1.21	$sh(3iz)$	1.22	$e^{7t}-5z$
1.23	$\cos(2z-i)$	1.24	$(z-5)^3$
1.25	$ch(z/2)$	1.26	$\cos(2z-1)$
1.27	$\sin 2z+i$	1.28	$Sin(z+1)$
1.29	$e^{5z}-i$	1.30	e^{3iz}

Task 2. Find images of functions using the property of linearity, Shift theorem and Delay theorem.

№	$f(t)$	№	$f(t)$
2.1	$sh\sqrt{3}t+e^{-2t}\sin 4t-(t-6)\cos(t-6)$	2.16	$\cos 9t-e^{4t}\sin 3t+(t-1)\sin 2(t-1)$
2.2	$\sin\sqrt{2}t-3e^{2t}sh4t+(t-4)\sin 3(t-4)$	2.17	$\sin 9t-e^{-t}sh2t-(t-7)sh(t-7)$
2.3	$t^7+e^{-t}ch5t-\cos^2(t-3)$	2.18	$cht-3e^{4t}cht+(t-5)^4$
2.4	$\cos\sqrt{5}t+3e^{-2t}t^3+\sin^2(t-3)$	2.19	$e^{6t}+4e^{2t}t^6-(t-6)ch(t-6)^7$

2.5	$sh\sqrt{2}t + e^{-3t}\sin 4t - (t-1)sh2(t-1)$	2.20	$8t^3 + e^{6t}\cos 3t + (t-5)e^{3(t-5)}$
2.6	$3 - 2e^{5t}\cos 3t + (t-8)^7$	2.21	$2t + e^{-3t}\cos t - (t-1)ch(t-1)$
2.7	$\sin 5t + 3e^{-2t}sh3t - (t-2)ch4(t-2)$	2.22	$2 + e^{2t}\sin 5t - (t-2)e^{(t-2)}$
2.8	$ch5t + e^{-t}ch4t - (t-9)e^{2(t-9)}$	2.23	$t^3 + 3e^{4t}sh3t - \cos^2(t-1)$
2.9	$\sin\sqrt{7}t + 5e^{6t}t^2 - (t-2)\cos 3(t-2)$	2.24	$2t^5 + e^{-3t}ch2t - \sin^2(t-1)$
2.10	$t + 7e^{-8t}\cos 4t + (t-8)\sin(t-8)$	2.25	$t^3 - 2e^{4t}\cos 3t + (t-1)e^{(t-1)}$
2.11	$4t^7 - 2e^{-3t}\sin\sqrt{2}t - (t-3)sh(t-3)$	2.26	$ch3t - 3e^{-t}\sin 2t + (t-2)ch(t-2)$
2.12	$2\cos\sqrt{5}t + e^{-2t}sh3t - 2(t-1)^6$	2.27	$t - e^{-4t}sh\sqrt{3}t - (t-5)^7$
2.13	$5 + 2e^{-3t}ch2t - (t-3)ch(t-3)$	2.28	$t^2 - e^{-4t}ch5t - \sin 2(t-3)$
2.14	$t^2 + e^{-3t}t^7 + (t-1)e^{2(t-1)}$	2.29	$7 - e^{4t}t^5 + (t-2)e^{3(t-2)}$
2.15	$e^{-7t} + 3e^{-4t}\cos 2t - (t-3)\cos 3(t-3)$	2.30	$t^2 - e^{2t}\cos 4t - (t-2)\cos 2(t-2)$

Task 3. Using theorems of image differentiation and original and image integration, find images of functions.

№	а) $f(t)$	б) $g(t)$	в) $h(t)$
3.1	$t\cos 3t$	$\int_0^t (\tau + \cos 3\tau)d\tau$	$\frac{1-\cos 3t}{t}$
3.2	$t\sin 2t$	$\int_0^t (\tau + \sin 2\tau)d\tau$	$\frac{\sin 3t}{t}$
3.3	$t^2\cos 2t$	$\int_0^t (\tau^2 + e^{-\tau})d\tau$	$\frac{e^{-2t} - e^{3t}}{t}$
3.4	$t^2\sin 3t$	$\int_0^t (\tau e^{3\tau})d\tau$	$\frac{\sin t\cos 3t}{t}$
3.5	te^{2t}	$\int_0^t (\tau^3 + 1)d\tau$	$\frac{\sin 2t - \cos 3t}{t}$
3.6	$(t+2)\sin 4t$	$\int_0^t (\tau\sin\tau)d\tau$	$\frac{e^t\sin 2t}{t}$
3.7	$t(2 - e^{-5t})$	$\int_0^t \cos 3\tau d\tau$	$\frac{3 - e^{-2t}}{t}$
3.8	$t(1 - \sin t)$	$\int_0^t ch2\tau d\tau$	$\frac{2}{t}(1 - \cos 2t)$
3.9	$t(3 - \cos 2t)$	$\int_0^t (\tau^3 e^{-2\tau})d\tau$	$\frac{2}{t}(1 - \sin 4t)$
3.10	$t^2 cht$	$\int_0^t \cos^2 2\tau d\tau$	$\frac{\sin 2t\cos 8t}{t}$

3.11	t^3e^t	$\int_0^t \sin^2 3\tau d\tau$	$\frac{1-e^{-3t}+t}{t}$
3.12	$t^2\cos 4t$	$\int_0^t (1-ch2\tau)d\tau$	$\frac{\sin 4t}{t}$
3.13	$t^2\sin 5t$	$\int_0^t \tau^2\cos 4\tau d\tau$	$\frac{e^{-5t}-3}{t}$
3.14	$(t+1)\cos 2t$	$\int_0^t \tau^2\sin 5\tau d\tau$	$\frac{\sin^2 2t}{t}$
3.15	t^2e^{3t}	$\int_0^t (\tau^4-3e^{-5\tau})d\tau$	$\frac{e^{-t}-3}{te^t}$
3.16	$tsht$	$\int_0^t \tau^2e^{-4\tau}d\tau$	$\frac{\sin 3t}{te^t}$
3.17	$tcht$	$\int_0^t e^{-5\tau}\sin\tau d\tau$	$\frac{\sin t-e^t}{t}$
3.18	t^2sht	$\int_0^t e^{-3\tau}\cos 3\tau d\tau$	$\frac{5-e^{-7t}}{t}$
3.19	$t^2(e^{2t}-\cos t)$	$\int_0^t (e^{-\tau}+2)\tau d\tau$	$\frac{e^{3t}\sin 4t}{t}$
3.20	$t^2(e^{-t}-\sin 5t)$	$\int_0^t \tau(1+\sin 2\tau)d\tau$	$\frac{\cos 2t-\cos 4t}{t}$
3.21	$t(5-sh2t)$	$\int_0^t (ch3\tau-\tau\sin 2\tau)d\tau$	$\frac{\sin 4t\cos t}{t}$
3.22	$t(2-ch3t)$	$\int_0^t (2-3\tau^3)d\tau$	$\frac{4-e^{2t}}{te^{6t}}$
3.23	$t^2(\cos t-1)$	$\int_0^t \tau(2-\cos 2\tau)d\tau$	$\frac{1-\cos 6t}{t}$
3.24	$t^2(2-\sin 3t)$	$\int_0^t (\tau sh3\tau-e^{6\tau})d\tau$	$\frac{e^{-2t}\sin 2t}{t}$
3.25	$t^3(e^t+2)$	$\int_0^t \tau^2(e^{2\tau}-\cos\tau)d\tau$	$\frac{e^{3t}-e^{-5t}}{t}$
3.26	$t^3\cos t$	$\int_0^t \tau^7e^{-8\tau}d\tau$	$\frac{\cos 6t-\cos 3t}{t}$
3.27	$t^3\sin t$	$\int_0^t e^{-9\tau}\tau^3 d\tau$	$\frac{e^{-3t}+2}{t}$
3.28	$t^2(t-\sin 5t)$	$\int_0^t \tau^2\sin 4\tau d\tau$	$\frac{\sin t}{te^t}$

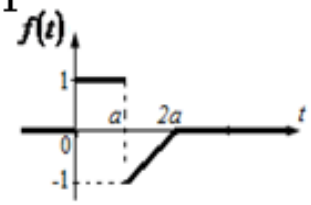
3.29	$t^2(\cos 4t + t)$	$\int_0^t \sin^2 3\tau d\tau$	$\frac{1-\cos t}{t}$
3.30	$t^2(\cos 2t - e^{2t})$	$\int_0^t \cos^2 \tau d\tau$	$\frac{1-e^{-2t}}{te^{-t}}$

Task 4. Find the convolution of originals $f(t)$ and $g(t)$ and the image of convolution.

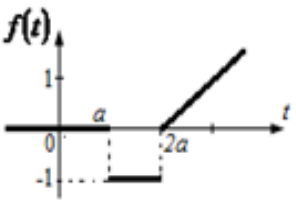
№	$f(t)$	$g(t)$	№	$f(t)$	$g(t)$
4.1	$2t-1$	$\cos t$	4.2	e^{2t}	$sh3t$
4.3	$t+5$	$ch3t$	4.4	$2t+1$	$\cos 2t$
4.5	e^{2t}	$\sin 3t$	4.6	sht	$ch2t$
4.7	$sh3t$	$\sin 2t$	4.8	e^{-t}	$sh2t$
4.9	cht	$sh3t$	4.10	e^{4t}	e^{-2t}
4.11	t	cht	4.12	$t-7$	$\sin 3t$
4.13	e^t	$ch2t$	4.14	$t+2$	$\sin t$
4.15	$\cos t$	$\sin 2t$	4.16	$\sin 2t$	$\sin t$
4.17	$\cos t$	$\cos 2t$	4.18	e^t	$\cos 2t$
4.19	e^{2t}	e^{-t}	4.20	$t-1$	e^{2t}
4.21	$\sin t$	$\cos t$	4.22	$4t-2$	$\sin 2t$
4.23	$\cos t$	$\cos t$	4.24	$t+1$	$\sin t$
4.25	e^t	e^{-t}	4.26	e^t	$\cos 2t$
4.27	$t-9$	$\cos t$	4.28	$\sin t$	$\sin t$
4.29	$ch6t$	$\sin t$	4.30	e^t	$sh2t$

Task 5. Find the image by an original graph.

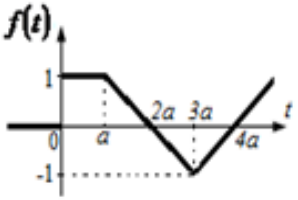
5.1



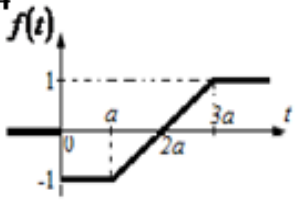
5.2



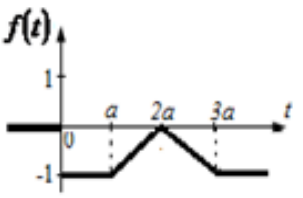
5.3



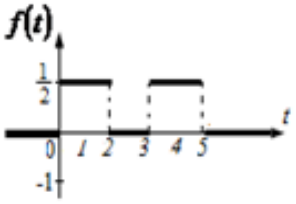
5.4



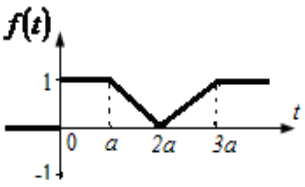
5.5



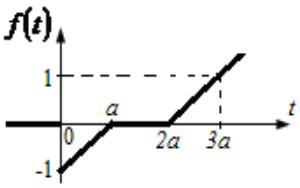
5.6



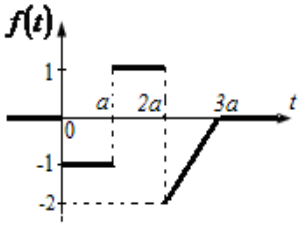
5.7



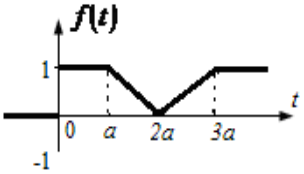
5.8



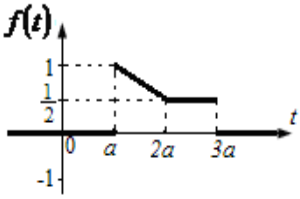
5.9



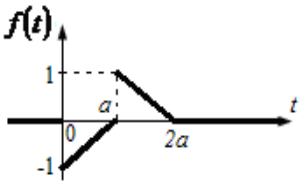
5.10



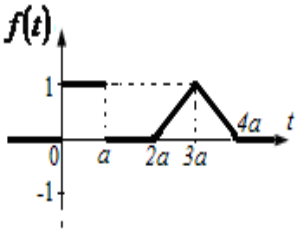
5.11



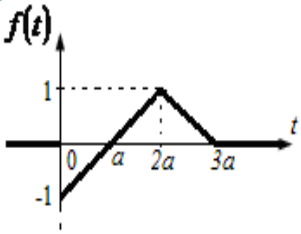
5.12



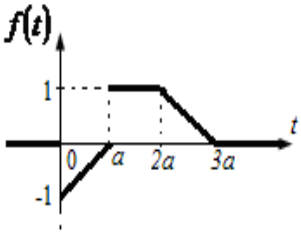
5.13



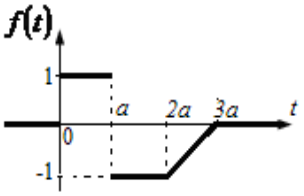
5.14



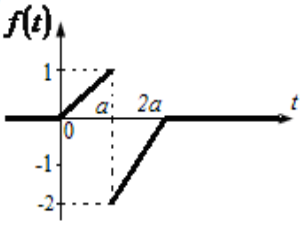
5.15



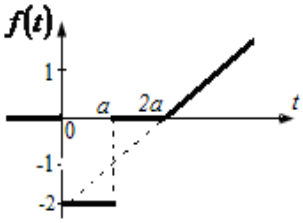
5.16



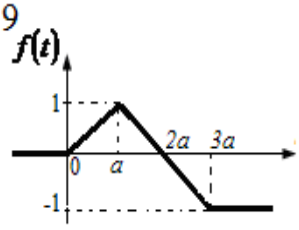
5.17



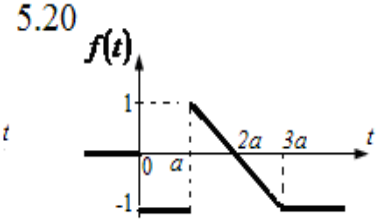
5.18



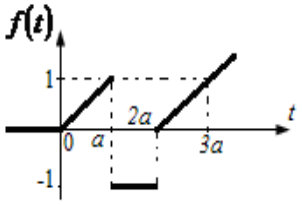
5.19



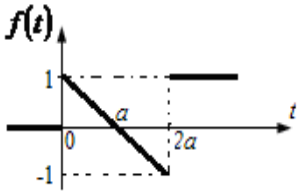
5.20



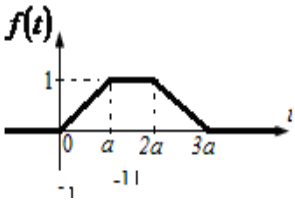
5.21



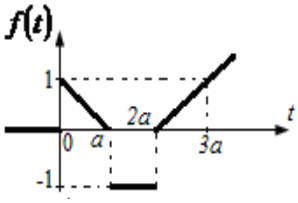
5.22



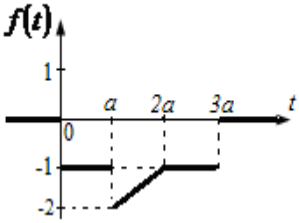
5.23



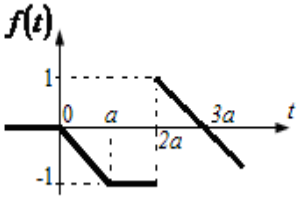
5.24



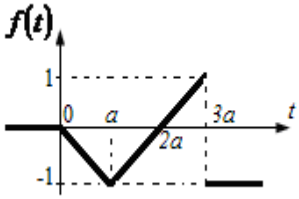
5.25



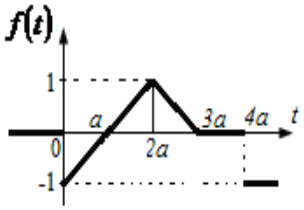
5.26



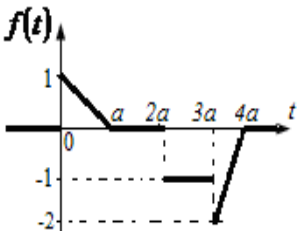
5.27



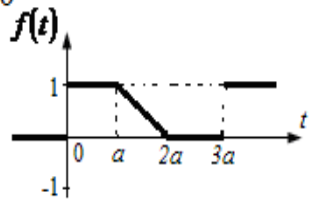
5.28



5.29



5.30



Task 6. Using Borel's theorem or Duhamel's formulas, find an original, corresponding to a given image.

№	$F(p)$	№	$F(p)$	№	$F(p)$
6.1	$\frac{p}{p^4-1}$	6.2	$\frac{p^2}{(p^2+4)(p^2+9)}$	6.3	$\frac{p}{(p^2+1)(p^2+4)}$
6.4	$\frac{1}{p^4(p-1)}$	6.5	$\frac{2}{p^2(p^2+4)}$	6.6	$\frac{p}{(p-3)(p-2)}$
6.7	$\frac{1}{p^3(p+5)}$	6.8	$\frac{p}{(p+3)(p+7)}$	6.9	$\frac{1}{p(p^2+36)}$
6.10	$\frac{1}{p(p+6)}$	6.11	$\frac{5p}{(p+1)(p^2-4)}$	6.12	$\frac{2p}{(p-3)(p-1)}$
6.13	$\frac{2}{p(p^2+3)}$	6.14	$\frac{p}{p^2(p^2+4)}$	6.15	$\frac{3p}{(p+5)(p+7)}$
6.16	$\frac{1}{p^2(p-1)}$	6.17	$\frac{2p}{(p+1)(p^2+25)}$	6.18	$\frac{1}{p(p^2+16)}$
6.19	$\frac{5p}{(p+4)(p-5)}$	6.20	$\frac{p^2}{(p^2+9)^2}$	6.21	$\frac{1}{(p+2)(p-6)}$
6.22	$\frac{2}{p^2-4}$	6.23	$\frac{8p}{(p+1)(p+2)}$	6.24	$\frac{p}{(p+1)(p^2+1)}$
6.25	$\frac{3p}{(p^2+1)(p+9)}$	6.26	$\frac{1}{p(p+2)}$	6.27	$\frac{p^2}{(p^2+4)^2}$
6.28	$\frac{2p}{(p-1)(p-2)}$	6.29	$\frac{p}{(p+3)(p+4)}$	6.30	$\frac{1}{p^2-9}$

Task 7. Using the decomposition theorem, find an original for a given image.

№	$F(p)$	№	$F(p)$
7.1	$\frac{2p+3}{(p-1)(p^2-p+1)}$	7.2	$\frac{2p}{(p^2+1)(p^2+9)}$
7.3	$\frac{2p+1}{(p^2+2)(p^2+1)}$	7.4	$\frac{3}{(p+1)(p^2+2p+2)}$
7.5	$\frac{1}{p^3+2p^2+2p}$	7.6	$\frac{4p+1}{p^2(p^2+3)}$

7.7	$\frac{3p}{p^3-8}$	7.8	$\frac{p+1}{p(p-1)(p-2)(p-3)}$
7.9	$\frac{3p-1}{(p^2+4)(p^2+9)}$	7.10	$\frac{3p}{(p+1)^2(p+3)}$
7.11	$\frac{p^2+2}{p(p+4)(p+6)}$	7.12	$\frac{p-1}{(p-2)(p^2+4)}$
7.13	$\frac{p+1}{(p^2+2)^2 p}$	7.14	$\frac{2p-1}{p(p^2+1)}$
7.15	$\frac{p-1}{(2p+1)(2p+2)}$	7.16	$\frac{p-1}{p^2(p^2-4)}$
7.17	$\frac{p+5}{(p+1)(p^2-2p+5)}$	7.18	$\frac{3p+1}{p^2(p^2+1)}$
7.19	$\frac{1-p}{p(p^2+3p+3)}$	7.20	$\frac{2-3p}{(p-1)(p^2-4p+5)}$
7.21	$\frac{p+1}{(p-3)(p^2+3p+4)}$	7.22	$\frac{2p+1}{p^2(p^2-2p+1)}$
7.23	$\frac{1}{p^3(p+2p+1)}$	7.24	$\frac{3p+1}{p^2(p^2+1)}$
7.25	$\frac{p+1}{(p-1)(p^2+4p+4)}$	7.26	$\frac{2p+1}{p(p+1)(p^2+3)}$
7.27	$\frac{p-1}{p(p+1)(p+2)}$	7.28	$\frac{3p+2}{(p+1)(p^2+3p+1)}$
7.29	$\frac{p+1}{(p-1)(p+2)^2}$	7.30	$\frac{p+3}{(p+1)^2(p+2)^2}$

Task 8. Solve linear differential

$$\alpha y'' + \beta y' + \gamma y = f(t), \quad y(t_0) = A, \quad y'(t_0) = B$$

equation by the operator method.

№	α	β	γ	$f(t)$	t_0	A	B	№	α	β	γ	$f(t)$	t_0	A	B
8.1	1	1	1	$\cos t$	0	0,5	-1	8.2	1	1	2	$5\cos t$	0	2	-3
8.3	1	1	-2	$2e^{2t}$	0	1	-2	8.4	1	1	2	$\sin t$	0	0	-4
8.5	1	-2	2	e^{-t}	0	0	0	8.6	1	0	-9	$2-t$	0	0	1
8.7	1	-6	9	$\cos t$	0	1	2	8.8	1	0	1	1	π	1	0
8.9	1	1	0	$-2t$	2	2	-2	8.10	1	1	-6	e^{-t}	0	-1	5
8.11	1	0	9	$18e^{3t}$	0	0	1	8.12	1	0	1	$-2\sin t$	$\pi/2$	0	1
8.13	1	0	-4	$4t$	0	1	0	8.14	1	6	5	$12e^{t}$	0	0	5
8.15	1	6	13	$4e^{-3t}$	0	1	0	8.16	1	0	1	$2+e^{-t}$	0	0	1
8.17	1	2	1	$2e^{1-t}$	1	1	-1	8.18	1	1	0	$2t$	1	1	-1

8.19	1	-1	0	$-2t$	2	8	6	8.20	1	-6	5	$2e^{2t}$	0	1	0
8.21	1	4	4	$9e^{t}$	0	1	1	8.22	1	2	1	$1+\cos 2t$	0	0	0
8.23	1	-2	1	$1-\sin t$	0	1	0	8.24	1	-1	0	te^{t}	0	0	-1
8.25	1	-1	0	$3t^{2}$	0	0	-1	8.26	2	8	-4	$t^{2}e^{t}$	0	1	1
8.27	2	0	8	$2\cos^{2} t$	0	-1	2	8.28	1	-2	1	$1-e^{t}$	0	1	2
8.29	3	6	-9	$2e^{-3t}$	0	3	-1	8.30	1	-4	5	$\cos 2t$	$\pi/2$	-1	1

Task 9. Solve the system of linear differential equations

$$\begin{cases} a_1 x' + b_1 y' + c_1 x + d_1 y = f_1(t), \quad x(0) = A, \\ a_2 x' + b_2 y' + c_2 x + d_2 y = f_2(t), \quad y(0) = B. \end{cases}$$

by the operator method.

№	a_1	b_1	c_1	d_1	$f_1(t)$	a_2	b_2	c_2	d_2	$f_2(t)$	A	B
9.1	1	0	0	1	0	0	1	-2	-2	1	1	1
9.2	0	1	-1	-2	1	1	0	2	1	0	0	0
9.3	1	0	-2	-4	2	0	1	1	2	0	0	0
9.4	1	0	7	-1	-1	0	1	2	5	0	1	1
9.5	1	10	0	-1	-2	2	1	0	2	1	0	0
9.6	1	0	-1	1	0	0	1	4	2	-2	0	0
9.7	1	0	0	1	4	0	1	1	0	5	1	1
9.8	1	2	1	1	1	0	1	-1	1	0	1	0
9.9	1	1	2	0	2	1	1	-1	0	4	0	0
9.10	1	1	0	-1	0	2	1	0	2	-3	0	0
9.11	3	0	2	0	0	1	4	0	3	1	0	3
9.12	11	0	2	2	0	0	1	-2	1	2	1	1
9.13	1	0	1	-3	3	0	1	-1	-1	0	1	0
9.14	1	0	1	-3	5	0	1	-1	-1	4	0	0
9.15	1	0	-1	-1	2	0	1	-3	1	2	0	1
9.16	1	0	-1	-1	-2	0	1	-3	1	0	1	1
9.17	1	0	5	2	4	0	1	-1	7	3	1	0
9.18	1	2	2	0	-4	1	1	-1	0	5	0	1
9.19	1	0	1	-3	1	0	1	-1	-1	-1	1	2
9.20	1	0	-1	-2	1	0	1	-2	-1	1	4	0
9.21	1	0	2	4	2	0	1	1	-1	1	0	0
9.22	1	0	-1	-1	3	0	1	1	-3	-1	0	1
9.23	1	0	-2	-2	0	0	1	1	0	3	1	1
9.24	1	0	0	1	1	0	1	1	0	2	-1	1
9.25	4	1	3	0	1	1	3	0	3	1	0	0
9.26	1	5	-1	0	-2	2	1	-1	0	4	0	-1
9.27	2	4	-2	-1	1	0	-2	1	0	0	1	0
9.28	3	3	-3	0	6	3	0	2	1	5	0	-2
9.29	4	2	-4	-2	4	0	-1	0	2	1	2	0
9.30	5	1	-5	3	5	4	0	-2	0	2	0	3

Solution of a standard version

Task 1. For function *w=f(z)=f(x+iy)* it is required:

a) to separate real *u (x, y)* and imaginary *v (x, y)* parts;

b) to check whether the function is analytical.

Solution:

a) using the property of sum sine we have

$$f(z) = sh(5z+1) = sh(5(x+iy)+1) = sh(5x+1+i5y) = sh(5x+1)ch5iy +$$

$$+ ch(5x+1)sh5iy = \begin{vmatrix} ch(i5y) = \cos 5y \\ sh(i5y) = i \cdot \sin 5y \end{vmatrix} = sh(5x+1)\cos 5y + i \cdot ch(5x+1)\sin 5y.$$

Thus, for $f(z) = \sin(5z+1)$ we isolate real and imaginary parts:

$$u(x,y)=sh(5x+1)cos5y, \qquad v(x,y)=ch(5x+1)sin5y;$$

b) complex variable function $f(x+iy) = u(x,y) + iv(x,y)$ is *analytic* if it satisfies *Cauchy-Riemann's condition*:

$$u'_x = v'_y; \quad v'_x = -u'_y.$$

Then derivative of the analytic function is calculated by formula

$$f'(z) = u'_x(x,y) + i \cdot v'_x(x,y).$$

We check partial derivatives:

$$u'_x = (sh(5x+1)\cos 5y)'_x = 5 \cdot ch(5x+1)cos5y;$$

$$v'_y = (ch(5x+1)\sin 5y)'_y = 5 \cdot ch(5x+1)cos5y,$$

where the Cauchy-Riemann's condition is satisfied, then we calculate the derivative of an analytic function

$$f'(z) = u'_x + iv'_x = (sh(5x+1)\cos 5y)'_x + i(ch(5x+1)\sin 5y)'_x = 5 \cdot ch(5x+1)cos5y +$$

$$+ i \cdot 5sh(5x+1)\sin 5y.$$

Task 2. Find the image of function $f(t) = 2t^7 + e^{2t}ch6t - 5(t-9)^3$, using the properties of Laplace's transformation.

Solution: using the linearity theorem, we find the image of each term separately.

$2t^7 \div \frac{2 \cdot 7!}{p^8}$ (according to table 1), $e^{2t}ch6t \div \frac{p-2}{(p-2)^2 - 36}$ (by Shift theorem),

$5(t-9)^3 \div 5e^{-9p} \cdot \frac{3}{p^4} = \frac{15e^{-9p}}{p^4}$ (by Delay theorem);

$$f(t) = 2t^7 + e^{2t}ch6t - 5(t-9)^3 \div \frac{2 \cdot 7!}{p^8} + \frac{p-2}{(p-2)^2 - 36} - \frac{15e^{-9p}}{p^4}.$$

In Mathcad (see below) it is carried out by the operation *symbolics* on the toolbar.

$2\cdot t^7 + e^{2\cdot t}\cdot\cosh(6\cdot t) - 5\cdot(t-9)^3 \text{ laplace}, t \rightarrow \frac{10080}{s^8} + \frac{s-2}{(s-2)^2-36} - \frac{30}{s^4} + \frac{270}{s^3} - \frac{1215}{s^2} + \frac{3645}{s}$

Task 3. Using theorems of image differentiation and original and image integration, find the images of functions:

a) $f(t) = t^2\left(\sin 2t + 5e^{3t} - 1\right)$;

b) $f(t) = \int_0^t \left(\tau^2 + 3\cos^2 4\tau - 7\right) d\tau$;

c) $f(t) = \dfrac{ch2t + 3e^{-9t}}{t}$.

Solution:

a) first, we find the image of function $\sin 2t + 5e^{3t} - 1$ according to the table, then we find derivative of the second order by Theorem of image differentiation:

$$\sin 2t + 5e^{3t} - 1 \div \frac{2}{p^2+4} + \frac{5}{p-3} - \frac{1}{p};$$

$$f(t) = t^2\left(\sin 2t + 5e^{3t} - 1\right) \div (-1)^2\left(\frac{2}{p^2+4} + \frac{5}{p-3} - \frac{1}{p}\right)'' = \left(\frac{-4p}{\left(p^2+4\right)^2} - \frac{5}{(p-3)^2} + \frac{1}{p^2}\right)' =$$

$$= \frac{\left(-12p^2+16\right)}{\left(p^2+4\right)^3} + \frac{10}{(p-3)^3} - \frac{2}{p^3}.$$

Calculations in Mathcad (see below).

$$\sin(2\cdot t) + 5\cdot e^{3\cdot t} - 1 \text{ laplace} \rightarrow \frac{4\cdot s^3 + 5\cdot s^2 + 10\cdot s + 12}{s\cdot\left(s^3 - 3\cdot s^2 + 4\cdot s - 12\right)} \text{ parfrac} \rightarrow \frac{2}{s^2+4} - \frac{1}{s} + \frac{5}{s-3}$$

12/

$$\frac{d^2}{ds^2}\left(\frac{2}{s^2+4} - \frac{1}{s} + \frac{5}{s-3}\right) \rightarrow \frac{16\cdot s^2}{\left(s^2+4\right)^3} - \frac{2}{s^3} - \frac{4}{\left(s^2+4\right)^2} + \frac{10}{(s-3)^3}$$

b) we find the image of original $f(t) = t^2 + 3\cos^2 4t - 7$. From the linearity property we obtain:

$$t^2 + 3\cos^2 4t - 7 = t^2 + 3\cdot\frac{1+\cos 8t}{2} - 7 \div \frac{2!}{p^3} + \frac{3}{2}\left(\frac{1}{p} + \frac{p}{p^2+64}\right) - \frac{7}{p}.$$

Then, according to the rule of original integration, we have

$$f(t) = \int_0^t \left(\tau^2 + 3\cos^2 4\tau - 7\right) d\tau \div \frac{1}{p}\left[\frac{2!}{p^3} + \frac{3}{2}\left(\frac{1}{p} + \frac{p}{p^2+64}\right) - \frac{7}{p}\right].$$

The calculations in Mathcad (see below).

$$t^2 + 3\cdot\cos(4\cdot t)^2 - 7 \text{ laplace} \rightarrow -\frac{2\cdot\left(2\cdot s^4 + 175\cdot s^2 - 64\right)}{s^3\cdot\left(s^2+64\right)} \text{ parfrac} \rightarrow \frac{3\cdot s}{2\cdot\left(s^2+64\right)} - \frac{11}{2\cdot s} + \frac{2}{s^3}$$

$$\int_0^t \left(\tau^2 + 3\cdot\cos(4\cdot\tau)^2 - 7\right) d\tau \rightarrow \frac{3\cdot\sin(8\cdot t)}{16} - \frac{11\cdot t}{2} + \frac{t^3}{3}$$

$$\frac{3\cdot\sin(8\cdot t)}{16} - \frac{11\cdot t}{2} + \frac{t^3}{3} \text{ laplace } \rightarrow -\frac{2\cdot\left(2\cdot s^4 + 175\cdot s^2 - 64\right)}{s^4\cdot\left(s^2 + 64\right)} \text{ parfrac } \rightarrow \frac{3}{2\cdot\left(s^2 + 64\right)} - \frac{11}{2\cdot s^2} + \frac{2}{s^4}$$

c) it's obvious that $ch2t + 3e^{-9t} \div \frac{p}{p^2 - 4} + \frac{3}{p+9}$. From the rule of image integration, it follows that

$$f(t) = \frac{ch2t - e^{-9t}}{t} \div \int_p^{+\infty} \left(\frac{z}{z^2 - 4} - \frac{1}{z+9}\right) dz = \lim_{\beta\to+\infty} \int_p^{\beta} \left(\frac{z}{z^2 - 4} - \frac{1}{z+9}\right) dz =$$

$$= \lim_{\beta\to+\infty} \left(\frac{1}{2}\ln(z^2 - 4) - \ln(z+9)\right)\Big|_p^{\beta} = \lim_{\beta\to+\infty} \ln\frac{\sqrt{z^2 - 4}}{z+9}\Big|_p^{\beta} =$$

$$= \lim_{\beta\to+\infty} \left(\ln\frac{\sqrt{\beta^2 - 4}}{\beta+9} - \ln\frac{\sqrt{p^2 - 4}}{p+9}\right) = -\ln\frac{\sqrt{p^2 - 4}}{p+9} = \ln\frac{p+9}{\sqrt{p^2 - 4}}.$$

The calculations in Mathcad (see below).

$$\cosh(2\cdot t) - e^{-9t} \text{ laplace}, t \rightarrow \frac{s}{s^2 - 4} - \frac{1}{s+9}$$

$$-\int \frac{s}{s^2 - 4} - \frac{1}{s+9}\, ds \rightarrow \frac{-1}{2}\cdot\ln\left(s^2 - 4\right) + \ln(s+9)$$

Task 4. Find the convolution of originals $f(t) = ch5t$ and $g(t) = \cos 3t$ and an image of convolution.

Solution: according to the definition of convolution we have:

$$ch5t * \cos 3t = \int_0^t ch5(t-\tau)\cdot\cos 3\tau d\tau = \int_0^t \frac{e^{5(t-\tau)} + e^{-5(t-\tau)}}{2}\cos 3\tau d\tau.$$

The following integral is used to simplify the calculation of the convolution (see Reference material)

$$\int e^{ax}\cos bx dx = \frac{e^{ax}(a\cdot cosbx + b\cdot\sin bx)}{a^2 + b^2}.$$

$$ch5t * \cos 3t = \int_0^t \frac{e^{5(t-\tau)} + e^{-5(t-\tau)}}{2} \cos 3\tau d\tau = \frac{1}{2} e^{5t} \int_0^t e^{-5\tau} \cos 3\tau d\tau + \frac{1}{2} e^{-5t} \int_0^t e^{5\tau} \cos 3\tau d\tau = \left| \begin{matrix} a = \mp 5 \\ b = 3 \end{matrix} \right| =$$

$$= \frac{1}{2} \frac{e^{5t}(e^{-5\tau}(-5\cos 3\tau + 3\sin 3\tau)}{(-5)^2 + 3^3} \Bigg|_0^t + \frac{1}{2} \frac{e^{-5t}(e^{5\tau}(5\cos 3\tau + 3\sin 3\tau)}{5^2 + 3^3} \Bigg|_0^t =$$

$$= \frac{1}{2}(-\frac{5}{34}\cos 3t + \frac{3}{34}\sin 3t + \frac{5}{34}e^{5t} + \frac{5}{34}\cos 3t + \frac{3}{34}\sin 3t - \frac{5}{34}e^{-5t}) = \frac{3}{34}\sin 3t + \frac{5}{34}(\frac{e^{5t} - e^{-5t}}{2}) =$$

$$= \frac{3}{34}\sin 3t + \frac{5}{34} sh5t.$$

We find the image of obtained original:

$$\frac{3}{34}\sin 3t + \frac{5}{34} sh5t \div \frac{3}{34} \cdot \frac{3}{p^2 + 9} + \frac{5}{34} \frac{5}{p^2 - 25} = \frac{1}{34}\left(\frac{9}{p^2 + 9} + \frac{25}{p^2 - 25} \right) =$$

$$= \frac{p^2}{(p^2 - 25)(p^2 + 9)}.$$

The same image can be obtained by another way. It is known that $ch5t \div \frac{p}{p^2 - 25}$, $\cos 3t \div \frac{p}{p^2 + 9}$. On the basis of Borel's theorem, we have

$$ch5t * \cos 3t \div \frac{p}{p^2 - 25} \cdot \frac{p}{p^2 + 9} = \frac{p^2}{(p^2 - 25)(p^2 + 9)}.$$

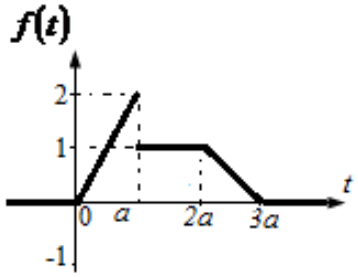
The calculations in Mathcad (see below).

$$\int_0^t \cosh[5\cdot(t - \tau)] \cdot \cos(3\cdot\tau)\, d\tau \rightarrow \frac{-3}{34}\cdot\sin(t) + \frac{6}{17}\cdot\sin(t)\cdot\cos(t)^2 + \left(\frac{5}{68}\cdot e^{10\cdot t} - \frac{5}{68}\right)\cdot e^{(-5)\cdot t}$$

$$\frac{-3}{34}\cdot\sin(t) + \frac{6}{17}\cdot\sin(t)\cdot\cos(t)^2 + \left(\frac{5}{68}\cdot e^{10\cdot t} - \frac{5}{68}\right)\cdot e^{(-5)\cdot t} \text{ laplace , t } \rightarrow \frac{9}{34\cdot(s^2 + 9)} + \frac{5}{68\cdot(s - 5)} - \frac{5}{68\cdot(s + 5)}$$

$$\frac{9}{34\cdot(s^2 + 9)} + \frac{5}{68\cdot(s - 5)} - \frac{5}{68\cdot(s + 5)} \text{ simplify } \rightarrow \frac{s^2}{(s^2 + 9)\cdot(s - 5)\cdot(s + 5)}$$

Task 5. Find an image by the original graph.



Solution: We denote points at which there is a discontinuity or a change of function by τ_k, jump of function at the discontinuity point by δ_k, the slope of k-th

section by a_k and the difference between the slopes by $\alpha_k = a_{k+1} - a_k$. The image of this type of functions can be found by formula:

$$F(p) = \sum_k \left(\frac{\delta_k}{p} + \frac{\alpha_k}{p^2} \right) \cdot e^{-\tau_k p}.$$

On the basis of the equation of a straight line passing through two points (t_1, y_1) and (t_2, y_2): $\frac{t - t_1}{t_2 - t_1} = \frac{y - y_1}{y_2 - y_1}$, we define the first sloping straight line by points $(a, 2)$ and $(0, 0)$, and the second one by points $(3a, 0)$ and $(2a, 1)$. Then, respectively, we have:

$y = \frac{2}{a} t$ и $y = -\frac{1}{a}(t - 3a)$.

Consequently, $a_1 = 0,\ a_2 = \frac{2}{a},\ a_3 = 0,\ a_4 = -\frac{1}{a},\ a_5 = 0$.

In our case: $\tau_1 = 0,\ \delta_1 = 0,\ \alpha_1 = \frac{2}{a}$; $\tau_2 = a,\ \delta_2 = 1 - 2 = -1,\ \alpha_2 = -\frac{2}{a}$;

$\tau_3 = 2a,\ \delta_3 = 0,\ \alpha_3 = -\frac{1}{a}$; $\tau_4 = 3a,\ \delta_4 = 0,\ \alpha_4 = \frac{1}{a}$.

Thus,

$$F(p) = \left(\frac{0}{p} + \frac{2}{ap^2} \right) \cdot e^0 + \left(\frac{-1}{p} - \frac{2}{ap^2} \right) \cdot e^{-ap} + \left(\frac{0}{p} - \frac{1}{ap^2} \right) \cdot e^{-2ap} + \left(\frac{0}{p} + \frac{1}{ap^2} \right) \cdot e^{-3ap} =$$

$$= \frac{2}{ap^2} - \left(\frac{1}{p} + \frac{2}{ap^2} \right) \cdot e^{-ap} - \frac{1}{ap^2} \cdot e^{-2ap} + \frac{1}{ap^2} \cdot e^{-3ap}.$$

Task 6. Using Borel's theorem or Duhamel's formula, find an original, corresponding to a given image.

a) $F(p) = \frac{12p}{(p+5)(p-3)}$;

b) $G(p) = \frac{1}{(p+5)(p-3)}$.

Solution: a) we use Duhamel's formula in solving practical problems

$$pF_1(p)F_2(p) \div f_1(t) f_2(0) + \int_0^t f_1(\tau) f_2'(t - \tau) d\tau.$$

We represent the image in the following form $F(p) = \frac{12p}{(p+5)(p-3)} = p \cdot \frac{1}{p+5} \cdot \frac{1}{p-3}$.

Since $\frac{1}{p+5} \div e^{-5t} = f_1(t)$, $\frac{1}{p-3} \div e^{3t} = f_2(t)$, then on the basis of Duhamel's theorem we obtain:

$$F(p)=\frac{12p}{(p+5)(p-3)}\div 12e^{-5t}\cdot e^{0}+12\int_0^t e^{-5\tau}\cdot e^{3(t-\tau)}d\tau=12e^{-5t}+12\int_0^t e^{-8\tau+3t}d\tau=$$

$$=12e^{-5t}-\frac{12}{8}e^{-8\tau+3t}\Big|_0^t=12e^{-5t}-\frac{12}{8}\left(e^{-5t}-e^{3t}\right)=\frac{15}{2}e^{-5t}+\frac{9}{2}e^{3t}.$$

The calculations in Mathcad (see below).

$$\frac{12\cdot s}{(s+5)\cdot(s-3)}\ \text{invlaplace},s\ \rightarrow \frac{15}{2}\cdot e^{(-5)\cdot t}+\frac{9}{2}\cdot e^{3\cdot t}$$

b) if $f(t)\div,\ g(t)\div G(p)$, then $F(p)\cdot G(p)\div f(t)*g(t)$.

Thus,

$$\frac{1}{(p-2)(p+5)}=\frac{1}{p-2}\cdot\frac{1}{p+5}\div e^{2t}*e^{-5t}=\int_0^t e^{2(t-\tau)}e^{-5\tau}d\tau=-\frac{1}{7}e^{2t-7\tau}\Big|_0^t=\frac{1}{7}e^{2t}\left(1-e^{-7t}\right).$$

The calculations in Mathcad (see below).

$$\int_0^t e^{2\cdot(t-\tau)}\cdot e^{-5\cdot\tau}\,d\tau\ \rightarrow \frac{-1}{7}\cdot e^{(-5)\cdot t}+\frac{1}{7}\cdot e^{2\cdot t}$$

$$\frac{1}{(s+5)\cdot(s-2)}\ \text{invlaplace},s\ \rightarrow \frac{-1}{7}\cdot e^{(-5)\cdot t}+\frac{1}{7}\cdot e^{2\cdot t}$$

Task 7. Using the decomposition theorem, find an original for given image $F(p)=\dfrac{5p+4}{(p-1)(p^2+p+3)}$.

Solution: function $F(p)$ is decomposed into a sum of partial fractions of the form $F(p)=\dfrac{A}{p-1}+\dfrac{Bp+C}{p^2+p+3}$.

We bring to a common denominator the fractions in the right-hand side of the last equality, then the numerator of the obtained fraction is equal to $5p+4$ and we can determine unknown coefficients: $A=\dfrac{9}{5},\ B=-\dfrac{9}{5},\ C=\dfrac{7}{5}$.

Consequently,

$$F(p)=\frac{5p+4}{(p-1)(p^2+p+3)}=\frac{\frac{9}{5}}{p-1}+\frac{-\frac{9}{5}p+\frac{7}{5}}{p^2+p+3}=\frac{1}{5}\left(\frac{9}{p-1}+\frac{-9p+7}{p^2+p+3}\right)=$$

$$=\frac{1}{5}\left(\frac{9}{p-1}+\frac{-9p+7}{\left(p+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}\right)=\frac{1}{5}\left(\frac{9}{p-1}+\frac{-9\left(p+\frac{1}{2}\right)+\frac{23}{2}}{\left(p+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}\right)=$$

$$=\frac{1}{5}\left(\frac{9}{p-1}-9\cdot\frac{p+\frac{1}{2}}{\left(p+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}+\frac{\frac{23}{2}}{\left(p+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}\right)=$$

$$=\frac{1}{5}\left(\frac{9}{p-1}-9\cdot\frac{p+\frac{1}{2}}{\left(p+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}+\frac{23}{\sqrt{11}}\cdot\frac{\frac{\sqrt{11}}{2}}{\left(p+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}\right)\div$$

$$\div\frac{1}{5}\left(9e^t-9e^{-\frac{1}{2}t}\cos\frac{\sqrt{11}}{2}t+\frac{23}{\sqrt{11}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{11}}{2}t\right).$$

Thus, the required original is

$$f(t)=\frac{1}{5}\left(9e^t-9e^{-\frac{1}{2}t}\cos\frac{\sqrt{11}}{2}t+\frac{23}{\sqrt{11}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{11}}{2}t\right).$$

In Mathcad it is carried out by the operation *symbolics* on the toolbar (see below).

$$\frac{5\cdot s+4}{(s-1)\cdot\left[\left(s^2+s\right)+3\right]}\ \text{invlaplace},s\ \rightarrow\frac{9}{5}\cdot e^t-\frac{9}{5}\cdot e^{\frac{-1}{2}\cdot t}\cdot\cos\left(\frac{1}{2}\cdot 11^{\frac{1}{2}}\cdot t\right)+\frac{23}{55}\cdot e^{\frac{-1}{2}\cdot t}\cdot 11^{\frac{1}{2}}\cdot\sin\left(\frac{1}{2}\cdot 11^{\frac{1}{2}}\cdot t\right)$$

$$\frac{5\cdot s+4}{(s-1)\cdot\left[\left(s^2+s\right)+3\right]}\ \text{convert},\text{parfrac},s\ \rightarrow\frac{9}{5\cdot(s-1)}-\frac{1}{5}\cdot\frac{(-7)+9\cdot s}{s^2+s+3}$$

$$\frac{9}{5\cdot(s-1)}-\frac{1}{5}\cdot\frac{(-7)+9\cdot s}{s^2+s+3}\ \text{invlaplace},s\ \rightarrow\frac{9}{5}\cdot e^t-\frac{9}{5}\cdot e^{\frac{-1}{2}\cdot t}\cdot\cos\left(\frac{1}{2}\cdot 11^{\frac{1}{2}}\cdot t\right)+\frac{23}{55}\cdot e^{\frac{-1}{2}\cdot t}\cdot 11^{\frac{1}{2}}\cdot\sin\left(\frac{1}{2}\cdot 11^{\frac{1}{2}}\cdot t\right)$$

Task 8. Solve the linear differential equation

$$\alpha y'' + \beta y' + \gamma y = f(t),\ \ y(t_0) = A,\ \ y'(t_0) = B,$$

by the operator method, where

$\alpha = 1,\quad \beta = -2,\quad \gamma = -1,\ \ f(t) = 3\cos 2t,\quad t_0 = \pi,\quad A = 5,\ \ B = -3.$

Solution: substituting the data into the equation, we obtain

$y'' - 2y' - y = 3\cos 2t,\ \ y(\pi) = 5,\ \ y'(\pi) = -3.$

We make an imaging equation using the operator equalities:

$$y(t) \div Y(p),\ \ y'(t) \div pY(p) - y(0) = pY(p),\ \ y''(t) = p^2Y(p) - 1,\ 3\cos 2t \div \frac{3p}{p^2+4}.$$

As a result, we obtain the operator equation for the differential equation:

$$p^2Y(p) - 1 - 2pY(p) - Y(p) = \frac{3p}{p^2+4}.$$

Hence

$$Y(p)\left(p^2 - 2p - 1\right) = \frac{3p}{p^2+4} + 1 \quad \text{or} \quad Y(p) = \frac{p^2+3p+4}{\left(p^2+4\right)\left(p^2-2p-1\right)}.$$

Having decomposed $Y(p)$ into partial fractions, we find an original by the image:

$$Y(p) = \frac{p^2+3p+4}{\left(p^2+4\right)\left(p^2-2p-1\right)} = \frac{1}{41}\left(\frac{-15p-24}{p^2+4} + \frac{15p+35}{p^2-2p-1}\right) = -\frac{15}{41}\cdot\frac{p}{p^2+4} - \frac{12}{41}\frac{2}{p^2+4} +$$

$$+\frac{15}{41}\cdot\frac{p-1}{(p-1)^2-2} + \frac{25\sqrt{2}}{41}\cdot\frac{\sqrt{2}}{(p-1)^2-2} \div -\frac{15}{41}\cos 2t - \frac{12}{41}\sin 2t + \frac{15}{41}e^t ch\sqrt{2}t + \frac{25}{41}e^t sh\sqrt{2}t.$$

Thus, required solution is

$$y(t) = -\frac{15}{41}\cos 2t - \frac{12}{41}\sin 2t + \frac{15}{41}e^t ch\sqrt{2}t + \frac{25}{41}e^t sh\sqrt{2}t\ .$$

In Mathcad it is carried out by the operation *symbolics* on the toolbar.

$$p^2\cdot Y(p) - 1 - 2\cdot p\cdot Y(p) - Y(p) - \frac{3\cdot p}{p^2+4}\ \text{solve}, Y(p) \ \rightarrow \frac{p^2+4+3\cdot p}{p^4+3\cdot p^2-2\cdot p^3-8\cdot p-4}$$

$$\frac{p^2+4+3\cdot p}{p^4+3\cdot p^2-2\cdot p^3-8\cdot p-4}\ \text{invlaplace}, p \ \rightarrow \frac{15}{41}\cdot e^t\cdot\cosh\left(2^{\frac{1}{2}}\cdot t\right) + \frac{25}{41}\cdot e^t\cdot 2^{\frac{1}{2}}\cdot\sinh\left(2^{\frac{1}{2}}\cdot t\right) - \frac{15}{41}\cdot\cos(2\cdot t) - \frac{12}{41}\cdot\sin(2\cdot t)$$

Task 9. Solve the system of linear differential equations

$$\begin{cases} a_1 x' + b_1 y' + c_1 x + d_1 y = f_1(t), & x(0) = A, \\ a_2 x' + b_2 y' + c_2 x + d_2 y = f_2(t), & y(0) = B. \end{cases}$$

by the operator method,

where $a_1=1,\ b_1=2,\ c_1=0,\ d_1=-1,\ f_1(t)=3,\ a_2=-1,\ b_2=0,\ c_2=4,\ d_2=1,$ $f_2(t)=5,\ A=1,\ B=0.$

Solution: substituting the data into the system of equations, we get

$$\begin{cases} x'+2y'-y=3, \\ -x'+4x+y=5 \end{cases}$$

for given initial conditions $x(0)=1, y(0)=0.$

We find the image:

$$x(t)\div X(p),\ x'(t)\div pX(p)-x(0)=pX(p)-1,\ 3\div\frac{3}{p},\ y(t)\div Y(p);$$

$$y'(t)\div pY(p)-y(0)=pY(p),\ 5\div\frac{5}{p}.$$

We obtain the system of linear operator equations:

$$\begin{cases} pX(p)-1+2\,pY(p)-Y(p)=\dfrac{3}{p}, \\ -(\,pX(p)-1)+4X(p)+Y(p)=\dfrac{5}{p}. \end{cases} \qquad \textbf{(*)}$$

Having solved the system with respect to $X(p)$ and $Y(p)$ by the method of determinants we have

$$\begin{cases} X(p)=\dfrac{p^2-5p+4}{p(p^2-4p+2)}, \\ Y(p)=\dfrac{2(p-3)}{p(p^2-4p+2)}. \end{cases}$$

Expanding into partial fractions, we get

$$\begin{cases} X(p)=\dfrac{2}{p}+\dfrac{-p+3}{p^2-4p+2}, \\ Y(p)=\dfrac{-3}{p}+\dfrac{3p-10}{p^2-4p+2}, \end{cases} \quad \text{or} \quad \begin{cases} X(p)=2\cdot\dfrac{1}{p}-\dfrac{p-2}{(p-2)^2-2}+\dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{(p-2)^2-2}, \\ Y(p)=-\dfrac{3}{p}+3\cdot\dfrac{p-2}{(p-2)^2-2}-\dfrac{2\sqrt{2}}{1}\cdot\dfrac{\sqrt{2}}{(p-2)^2-2}. \end{cases}$$

We carry out the inverse transformation, which is the required solution of the system of differential equations

$$\begin{cases} x(t)=2+e^{2t}ch\sqrt{2}t-\dfrac{1}{\sqrt{2}}\sin\sqrt{2}t, \\ y(t)=-3+3e^{2t}ch\sqrt{2}t+-2\sqrt{2}\sin\sqrt{2}t. \end{cases}$$

In Mathcad the system of operator equations (*) is solved immediately by the operation *symbolics* on the toolbar

$$\Delta := \begin{vmatrix} p & 2\cdot p - 1 \\ 4 - p & 1 \end{vmatrix} \qquad \Delta 1 := \begin{vmatrix} \left(\frac{3}{p} + 1\right) & 2\cdot p - 1 \\ \frac{5}{p} - 1 & 1 \end{vmatrix} \qquad \Delta 2 := \begin{vmatrix} p & \frac{3}{p} + 1 \\ 4 - p & \frac{5}{p} - 1 \end{vmatrix}$$

$$\Delta 2 \to 4\cdot\frac{p - 3}{p} \qquad \Delta 1 \to 2\cdot\frac{4 - 5\cdot p + p^2}{p} \qquad \Delta \to (-8)\cdot p + 2\cdot p^2 + 4$$

$$x(p) := \frac{\Delta 1}{\Delta}\ \text{invlaplace}, p \to 2 - e^{2\cdot t}\cdot\cosh\left(2^{\frac{1}{2}}\cdot t\right) + \frac{1}{2}\cdot e^{2\cdot t}\cdot 2^{\frac{1}{2}}\cdot\sinh\left(2^{\frac{1}{2}}\cdot t\right)$$

$$Y(p) := \frac{\Delta 2}{\Delta}\ \text{invlaplace}, p \to (-3) + 3\cdot e^{2\cdot t}\cdot\cosh\left(2^{\frac{1}{2}}\cdot t\right) - 2\cdot e^{2\cdot t}\cdot 2^{\frac{1}{2}}\cdot\sinh\left(2^{\frac{1}{2}}\cdot t\right)$$

Reference material

Properties of trigonometric functions.

1) $e^{\pm ix} = \cos x \pm i\cdot\sin x,$

$\sin ix = i\cdot shx, \quad \cos ix = chx, \quad shix = i\cdot\sin x, \quad chix = \cos x,$

2) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta,$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta,$

$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}, \quad \sin\alpha\sin\beta = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2},$

$\sin\alpha\cos\beta = \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2},$

3) $\int e^{ax}\cos bx dx = \frac{e^{ax}(a\cdot cosbx + b\cdot\sin bx)}{a^2+b^2},$

$\int e^{ax}\sin bx dx = \frac{e^{ax}(a\cdot\sin bx - b\cdot\cos bx)}{a^2+b^2}.$

Properties of Laplace's transformation.

If $f(t) \div F(p), \quad g(t) \div G(p)$, then

1) $Af(t) + Bg(t) \div AF(p) + BG(p)$ (linearity theorem);

2) $f(\lambda t) \div \frac{1}{\lambda}F\left(\frac{p}{\lambda}\right)$ (similarity theorem);

3) $e^{at}f(t) \div F(p-a)$ (shift theorem);

4) $f(t-\tau) \div e^{-p\tau}F(p)$ ($\tau > 0$ delay theorem);

5) $f'(\tau) \div pF(p) - f(0), \ f''(\tau) \div p^2F(p) - pf(0) - f'(0), \ldots,$

$f^{(n)}(\tau) \div p^n F(p) - p^{n-1} f(0) - ... - f^{(n-1)}(0)$ (theorem of original differentiation);

6) $\int_0^t f(z)dz \div \frac{F(p)}{p}$ (theorem of original integration);

7) $(-1)^n t^n f(t) \div F^{(n)}(p)$ (theorem of image differentiation);

8) $\frac{f(t)}{t} \div \int_p^\infty F(z)dz$ (theorem of image integration);

9) $f(t) * g(t) \div F(p) \cdot G(p)$, (theorem of image multiplication);

10) $f(t) * g(t) = \int_0^t f(\tau)g(t-\tau)d\tau = \int_0^t f(t-\tau)g(\tau)d\tau$ (convolution of functions);

11) $F(p) \cdot G(p) \div f'(t) * g(t) + f(0) \cdot g(t) = g'(t) * f(t) + g(0) \cdot f(t)$, (Duhamel's integral).

Table of original-image

№	$f(t)$	$F(p)$	№	$f(t)$	$F(p)$
1	1	$\frac{1}{p}$	8	$e^{at}\sin\omega t$	$\frac{\omega}{(p-a)^2-\omega^2}$
2	e^{at}	$\frac{1}{p-a}$	9	$e^{at}\cos\omega t$	$\frac{p-a}{(p-a)^2-\omega^2}$
3	t^n	$\frac{n!}{p^{n+1}}$	10	$t^n e^{at}$	$\frac{n!}{(p-a)^{n+1}}$
4	$\sin\omega t$	$\frac{\omega}{p^2+\omega^2}$	11	$t\sin\omega t$	$\frac{2\omega p}{(p^2+\omega^2)^2}$
5	$\cos\omega t$	$\frac{p}{p^2+\omega^2}$	10	$t\cos\omega t$	$\frac{p^2-\omega^2}{(p^2+\omega^2)^2}$
6	$sh\omega t$	$\frac{\omega}{p^2-\omega^2}$	11	$tsh\omega t$	$\frac{2\omega p}{(p^2-\omega^2)^2}$
7	$ch\omega t$	$\frac{p}{p^2-\omega^2}$	12	$tch\omega t$	$\frac{p^2-\omega^2}{(p^2-\omega^2)^2}$

Appendices

1. Table of values $q = q(\gamma, n)$

n \ γ	0,95	0,99	0,999	n \ γ	0,95	0,99	0,999
5	1,37	2,67	5,64	20	0,37	0,58	0,88
6	1,09	2,01	3,88	25	0,32	0,49	0,73
7	0,92	1,62	2,98	30	0,28	0,43	0,63
8	0,80	1,38	2,42	35	0,26	0,38	0,56
9	0,71	1,20	2,06	40	0,24	0,35	0,50
10	0,65	1,08	1,80	45	0,22	0,32	0,46
11	0,59	0,98	1,60	50	0,21	0,30	0,43
12	0,55	0,90	1,45	60	0,188	0,269	0,38
13	0,52	0,83	1,33	70	0,174	0,245	0,34
14	0,48	0,78	1,23	80	0,161	0,226	0,31
15	0,46	0,73	1,15	90	0,151	0,211	0,29
16	0,44	0,70	1,07	100	0,143	0,198	0,27
17	0,42	0,66	1,01	150	0,115	0,160	0,211
18	0,40	0,63	0,96	200	0,099	0,136	0,185
19	0,39	0,60	0,92	300	0,079	0,110	0,152

2. Critical points of Pearson's distribution χ^2

Number of freedom degrees k	Significance level α					
	0,01	0,025	0,05	0,95	0,975	0,89
1	6,6	5,0	3,8	0,0039	0,00098	0,00016
2	9,2	7,4	6,0	0,103	0,051	0,020
3	11,3	9,4	7,8	0,352	0,216	0,115
4	13,3	11,1	9,5	0,711	0,484	0,297
5	15,1	12,8	11,1	1,15	0,831	0,554
6	16,8	14,4	12,6	1,64	1,24	0,872
7	18,5	16,0	14,1	2,17	1,69	1,24
8	20,1	17,5	15,5	2,73	2,18	1,65
9	21,7	19,0	16,9	3,33	2,70	2,09
10	23,2	20,5	18,3	3,94	3,25	2,56
11	24,7	21,9	19,7	4,57	3,82	3,05
12	26,2	23,3	21,0	5,23	4,40	3,57
13	27,7	24,7	22,4	5,89	5,01	4,11
14	29,1	26,1	23,7	6,57	5,63	4,66
15	30,6	27,5	25,0	7,26	6,26	5,23
16	32,0	28,8	26,3	7,96	6,91	5,81
17	33,4	30,2	27,6	8,67	7,56	6,41
18	34,8	31,5	28,9	9,39	8,23	7,01
19	36,2	32,9	30,1	10,1	8,91	7,63
20	37,6	34,2	31,4	10,9	9,59	8,26
21	38,9	35,5	32,7	11,6	10,3	8,90
22	40,3	36,8	33,9	12,3	11,0	9,54
23	41,6	38,1	35,2	13,1	11,7	10,2

Bibliography

1 Khasseinov K.A. Canons of Mathematics.– Moscow: Nauka, 2007.– 592 p. – Almaty, Akbar, 2011 – 592 p., 2nd edition.

2 Хасеинов К.А. Каноны математики. Учебник.– Алматы, 2003.– 686 с.

3 Мустахишев К.М., Ералиев С.Е., Атабай Б.Ж. Математика (полный курс). –Алматы: NSN-Company, 2009. -429 с.

4 Гмурман В.Е. Теория вероятностей и математическая статистика.- Учебное пособие для вузов.- М.: ВШ, 2013.-479 с.

5 Гмурман В.Е. Руководство к решению задач по теории вероятностей и математической статистике.- М.: ВШ, 2004. -400 с.

6 Лунгу К.Н. и др. Сборник задач по высшей математике. -М.: Айрис-пресс, 2011. -576 с.

7 Сборник индивидуальных заданий по высшей математике: В 3ч. /А.П. Рябушко, В.В. Бархатов и др. /Под ред. А.П. Рябушко. –Минск: Вышэйшая школа, 2007. Ч.4. -356 с.

8 Чудесенко В.Ф. Сборник заданий по специальным курсам высшей математики. Учебники для вузов. -Изд.: Лань, 2005. -128 с.

9 Kim R.E., Masanova A.Zh. Computer solution of problems in operational calculus and probability theory. Methodological Guidelines and Tasks for carrying out calculation-graphical work for students of specialty 5B071900 – Radio engineering, electronics and telecommunications. Part 1 – 2. – Almaty: AUPET, 2016. – 38 p.

Content

Calculation-graphical work (CGW) №3. Mathematical Statistics
Theoretical questions 3
Tasks for calculation 3
Solution of a standard version 12
Calculation-graphical work (CGW) №4. Operational calculus
Theoretical questions 20
Tasks for calculation 20
Solution of a standard version 28
Reference material 37
Appendices 39
Bibliography 40

Summary plan 2016, pos.134

Kim Regina Evgenievna
Masanova Aida Zhaylauovna

COMPUTER SOLUTION OF PROBLEMS IN OPERATIONAL CALCULUS AND PROBABILITY THEORY

Methodological Guidelines and Tasks
for carrying out calculation-graphical work for students of specialty
5B071900 – Radio engineering, electronics and telecommunications
Part 3– 4

Editor V.S. Kozlov
Specialist in standardization N.K. Moldabekova

Signed for printing _____
Edition 60 copies
Volume 2,5 quires

Format 60x84 1/16
Typographical paper №1
Order_____ Price 1250 tg

Photocopying Department of
Non-Profit Joint Stock Company
«Almaty University of Power Engineering and Telecommunications»
050013, 126, Baytursynov str., Almaty