Non-Profit Joint Stock Company

ALMATY UNIVERSITY OF POWER ENGINEERING AND TELECOMMUNICATIONS

The department of Higher mathematics

SOLUTION OF DIFFERENTIAL EQUATION BY OPERATIONAL CALCULUS APPLICATION

Methodological Guidelines and Tasks
for carrying out the calculation-graphical work for students of speciality
5B071800 - Electrical power engineering

Almaty 2015

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Methodological Guidelines and Tasks for carrying out the calculation-graphical work contain exemplary calculations №1 and № 2 of discipline “Solution of differential equation by operational calculus application”. The present material corresponds to the section "Solution of differential equation by operational calculus application", provided by the curriculum for students of speciality 5B071800 - Electrical power engineering is compiled in accordance with program of discipline “Solution of differential equation by operational calculus application”.

The basic theoretical questions of the program and the solution of an exemplary embodiment are given.

Tables - 22, figures - 7, bibl.- 9.

Reviewer: candidate of sciences in philology, V.S.Kozlov

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Preface

Mathematics plays an important role in engineering studies. It is not only a quantitative calculation device, but also mathematics is the means of accurate research and extremely precise formulation of concepts and problems. Mathematical methods have become an integral part of any technical discipline. These facts lead to the need to strengthen the applied orientation of mathematics course and improve basic mathematical training.

The present Methodological Guidelines and Tasks for carrying out the calculation-graphical work contains two chapters: "Operational calculus. Solution of differential equations by operational methods. Differential equations solution by method of computer mathematics" and "1-st and 2-nd types curvilinear integrals. 1-st and 2-nd types surface integrals. Elements of the field theory." for discipline "Solution of differential equation by operational calculus application".

The content and volume of each module is defined subject to age-related psychological and pedagogical individualities of students and effective schedule of assessment of academic progress in AUPET. The sequence of presentation of teaching material corresponds to interdisciplinary connections.

The aim of study of discipline "Solution of differential equation by operational calculus application" is knowledge acquisition and mastering of basic laws, formulas, theorem and methods of mathematical investigations.

CGW is performed in a separate thin notebook. In the number of each task the second digit indicates the variant.

Theoretical questions.

Module 1. Operational calculus. Solution of differential equations by operational methods. Differential equations solution by method of computer mathematics.

1. Laplace transformation. Originals and images.
2. Basic theorems(properties): linearity, similarity, of shifting, delay.
3. Theorems of differentiation and integration of the images and originals.
4. Convolution functions. The theorem of multiplication of images. Formula Duhamel.
5. Inverse Laplace's transform. Finding original by a known image.
6. Application of the Laplace transform in the solution of differential equations and their systems.

Module 2. 1-st and 2-nd types curvilinear integrals. 1-st and 2-nd types surface integrals. Elements of the field theory.

1. Calculation of 1-st and 2-nd types curvilinear integrals.
2. Green's formula.
3. Calculation of 1-st and 2-nd types surface integrals.
4. Scalar fields: the directional derivative, gradient.
5. The vector fields: the divergence, circulation, curl, flux.
6. Ostrogradskii-Gauss and Stokes formula.
7. Solenoidal, potential and harmonic field.

Calculation-graphical work №1. Operational calculus. Solution of differential equations by operational methods. Differential equations solution by method of computer mathematics.

Calculated tasks

Task 1. Is the function original or not and why?

№	a) $f(t)$	b) $f(t)$	№	c) $f(t)$	d) $f(t)$
1.1	$(t+1)\cdot\eta(t)$	$e^{t^3}\cdot\eta(t)$	1.2	$3^t\cdot\eta(t)$	$tg2t\cdot\eta(t)$
1.3	$\frac{t+1}{t^3-1}\cdot\eta(t)$	$t\sin t\cdot\eta(t)$	1.4	$t^2\cdot\eta(t)$	$\frac{1}{t-5}\cdot\eta(t)$
1.5	$e^{4t^2}\cdot\eta(t)$	$e^{-t}\cdot\eta(t)$	1.6	$\frac{3t+1}{t-3}\cdot\eta(t)$	$\frac{1}{t-2}\cdot\eta(t)$
1.7	$2t\cdot\eta(t)$	$ctgt\cdot\eta(t)$	1.8	$2t^2\cdot\eta(t)$	$e^{2t^2}\cdot\eta(t)$
1.9	$e^{7t}\cdot\eta(t)$	$ch3t\cdot\eta(t)$	1.10	$e^{5t}\sin t\cdot\eta(t)$	$\cos t\cdot\eta(t)$
1.11	$e^{8t}\cdot\eta(t)$	$3t^5\cdot\eta(t)$	1.12	$sh5t\cdot\eta(t)$	$t\cdot\eta(t)$
1.13	$t\cdot\eta(t)$	$tgt\cdot\eta(t)$	1.14	$\cos 4t\cdot\eta(t)$	$te^{-t}\cdot\eta(t)$
1.15	$\frac{t}{t-4}\cdot\eta(t)$	$2^{3^t}\cdot\eta(t)$	1.16	$\sin 9t\cdot\eta(t)$	$\frac{1}{t^2-1}\cdot\eta(t)$
1.17	$e^{t^2}\cdot\eta(t)$	$\frac{2}{t-5}\cdot\eta(t)$	1.18	$e^{3t}\sin t\cdot\eta(t)$	$5t\cdot\eta(t)$
1.19	$t^2\cos t\cdot\eta(t)$	$tg8t\cdot\eta(t)$	1.20	$sh2t\cdot\eta(t)$	$te^{t^3}\cdot\eta(t)$
1.21	$\frac{1}{t-3}\cdot\eta(t)$	$\frac{7}{t-3}\cdot\eta(t)$	1.22	$e^{7t}\cos t\cdot\eta(t)$	$e^{-3t}\cdot\eta(t)$
1.23	$\cos t\cdot\eta(t)$	$e^{3t^2}\cdot\eta(t)$	1.24	$2^t\cdot\eta(t)$	$ctg4t\cdot\eta(t)$
1.25	$\frac{1}{t-1}\cdot\eta(t)$	$\frac{3}{t^2-4}\cdot\eta(t)$	1.26	$\frac{1}{t-9}\cdot\eta(t)$	$ch8t\cdot\eta(t)$
1.27	$\sin 2t\cdot\eta(t)$	$te^{4t}\cdot\eta(t)$	1.28	$(2t+1)\cdot\eta(t)$	$t\sin 8t\cdot\eta(t)$
1.29	$e^{5t}\cos t\cdot\eta(t)$	$tg4t\cdot\eta(t)$	1.30	$t^3\cos t\cdot\eta(t)$	$3t^4\cdot\eta(t)$

Task 2. Find the image following functions, using theorems linearity, similarity, of shifting, delay.

№	a) $f(t)$	b) $f(t)$	c) $f(t)$	d) $f(t)$
2.1	$t-3e^{2t}-\sin 3t$	$e^{-3t}\cos t$	$(t-1)ch(t-1)$	$\sin t\cdot\sin 3t$
2.2	$2+3t^3-5\cos 2t$	$e^{2t}\sin 5t$	$(t-2)e^{(t-2)}$	$\cos t\cdot\cos 2t$
2.3	$t^3+4e^{-3t}-ch\sqrt{5}t$	$3e^{4t}sh3t$	$\cos^2(t-1)$	$\sin t\cdot\cos 3t$
2.4	$3+2t^5+\cos\sqrt{2}t$	$e^{-3t}ch2t$	$\sin^2(t-1)$	$sht\cdot cht$

2.5	$t^3-\sin\sqrt{2}t+e^{4t}$	$2e^{4t}\cos 3t$	$(t-1)e^{(t-1)}$	$e^{3t}\cdot\sin^2 t$
2.6	$3+ch3t-4\sin 4t$	$3e^{-t}\sin 2t$	$(t-2)ch(t-2)$	$e^{-t}\cdot\cos^2 t$
2.7	$t-sh\sqrt{3}t-2e^{-4t}$	$e^{7t}sht$	$(t-5)^7$	$\sin t\cdot ch3t$
2.8	$t^2-e^{-2t}-\sin 2t$	$e^{-4t}ch5t$	$(t-3)\sin 2(t-3)$	$\cos t\cdot sh2t$
2.9	$7-e^{4t}+\sqrt{6}\cdot t^5$	$e^{-3t}t^7$	$(t-2)e^{3(t-2)}$	$ch2t\cdot sh3t$
2.10	$t^2-\sin 3t-e^{-5t}$	$e^{2t}\cos 4t$	$(t-2)\cos 2(t-2)$	$\sin 2t\cdot\sin 5t$
2.11	$1+4t^7-\sin 7t$	$2e^{-3t}\sin\sqrt{2}t$	$(t-3)sh(t-3)$	$\cos 2t\cdot\cos 3t$
2.12	$t^7+2\cos\sqrt{5}t+2e^{6t}$	$e^{-2t}sh3t$	$(t-1)^6$	$\sin 4t\cdot\cos 2t$
2.13	$5+2sh7t-\cos 4t$	$2e^{-3t}ch2t$	$(t-3)ch(t-3)$	$sh2t\cdot ch3t$
2.14	$t^2+3ch\sqrt{3}t+4e^{-2t}$	$e^{-3t}t^7$	$(t-1)e^{2(t-1)}$	$e^{-2t}\cdot\sin^2 t$
2.15	$6+e^{-7t}+4\sin 5t$	$3e^{-4t}\cos 2t$	$(t-3)\cos 3(t-3)$	$e^{-3t}\cdot\cos^2 t$
2.16	$\cos 9t-4e^{-t}+3$	$e^{4t}\sin 3t$	$(t-1)\sin 2(t-1)$	$\sin 2t\cdot cht$
2.17	$\sin 9t-t-\cos 3t$	$-e^{-t}sh2t$	$(t-7)sh(t-7)$	$\cos 2t\cdot sh3t$
2.18	$2-cht-3\sin 3t$	$3e^{4t}cht$	$(t-5)^4$	$\sin 4t\cdot\sin 6t$
2.19	$e^{6t}+5-4t^7$	$4e^{2t}t^6$	$(t-6)ch(t-6)$	$\cos t\cdot\cos 6t$
2.20	$8t^3+5e^{-6t}+sh3t$	$e^{6t}\cos 3t$	$(t-5)e^{3(t-5)}$	$\sin 3t\cdot\cos 5t$
2.21	$t-sh\sqrt{3}t-2e^{-4t}$	$e^{-2t}\sin 4t$	$(t-6)\cos(t-6)$	$sh3t\cdot ch4t$
2.22	$7+\sin\sqrt{2}t-2\cos\sqrt{2}t$	$3e^{2t}sh4t$	$(t-4)\sin 3(t-4)$	$e^{3t}\cdot\sin^2 t$
2.23	$t^7+2\sin\sqrt{3}t-e^{3t}$	$e^{-t}ch5t$	$\cos^2(t-3)$	$e^{4t}\cdot\cos^2 t$
2.24	$\cos\sqrt{5}t+t^2+7$	$3e^{-2t}t^3$	$\sin^2(t-3)$	$\sin 3t\cdot ch2t$
2.25	$sh\sqrt{2}t+e^{5t}-\cos 7t$	$e^{-3t}\sin 4t$	$(t-1)sh2(t-1)$	$\cos 4t\cdot sh5t$
2.26	$3-e^{-4t}+\sin 9t$	$2e^{5t}\cos 3t$	$(t-8)^7$	$\sin 8t\cdot\sin 2t$
2.27	$\sin 5t+\cos 2t-t^{11}$	$3e^{-2t}sh3t$	$(t-2)ch4(t-2)$	$\cos 4t\cdot\cos 3t$
2.28	$ch5t+e^{-3t}-5t^4$	$e^{-t}ch4t$	$(t-9)e^{2(t-9)}$	$\sin 5t\cdot\cos 2t$
2.29	$\sin\sqrt{7}t+\cos t-e^{-2t}$	$5e^{6t}t^2$	$(t-2)\cos 3(t-2)$	$sh4t\cdot ch6t$
2.30	$3-\sin 2t+3t^5$	$7e^{-8t}\cos 4t$	$(t-8)\sin(t-8)$	$ch^2 2t$

Task 3. Find the image of the function f(t), using the theorem of differentiation of the original.

№	$f(t)$	№	$f(t)$	№	$f(t)$	№	$f(t)$
3.1	$\sin^2 t$	3.2	$\cos^2 t$	3.3	$t\sin 2t$	3.4	$t\cos 2t$
3.5	$sh^2 t$	3.6	$ch^2 t$	3.7	$tsh2t$	3.8	$tch2t$
3.9	te^{2t}	3.10	$\sin^2 4t$	3.11	$\cos^2 4t$	3.12	$e^t\cos 3t$
3.13	$e^t\sin 3t$	3.14	$t\sin 4t$	3.15	$t\cos 6t$	3.16	$\sin^3 t$

3.17	$\cos^3 t$	3.18	te^{4t}	3.19	$e^{2t}\cos t$	3.20	$e^{3t}\sin 2t$
3.21	$tsh3t$	3.22	$tch4t$	3.23	$t\sin 5t$	3.24	$\sin^2 6t$
3.25	$t\cos 3t$	3.26	te^{-5t}	3.27	$\sin^2 5t$	3.28	$t\sin 6t$
3.29	$e^t ch2t$	3.30	$e^t sh2t$	3.31	$\sin^3 4t$	3.32	$t\cos 7t$

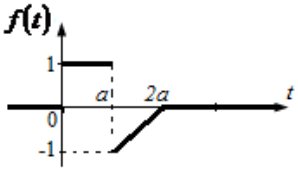
Task 4. Find the image following functions, using theorems of differentiation image and integration of the original and image .

№	a)	b)	c)
4.1	$t\cos 3t$	$\int_0^t (\tau+\cos 3\tau)d\tau$	$\frac{1-\cos 3t}{t}$
4.2	$t\sin 2t$	$\int_0^t (\tau+\sin 2\tau)d\tau$	$\frac{\sin 3t}{t}$
4.3	$t^2\cos 2t$	$\int_0^t (\tau^2+e^{-\tau})d\tau$	$\frac{e^{-2t}-e^{3t}}{t}$
4.4	$t^2\sin 3t$	$\int_0^t (\tau e^{3\tau})d\tau$	$\frac{\sin t\cos 3t}{t}$
4.5	te^{2t}	$\int_0^t (\tau^3+1)d\tau$	$\frac{\sin 2t-\cos 3t}{t}$
4.6	$(t+2)\sin 4t$	$\int_0^t (\tau\sin\tau)d\tau$	$\frac{e^t\sin 2t}{t}$
4.7	$t(2-e^{-5t})$	$\int_0^t \cos 3\tau d\tau$	$\frac{3-e^{-2t}}{t}$
4.8	$t(1-\sin t)$	$\int_0^t ch2\tau d\tau$	$\frac{2}{t}(1-\cos 2t)$
4.9	$t(3-\cos 2t)$	$\int_0^t (\tau^3 e^{-2\tau})d\tau$	$\frac{2}{t}(1-\sin 4t)$
4.10	$t^2 cht$	$\int_0^t \cos^2 2\tau d\tau$	$\frac{\sin 2t\cos 8t}{t}$
4.11	$t^3 e^t$	$\int_0^t \sin^2 3\tau d\tau$	$\frac{1-e^{-3t}+t}{t}$
4.12	$t^2\cos 4t$	$\int_0^t (1-ch2\tau)d\tau$	$\frac{\sin 4t}{t}$
4.13	$t^2\sin 5t$	$\int_0^t \tau^2\cos 4\tau d\tau$	$\frac{e^{-5t}-3}{t}$

4.14	$(t+1)\cos 2t$	$\int_0^t \tau^2 \sin 5\tau d\tau$	$\frac{\sin^2 2t}{t}$
4.15	$t^2 e^{3t}$	$\int_0^t \left(\tau^4 - 3e^{-5\tau}\right) d\tau$	$\frac{e^{-t} - 3}{te^t}$
4.16	$tsht$	$\int_0^t \tau^2 e^{-4\tau} d\tau$	$\frac{\sin 3t}{te^t}$
4.17	$tcht$	$\int_0^t e^{-5\tau} \sin \tau d\tau$	$\frac{\sin t - e^t}{t}$
4.18	$t^2 sht$	$\int_0^t e^{-3\tau} \cos 3\tau d\tau$	$\frac{5 - e^{-7t}}{t}$
4.19	$t^2\left(e^{2t} - \cos t\right)$	$\int_0^t \left(e^{-\tau} + 2\right)\tau d\tau$	$\frac{e^{3t} \sin 4t}{t}$
4.20	$t^2\left(e^{-t} - \sin 5t\right)$	$\int_0^t \tau(1 + \sin 2\tau) d\tau$	$\frac{\cos 2t - \cos 4t}{t}$
4.21	$t(5 - sh2t)$	$\int_0^t (ch3\tau - \tau \sin 2\tau) d\tau$	$\frac{\sin 4t \cos t}{t}$
4.22	$t(2 - ch3t)$	$\int_0^t \left(2 - 3\tau^3\right) d\tau$	$\frac{4 - e^{2t}}{te^{6t}}$
4.23	$t^2(\cos t - 1)$	$\int_0^t \tau(2 - \cos 2\tau) d\tau$	$\frac{1 - \cos 6t}{t}$
4.24	$t^2(2 - \sin 3t)$	$\int_0^t \left(\tau sh3\tau - e^{6\tau}\right) d\tau$	$\frac{e^{-2t} \sin 2t}{t}$
4.25	$t^3\left(e^t + 2\right)$	$\int_0^t \tau^2\left(e^{2\tau} - \cos \tau\right) d\tau$	$\frac{e^{3t} - e^{-5t}}{t}$
4.26	$t^3 \cos t$	$\int_0^t \tau^7 e^{-8\tau} d\tau$	$\frac{\cos 6t - \cos 3t}{t}$
4.27	$t^3 \sin t$	$\int_0^t e^{-9\tau} \tau^3 d\tau$	$\frac{e^{-3t} + 2}{t}$
4.28	$t^2(t - \sin 5t)$	$\int_0^t \tau^2 \sin 4\tau d\tau$	$\frac{\sin t}{te^t}$
4.29	$t^2(\cos 4t + t)$	$\int_0^t \sin^2 3\tau d\tau$	$\frac{1 - \cos t}{t}$
4.30	$t^2\left(\cos 2t - e^{2t}\right)$	$\int_0^t \cos^2 \tau d\tau$	$\frac{1 - e^{-2t}}{te^{-t}}$

Task 5. Find the image, using the original graph.

5.1



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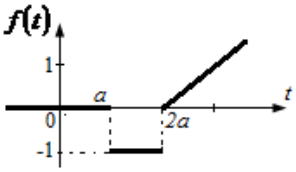

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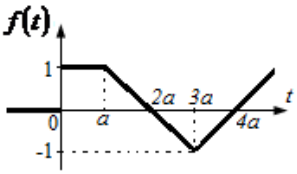

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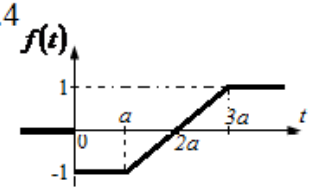

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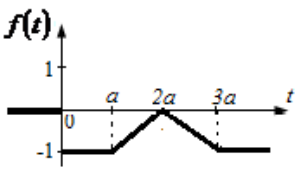

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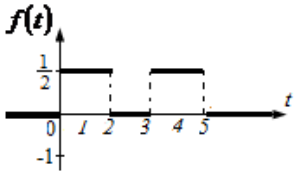

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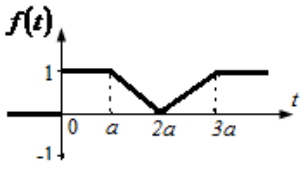

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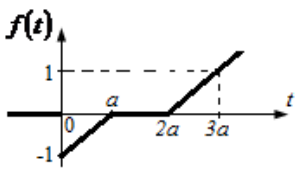

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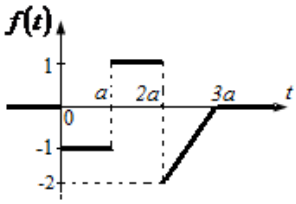

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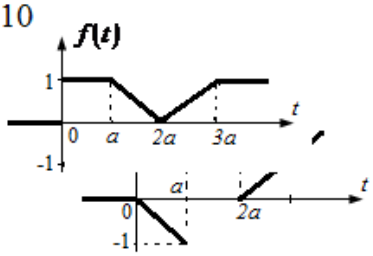

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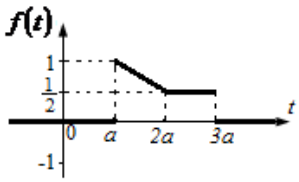

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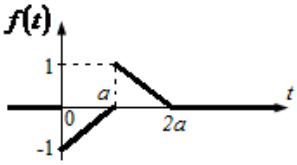

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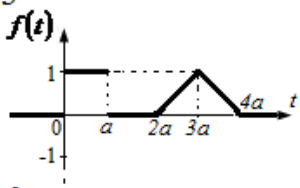

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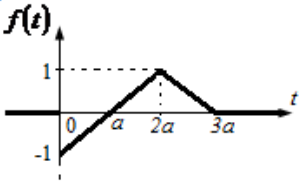

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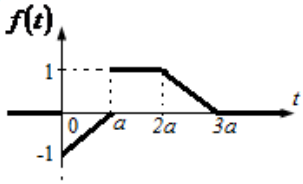

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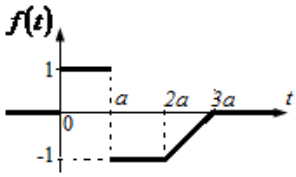

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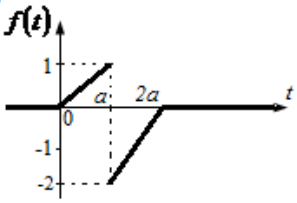

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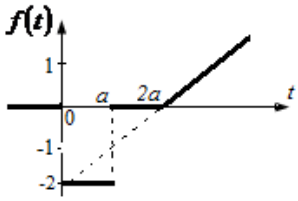

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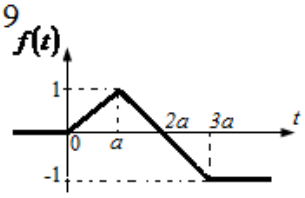

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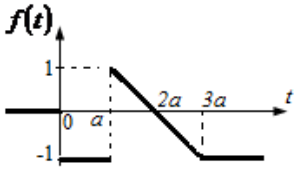

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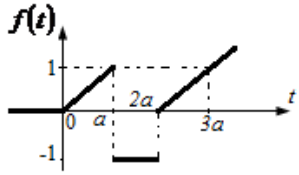

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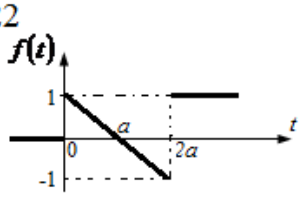

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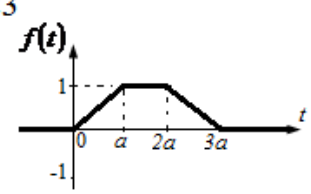

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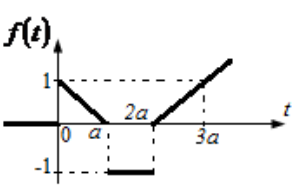

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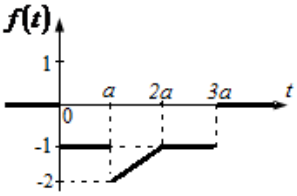

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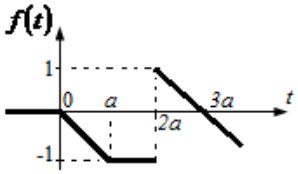

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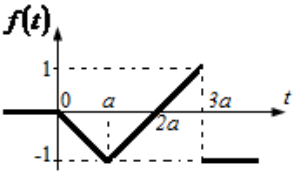

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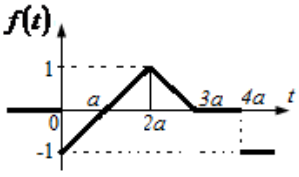

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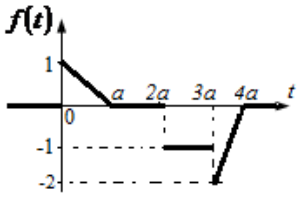

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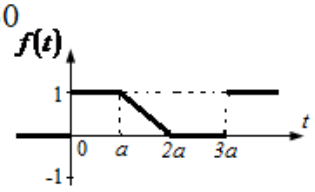

Task 6. Find the convolution functions $f(t)$ and $g(t)$ and its image.

№	$f(t)$	$g(t)$	№	$f(t)$	$g(t)$
6.1	$\sin t$	$\cos t$	6.2	t	$\sin 2t$
6.3	$\cos t$	$\cos t$	6.4	$t+1$	$\sin t$
6.5	e^{t}	e^{-t}	6.6	e^{t}	$\cos 2t$
6.7	t	$\cos t$	6.8	$\sin t$	$\sin t$
6.9	t	$\sin t$	6.10	e^{t}	$sh2t$
6.11	t	cht	6.12	t	$\sin 3t$
6.13	e^{t}	$ch2t$	6.14	$t+2$	$\sin t$
6.15	$\cos t$	$\sin 2t$	6.16	$\sin 2t$	$\sin t$
6.17	$\cos t$	$\cos 2t$	6.18	e^{t}	$\cos 2t$
6.19	e^{2t}	e^{-t}	6.20	$t-1$	e^{2t}
6.21	$t-1$	$\cos t$	6.22	e^{2t}	$sh3t$
6.23	$t-2$	$ch3t$	6.24	1	$\cos 2t$
6.25	e^{2t}	$\sin 3t$	6.26	sht	$ch2t$
6.27	1	$\sin 2t$	6.28	e^{-t}	$sh2t$
6.29	cht	$sh3t$	6.30	e^{4t}	e^{-2t}

Task 7. Find the original $f(t)$ function $F(p)$, applying the theorem of multiplication of images.

№	$F(p)$	№	$F(p)$	№	$F(p)$
7.1	$\frac{1}{(p+1)(p-2)}$	7.2	$\frac{2}{p^2(p^2+1)}$	7.3	$\frac{1}{p(p^2+3)}$
7.4	$\frac{2}{p^2-4}$	7.5	$\frac{1}{p(p+2)}$	7.6	$\frac{p}{(p+1)(p^2+1)}$

7.7	$\frac{3p}{(p^2+1)(p+9)}$	7.8	$\frac{1}{(p+3)(p+4)}$	7.9	$\frac{p^2}{(p^2+4)^2}$
7.10	$\frac{2}{(p-1)(p-2)}$	7.11	$\frac{1}{(p+1)(p^2-4)}$	7.12	$\frac{1}{p^2-9}$
7.13	$\frac{2}{p(p^2+3)}$	7.14	$\frac{p}{p^2(p^2+4)}$	7.15	$\frac{1}{(p+5)(p+7)}$
7.16	$\frac{1}{p^2(p-1)}$	7.17	$\frac{2p}{(p+1)(p^2+25)}$	7.18	$\frac{1}{p(p^2+16)}$
7.19	$\frac{5}{(p+4)(p-5)}$	7.20	$\frac{p^2}{(p^2+9)^2}$	7.21	$\frac{1}{(p+2)(p-6)}$
7.22	$\frac{1}{p^4(p-1)}$	7.23	$\frac{1}{(p+1)(p+2)}$	7.24	$\frac{1}{(p-3)(p-2)}$
7.25	$\frac{1}{p^3(p+5)}$	7.26	$\frac{2}{p^2(p^2+4)}$	7.27	$\frac{1}{p(p^2+36)}$
7.28	$\frac{1}{p(p+6)}$	7.29	$\frac{1}{(p+3)(p+7)}$	7.30	$\frac{1}{(p-3)(p-1)}$

Task 8. Find the original for a given image.

№	$F(p)$	№	$F(p)$
8.1	$\frac{p+1}{(p-3)(p^2+3p+4)}$	8.2	$\frac{2p+1}{p^2(p^2-2p+1)}$
8.3	$\frac{1}{p^3(p+2p+1)}$	8.4	$\frac{3p+1}{p^2(p^2+1)}$
8.5	$\frac{p+1}{(p-1)(p^2+4p+4)}$	8.6	$\frac{2p+1}{p(p+1)(p^2+3)}$
8.7	$\frac{p-1}{p(p+1)(p+2)}$	8.8	$\frac{3p+2}{(p+1)(p^2+3p+1)}$
8.9	$\frac{p+1}{(p-1)(p+2)^2}$	8.10	$\frac{p+3}{(p+1)^2(p+2)^2}$
8.11	$\frac{p^2+2}{p(p+4)(p+6)}$	8.12	$\frac{p-1}{(p-2)(p^2+4)}$
8.13	$\frac{p+1}{(p^2+2)^2 p}$	8.14	$\frac{2p-1}{p(p^2+1)}$
8.15	$\frac{p-1}{(2p+1)(2p+2)}$	8.16	$\frac{p-1}{p^2(p^2-4)}$

8.17	$\dfrac{p+5}{(p+1)(p^2-2p+5)}$	8.18	$\dfrac{3p+1}{p^2(p^2+1)}$
8.19	$\dfrac{1-p}{p(p^2+3p+3)}$	8.20	$\dfrac{2-3p}{(p-1)(p^2-4p+5)}$
8.21	$\dfrac{2p+3}{(p-1)(p^2-p+1)}$	8.22	$\dfrac{2p}{(p^2+1)(p^2+9)}$
8.23	$\dfrac{2p+1}{(p^2+2)(p^2+1)}$	8.24	$\dfrac{3}{(p+1)(p^2+2p+2)}$
8.25	$\dfrac{1}{p^3+2p^2+2p}$	8.26	$\dfrac{4p+1}{p^2(p^2+3)}$
8.27	$\dfrac{3p}{p^3-8}$	8.28	$\dfrac{p+1}{p(p-1)(p-2)(p-3)}$
8.29	$\dfrac{3p-1}{(p^2+4)(p^2+9)}$	8.30	$\dfrac{3p}{(p+1)^2(p+3)}$

Task 9. Solve the Cauchy problem by the method of operational calculus.

9.1	$y''+y=2\cos t,\ y(0)=0,\ y'(0)=-1$	9.2	$y''-2y'+5y=1-t,\ y(0)=y'(0)=0$
9.3	$y''-y'=te^t,\ y(0)=y'(0)=0$	9.4	$y''+2y'+y=t^2,\ y(0)=1,\ y'(0)=0$
9.5	$y''+4y=t,\ y(0)=1,\ y'(0)=0$	9.6	$y''+3y'=20e^{2t},\ y(0)=1,\ y'(0)=7$
9.7	$y''+y'=sht,\ y(0)=2,\ y'(0)=1$	9.8	$y''+2y'=2+e^t,\ y(0)=1,\ y'(0)=2$
9.9	$y''-2y'-y=e^{3t},\ y(0)=y'(0)=0$	9.10	$y''+2y'=t\sin t,\ y(0)=y'(0)=0$
9.11	$y''+y'=\cos t,\ y(0)=2,\ y'(0)=0$	9.12	$y''+y=2\sin t,\ y(0)=1,\ y'(0)=-1$
9.13	$y''+y=e^t,\ y(0)=0,\ y'(0)=2$	9.14	$y''+y'+y=t^2+t,\ y(0)=1,\ y'(0)=-3$
9.15	$y''+6y'+9y=9e^{3t},\ y(0)=y'(0)=0$	9.16	$y''-y'+y=e^{-t},\ y(0)=0,\ y'(0)=1$
9.17	$y''+y=\sin t+\cos 2t, y(0)=y'(0)=1$	9.18	$y''+y=0,\ y(0)=0,\ y'(0)=-1$
9.19	$y''-2y'+2y=1,\ y(0)=y'(0)=0$	9.20	$y''-2y'+y=t-\sin t,\ y(0)=y'(0)=0$
9.21	$y''-9y=2-t,\ y(0)=0,\ y'(0)=1$	9.22	$y''+5y'+4y=e^{-t},\ y(0)=0,\ y'(0)=2$
9.23	$2y''-y=\sin 3t,\ y(0)=-2,\ y'(0)=1$	9.24	$y''+4y'+4y=t^3e^{2t}, y(0)=1, y'(0)=2$
9.25	$y''-2y'-3y=2t,\ y(0)=y'(0)=1$	9.26	$y''+y=6e^{-t},\ y(0)=3,\ y'(0)=1$
9.27	$y''-y'-6y=e^{-t},$ $y(0)=1,\ y'(0)=0$	9.28	$y''+4y=(8t+4)e^{2t},$ $y(0)=4, y'(0)=-1$
9.29	$y''-2y'=e^t(t^2+t-3),$ $y(0)=y'(0)=2$	9.30	$y''-y=4\sin t+5\cos 2t,$ $y(0)=-1, y'(0)=-2$

Task 10. Find the solution of the Cauchy problem using the formula Duhamel.

10.1	$x''+x=tgt,\ x(0)=1,\ x'(0)=2$	10.2	$x''=\frac{2}{1+t^2},\ x(0)=x'(0)=0$
10.3	$x''=t\ln^2 t,\ x(0)=x'(0)=0$	10.4	$x''-x=\frac{2}{1+\cos^2 t},\ x(0)=x'(0)=0$
10.5	$x''+x=\frac{1}{2+\cos t},\ x(0)=x'(0)=0$	10.6	$x''-2x=tht,\ x(0)=x'(0)=0$
10.7	$x''=arctg2t,\ x(0)=x'(0)=0$	10.8	$x''+x=\frac{1}{1+tg^2 t},\ x(0)=x'(0)=0$
10.9	$x''+x'=\frac{1}{1+e^t},\ x(0)=x'(0)=0$	10.10	$x''-x=\frac{1}{1+tg^2 t},\ x(0)=x'(0)=0$
10.11	$x''+x=tht,\ x(0)=x'(0)=0$	10.12	$x''=3t\ln^2 t,\ x(0)=x'(0)=0$
10.13	$x''+x=\frac{1}{1+\cos^2 t},\ x(0)=x'(0)=0$	10.14	$x''+x=\frac{1}{1+\sin^2 t},\ x(0)=x'(0)=0$
10.15	$x''=3t\ln^2 t,\ x(0)=x'(0)=0$	10.16	$x''=\frac{1}{1+t},\ x(0)=x'(0)=0$
10.17	$x''-x=\frac{2}{1+\sin t},\ x(0)=x'(0)=0$	10.18	$x''-x=\frac{1}{1+\cos t},\ x(0)=x'(0)=0$
10.19	$x''=arctg3t,\ x(0)=x'(0)=0$	10.20	$x''=\frac{3}{1+t^2},\ x(0)=x'(0)=0$
10.21	$x''-x'=\frac{2}{1+e^t},\ x(0)=x'(0)=0$	10.22	$x''-x=tgt,\ x(0)=x'(0)=0$
10.23	$x''-x=\frac{1}{1+\sin^2 t},\ x(0)=x'(0)=0$	10.24	$x''-x=\frac{1}{1+\cos 2t},\ x(0)=x'(0)=0$
10.25	$x''-x=\frac{4}{1+tg^2 t},\ x(0)=x'(0)=0$	10.26	$x''=2t\ln^2 t,\ x(0)=x'(0)=0$
10.27	$x''-x=\frac{1}{1+\cos 2t},\ x(0)=x'(0)=0$	10.28	$x''-x=\frac{1}{1+\sin t},\ x(0)=x'(0)=0$
10.29	$x''+x=\frac{2}{2+\cos^2 t},\ x(0)=x'(0)=0$	10.30	$x''+x=\frac{3}{1+\sin^2 t},\ x(0)=x'(0)=0$

Task 11. Solve the system of differential equations by operational method.

11.1	$\begin{cases}\dot{x}=-2x+y+2,\ x(0)=1\\ \dot{y}=3x,\ y(0)=0\end{cases}$	11.2	$\begin{cases}\dot{x}=y-1,\ x(0)=1\\ \dot{y}=-x-2y,\ y(0)=-1\end{cases}$
11.3	$\begin{cases}\dot{x}=3x+4y,\ x(0)=1\\ \dot{y}=4x-3y,\ y(0)=1\end{cases}$	11.4	$\begin{cases}\dot{x}=y+3,\ x(0)=1\\ \dot{y}=x+2,\ y(0)=0\end{cases}$

11.5	$\begin{cases}\dot{x}=y-1,\ x(0)=1\\ \dot{y}=-x+2,\ y(0)=0\end{cases}$	11.6	$\begin{cases}\dot{x}=-2x+y,\ x(0)=0\\ \dot{y}=3x,\ y(0)=1\end{cases}$
11.7	$\begin{cases}\dot{x}=3y,\ x(0)=2\\ \dot{y}=3x+1,\ y(0)=0\end{cases}$	11.8	$\begin{cases}\dot{x}=x+2y+1,\ x(0)=0\\ \dot{y}=4x-y,\ y(0)=1\end{cases}$
11.9	$\begin{cases}\dot{x}=3x+5y+2,\ x(0)=0\\ \dot{y}=3x+y+1,\ y(0)=2\end{cases}$	11.10	$\begin{cases}\dot{x}=3y+2,\ x(0)=-1\\ \dot{y}=x+2y,\ y(0)=1\end{cases}$
11.11	$\begin{cases}\dot{x}=-x+3y+1,\ x(0)=1\\ \dot{y}=x+y,\ y(0)=2\end{cases}$	11.12	$\begin{cases}\dot{x}=x+3y,\ x(0)=1\\ \dot{y}=x-y,\ y(0)=0\end{cases}$
11.13	$\begin{cases}\dot{x}=2x+2y+2,\ x(0)=0\\ \dot{y}=4y+1,\ y(0)=1\end{cases}$	11.14	$\begin{cases}\dot{x}=x+4y,\ x(0)=1\\ \dot{y}=2x-y+9,\ y(0)=0\end{cases}$
11.15	$\begin{cases}\dot{x}=x+y,\ x(0)=1\\ \dot{y}=4x+y+1,\ y(0)=0\end{cases}$	11.16	$\begin{cases}\dot{x}=2y,\ x(0)=2\\ \dot{y}=2x,\ y(0)=2\end{cases}$
11.17	$\begin{cases}\dot{x}=2x+2y,\ x(0)=3\\ \dot{y}=-4x,\ y(0)=1\end{cases}$	11.18	$\begin{cases}\dot{x}=2y+1,\ x(0)=-1\\ \dot{y}=2x+3,\ y(0)=0\end{cases}$
11.19	$\begin{cases}\dot{x}=x+4y+1,\ x(0)=0\\ \dot{y}=2x+3y,\ y(0)=1\end{cases}$	11.20	$\begin{cases}\dot{x}=-3x-4y+1,\ x(0)=-1\\ \dot{y}=4x-2y,\ y(0)=0\end{cases}$
11.21	$\begin{cases}\dot{x}=x+y+1,\ x(0)=0;\\ \dot{y}=4x-y,\ y(0)=1.\end{cases}$	11.22	$\begin{cases}\dot{x}=-y,\ x(0)=1\\ \dot{y}=2x+2y,\ y(0)=1\end{cases}$
11.23	$\begin{cases}\dot{x}=x+2y,\ x(0)=0\\ \dot{y}=2x+y+1,\ y(0)=5\end{cases}$	11.24	$\begin{cases}\dot{x}=-2x+6y+1,\ x(0)=0\\ \dot{y}=2x+2,\ y(0)=1\end{cases}$
11.25	$\begin{cases}\dot{x}=-2x+y,\ x(0)=0\\ \dot{y}=3x,\ y(0)=1\end{cases}$	11.26	$\begin{cases}\dot{x}=-2x-2y,\ x(0)=3\\ \dot{y}=-4x,\ y(0)=1\end{cases}$
11.27	$\begin{cases}\dot{x}=x+2y,\ x(0)=0\\ \dot{y}=x-y+1,\ y(0)=1\end{cases}$	11.28	$\begin{cases}\dot{x}=3x+2y,\ x(0)=0\\ \dot{y}=x-y,\ y(0)=1\end{cases}$
11.29	$\begin{cases}\dot{x}=2y,\ x(0)=2\\ \dot{y}=2x+3y+1,\ y(0)=1\end{cases}$	11.30	$\begin{cases}\dot{x}=2x+8y+1,\ x(0)=2\\ \dot{y}=3x+4y,\ y(0)=1\end{cases}$

Decision of an exemplary embodiment.

Task 1. Is the function original or not and why?

а) $f_1(t)=3e^{5t}\cdot\eta(t)$, б) $f_2(t)=\dfrac{1}{t-8}$, в) $f_3(t)=3^{2^t}$.

Solution: Any complex or real-valued function $f(t)$ of a real argument *t* called original-function if satisfying the following three conditions:

1) $f(t)$-piecewise continuous function for $t\geq 0$;We assume that the function f(t) = 0 for all t < 0 and $f(0)=f(0+0)$;

2) $f(t)=0$ for $t<0$;

3) $f(t)$ grows no faster than an exponential function: there are constants $M > 0,\ s_0 \geq 0$, that $|f(t)| \leq Me^{s_0 t}$ for $t > 0$.

The unit function:

$$\eta(t) = \begin{cases} 1, & t \geq 0, \\ 0, & t > 0, \end{cases}$$

so-called Heaviside function is a function-orginal.

If $f(t)$ satisfies the conditions 1 and 2, then $f(t) \cdot \eta(t)$ satisfies all three conditions of the original function.

a) The function $f_1(t)$ is the original, because it satisfies the conditions 1 and 3: $M = 3 > 0,\ s_0 = 5 > 0$.

b) The function $f_2(t)$ is not original, because $t = 8$ is a point of discontinuity of the second type, i.e. condition 1 is not satisfied.

c) The function $f_3(t)$ is not original, because it is growing faster than the exponential function for all M and s_0.

Tasks 2-4. Find the image $F(p)$ of the function $f(t)$, using the properties of the Laplace transform:

a) $3 + 6e^{5t} + t^7$, b) $e^{5t}\cos 3t$, c) $(t-4)^5$, d) $\sin 7t \cdot \sin 5t$, e) $t^2 e^t$, f) $\int_0^t e^{2t} dt$, g) $\frac{\sin t}{t}$, h) $\cos^2 6t$.

Solution:

a) Using the Table 1and theorem linearity, we have:

$$f(t) = 3 + 6e^{5t} + t^7 \div \frac{3}{p} + \frac{6}{p-5} + \frac{7!}{p^8} = F(p).$$

b) By the correspondence table: $\cos 3t \div \frac{p}{p^2 + 9}$ and by the theorem shift:

$$e^{5t}\cos 3t \div \frac{p-5}{(p-5)^2 + 9}.$$

c) By the table 1: $t^5 \div \frac{5!}{p^6}$ and by the theorem delay we have:

$$(t-4)^5 \div \frac{5!}{p^6} \cdot e^{-5p}.$$

d) Transform this function: $\sin 7t \cdot \sin 5t = \frac{1}{2}(\cos 2t - \cos 12t)$. Now, using the table and theorem linearity, we have:

$$\sin 7t \cdot \sin 5t = \frac{1}{2}(\cos 2t - \cos 12t) \div \frac{1}{2} \cdot \frac{p}{p^2+4} - \frac{1}{2} \cdot \frac{p}{p^2+144}.$$

e) By the table 1: $e^t \div \frac{1}{p-1}$ and by the theorem on the differentiation of the image: $t^2 e^t \div (-1)^2 \cdot \left(\frac{1}{p-1}\right)'' = \frac{2}{(p-1)^3}$.

f) We have: $e^{2t} \div \frac{1}{p-2}$.

By the theorem on the integration of the original:

$$\int_0^t e^{2t} dt \div \frac{\frac{1}{p-2}}{p} = \frac{1}{p(p-2)}.$$

g) So how $\sin t \div \frac{1}{p^2+1}$, then by the theorem on the integration of the image we have:

$$\frac{\sin t}{t} \div \int_p^\infty \frac{dp}{p^2+1} = arctgp\Big|_p^\infty = \frac{\pi}{2} - arctgp.$$

h) So how $f(t) = \cos^2 6t$, then $f'(t) = -6\sin 12t \div -\frac{72}{p^2+144}$. By the theorem of differentiation of the original $f'(t) \div pF(p) - f(0) = \left| f(0) = \cos 0 = 1 \right| = pF(p) - 1$. Thus, $-\frac{72}{p^2+144} = pF(p) - 1$, from whence $F(p) = \frac{1}{p} - \frac{72}{p(p^2+144)}$.

Task 5. For a given the original graph find the image.

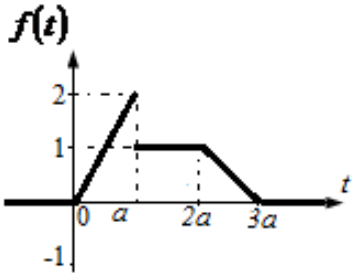

Figure 1

Solution: we denote points, in which happens gap or change of function through the point τ_k, jump of the function at the point of discontinuity through the point δ_k, angular coefficient plot k through the point α_k and $\alpha_k = a_{k+1} - a_k$ Images Of this type functions can be found by the formula:

$$F(p)=\sum_k\left(\frac{\delta_k}{p}+\frac{\alpha_k}{p^2}\right)\cdot e^{-\tau_k p}.$$

On the basis of the equations of the line passing through two points (t_1, y_1) and (t_2, y_2): $\frac{t-t_1}{t_2-t_1}=\frac{y-y_1}{y_2-y_1}$, define the first slope by points $(a, 2)$ and $(0, 0)$,the second slope by points $(3a, 0)$ and $(2a, 1)$. Then correspondingly, we have:

$y=\frac{2}{a}t$ и $y=-\frac{1}{a}(t-3a)$. Consequently, $a_1=0,\ a_2=\frac{2}{a},\ a_3=0,\ a_4=-\frac{1}{a},\ a_5=0$.

In our case: $\tau_1=0,\ \delta_1=0,\ \alpha_1=\frac{2}{a};\quad \tau_2=a,\ \delta_2=1-2=-1,\ \alpha_2=-\frac{2}{a};$

$\tau_3=2a,\ \delta_3=0,\ \alpha_3=-\frac{1}{a};\quad \tau_4=3a,\ \delta_4=0,\ \alpha_4=\frac{1}{a}.$

Thus,

$$F(p)=\left(\frac{0}{p}+\frac{2}{ap^2}\right)\cdot e^0+\left(\frac{-1}{p}-\frac{2}{ap^2}\right)\cdot e^{-ap}+\left(\frac{0}{p}-\frac{1}{ap^2}\right)\cdot e^{-2ap}+\left(\frac{0}{p}+\frac{1}{ap^2}\right)\cdot e^{-3ap}=$$

$$=\frac{2}{ap^2}-\left(\frac{1}{p}+\frac{2}{ap^2}\right)\cdot e^{-ap}-\frac{1}{ap^2}\cdot e^{-2ap}+\frac{1}{ap^2}\cdot e^{-3ap}.$$

Task 6. Find the convolution of functions t and e^t and its image.

Solution: convolution of functions $f(t)$ and $g(t)$ found by the formulae:

$$f(t)*g(t)=\int_0^t f(\tau)g(t-\tau)d\tau.$$

Therefore $t*e^t=\int_0^t \tau e^{t-\tau}d\tau=\int_0^t (t-\tau)e^{\tau}d\tau=\left(te^{\tau}-\tau e^{\tau}+e^{\tau}\right)\Big|_0^t=e^t-t-1.$

Image convolution we find by table and the linearity theorem:

$$t*e^t=e^t-t-1\div\frac{1}{p-1}-\frac{1}{p^2}-\frac{1}{p}=\frac{1}{p^2(p-1)}.$$

Task 7. Find the $f(t)$ original $\frac{1}{(p-2)(p+5)}$ functions, applying the theorem of multiplication of images.

Solution: by the theorem of multiplication: if $f(t)\div F(p),\ g(t)\div G(p)$, then $F(p)\cdot G(p)\div f(t)*g(t)$. Thus,

$$\frac{1}{(p-2)(p+5)}=\frac{1}{p-2}\cdot\frac{1}{p+5}\div e^{2t}*e^{-5t}=\int_0^t e^{2(t-\tau)}e^{-5\tau}d\tau=-\frac{1}{7}e^{2t-7\tau}\Big|_0^t=\frac{1}{7}e^{2t}\left(1-e^{-7t}\right).$$

Task 8. Find the original $f(t)$ for a given image $F(p)$:

a) $F(p)=\frac{p^2}{(p-2)(p^2+p+1)}$, b) $F(p)=\frac{1}{p(p^2+36)}$.

Solution:

a) Decompose the rational function of the sum of simple fractions:

We are looking for coefficients *A, B* and *C:*

$$F(p)=\frac{p^2}{(p-2)(p^2+p+1)}=\frac{A}{p-2}+\frac{Bp+C}{p^2+p+1}.$$

Multiplying the equation by its denominator:

$$p^2=A(p^2+p+1)+(Bp+C)(p-2).$$

Now we can substitute to get *A:*

$$p=2:\ 4=7A\ \rightarrow A=\frac{4}{7}.$$

The *B* coefficient can be obtained by comparison of p^2 factors:

$$p^2:\ 1=A+B\ \rightarrow\ B=\frac{3}{7}.$$

The *C* coefficient can be obtained by comparison of p^0 factors:

$$p^0:\ 0=A-2C\ \rightarrow\ C=\frac{2}{7}.$$

Decomposed *F(p)*:

$$F(p)=\frac{4}{7}\cdot\frac{1}{p-2}+\frac{1}{7}\cdot\frac{3p+2}{p^2+p+1}=\frac{4}{7}\cdot\frac{1}{p-2}+\frac{3}{7}\cdot\frac{p+\frac{2}{3}}{\left(p+\frac{1}{2}\right)^2+\frac{3}{4}}=$$

$$=\frac{4}{7}\cdot\frac{1}{p-2}+\frac{3}{7}\cdot\frac{p+\frac{1}{2}}{\left(p+\frac{1}{2}\right)^2+\frac{3}{4}}-\frac{1}{7\sqrt{3}}\cdot\frac{\frac{\sqrt{3}}{2}}{\left(p+\frac{1}{2}\right)^2+\frac{3}{4}}.$$

So the original function : $f(t)=\frac{4}{7}e^{2t}+\frac{3}{7}e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t-\frac{1}{7\sqrt{3}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t.$

b) By table $\frac{1}{p^2+36}\div\frac{1}{6}\sin 6t$, by the theorem on the integration of the original:

$$\frac{1}{p(p^2+36)}\div\int_0^t\frac{1}{6}\sin 6t\,dt=-\frac{1}{36}\cos 6t\Big|_0^t=\frac{1}{36}-\cos 6t.$$

Task 9. Solve the Cauchy problem for LIDE with constant coefficients by operational calculus methods:

$$y''+4y=\sin 2t,\quad y(0)=0,\quad y'(0)=1.$$

Solution: let $y(t)\div Y(p)$, then by the theorem on the differentiation of the original:

$$y''(t) = p^2Y(p) - py(0) - y'(0) = p^2Y(p) - 1, \text{ a } \sin 2t \div \frac{2}{p^2+4}.$$

The operator equation has the form: $p^2Y(p) - 1 + 4Y(p) = \frac{2}{p^2+4}$.

From whence: $Y(p)(p^2+4) = \frac{2}{p^2+4} + 1,$

$$Y(p) = \frac{2}{(p^2+4)^2} + \frac{1}{p^2+4}.$$

Original $\frac{2}{(p^2+4)^2}$ can be found by the theorem of the multiplication theorem:

$$\frac{2}{(p^2+4)^2} = \frac{2}{p^2+4} \cdot \frac{1}{p^2+4} \div \sin 2t * \frac{1}{2}\sin 2t = \frac{1}{2}\int_0^t \sin 2\tau \sin 2(t-\tau) d\tau =$$

$$= 2\left(\frac{1}{16}\sin 2t - \frac{t}{8}\cos 2t\right).$$

Thus, the desired solution:

$$y(t) = \frac{1}{8}\sin 2t - \frac{t}{4}\cos 2t + \frac{1}{2}\sin 2t = \frac{5}{8}\sin 2t - \frac{t}{4}\cos 2t.$$

Task 10. Find the solution of the Cauchy problem with the formula Duhamel:

$$x'' - 4x = \frac{1}{2+e^{2t}}, \quad x(0) = x'(0) = 0.$$

Solution: we form the auxiliary equation $x'' - 4x = 1$ and decide its by operator method. The operator equation: $p^2X_1 - 4X_1 = \frac{1}{p}$. Its decision : $X_1 = \frac{1}{p(p^2-4)}$.

By known methods find the original that solutions:

$$X_1 \div x_1 = -\frac{1}{4} + \frac{1}{4}ch2t.$$

To determine the solutions of this equations we use the formula:

$$x(t) = \int_0^t f(\tau)\frac{1}{2}x_1'(t-\tau)d\tau.$$

In that $x_1'(t) = \frac{1}{2}sh2t = \frac{e^{2t} - e^{-2t}}{4}$, a $f(t) = \frac{1}{2+e^{2t}}$,

then

$$x(t) = \int_0^t \left(\frac{1}{2+e^{2t}} \cdot \frac{e^{2(t-\tau)} - e^{-2(t-\tau)}}{4}\right)d\tau = \ \ldots =$$

$$= \frac{e^{2t}}{4}\left(-\frac{1}{4}e^{-2t} + \frac{1}{8}\ln\left|\frac{2e^{-2t}+1}{3}\right| + \frac{1}{4}\right) - \frac{e^{-2t}}{8}\ln\left|\frac{e^{2t}+2}{3}\right|.$$

Task 11. Solve the system of differential equations by operational method:

$$\begin{cases} \dot{x} = 4x - 3y + \sin t, \\ \dot{y} = 2x - y - 2\cos t. \end{cases} \quad x(0) = -1, \ \ y(0) = 0.$$

Solution: let $x(t) \div X(p)$, $y(t) \div Y(p)$. We form a system of operator equations, using the properties of the Laplace transform, table 1 and the initial conditions:

$$\begin{cases} pX(p) + 1 = 4X(p) - 3Y(p) + \dfrac{1}{p^2+1} \\ pY(p) = 2X(p) - Y(p) - \dfrac{2p}{p^2+1} \end{cases} \rightarrow \begin{cases} (p-4)X(p) + 3Y(p) = \dfrac{-p^2}{p^2+1} \\ -2X(p) + (p+1)Y(p) = -\dfrac{2p}{p^2+1}. \end{cases}$$

We solve it by Cramer's rule:

$$\Delta = \begin{vmatrix} p-4 & 3 \\ -2 & p+1 \end{vmatrix} = p^2 - 3p + 2 = (p-1)(p-2);$$

$$\Delta_x = \begin{vmatrix} \dfrac{-p^2}{p^2+1} & 3 \\ -\dfrac{2p}{p^2+1} & p+1 \end{vmatrix} = \frac{-p^3 - p^2 + 6p}{p^2+1}, \ \Delta_y = \begin{vmatrix} p-4 & \dfrac{-p^2}{p^2+1} \\ -2 & -\dfrac{2p}{p^2+1} \end{vmatrix} = \frac{-4p^2 + 8p}{p^2+1};$$

$$X(p) = \frac{\Delta_x}{\Delta} = \frac{-p^3 - p^2 + 6p}{(p^2+1)(p-1)(p-2)}, \ \ Y(p) = \frac{\Delta_y}{\Delta} = \frac{-4p^2 + 8p}{(p^2+1)(p-1)(p-2)};$$

$$\frac{-p^3 - p^2 + 6p}{(p^2+1)(p-1)(p-2)} = \frac{A_1}{p-1} + \frac{B_1}{p-2} + \frac{C_1 p + D_1}{p^2+1};$$

$$\frac{-4p^2 + 8p}{(p^2+1)(p-1)(p-2)} = \frac{A_2}{p-1} + \frac{B_2}{p-2} + \frac{C_2 p + D_2}{p^2+1}.$$

We find by the known methods:

$$A_1 = -1, \ B_1 = 0, \ C_1 = 1, \ D_1 = -1, \ A_2 = -2, \ B_2 = 0, \ C_2 = 2, \ D_2 = -2.$$

So,

$$X(p) = \frac{-1}{p-1} + \frac{p-1}{p^2+1} \div -e^t + \cos t - \sin t,$$

$$Y(p) = -\frac{2}{p-1} + 2 \cdot \frac{p-1}{p^2+1} \div -2e^t + 2\cos t - 2\sin t.$$

Answer: $x(t) = -e^t + \cos t - \sin t, \qquad y(t) = 2\left(-e^t + \cos t - \sin t\right).$

Reference material.

Properties of Laplace transform. Let $f(t) \div F(p)$, $g(t) \div G(p)$:

1) $Af(t) + Bg(t) \div AF(p) + BG(p)$ (theorem linearity);

2) $f(\lambda t) \div \frac{1}{\lambda} F\left(\frac{p}{\lambda}\right)$ (theorem similarity);

3) $e^{at} f(t) \div F(p-a)$ (theorem of shifting);

4) $f(t-\tau) \div e^{-p\tau} F(p)$ ($\tau > 0$ theorem delay);

5) $f'(\tau) \div pF(p) - f(0)$, $f''(\tau) \div p^2 F(p) - pf(0) - f'(0)$, ... ,

$f^{(n)}(\tau) \div p^n F(p) - p^{n-1} f(0) - \ldots - f^{(n-1)}(0)$ (theorem of differentiation of the original);

6) $\int_0^t f(z)dz \div \frac{F(p)}{p}$ (theorem of integration of the original);

7) $(-1)^n t^n f(t) \div F^{(n)}(p)$ (theorem of differentiation of the image);

8) $\frac{f(t)}{t} \div \int_p^\infty F(z)dz$ (theorem of integration of the images);

9) $f(t) * g(t) = \int_0^t f(\tau) g(t-\tau) d\tau = \int_0^t f(t-\tau) g(\tau) d\tau \div F(p) \cdot G(p)$,

(theorem of multiplication of images);

10) $F(p) \cdot G(p) \div f'(t) * g(t) + f(0) \cdot g(t) = g'(t) * f(t) + g(0) \cdot f(t)$,

(Duhamel integrals).

Table of correspondence of original-image.

Table 1

№	$f(t)$	$F(p)$	№	$f(t)$	$F(p)$
1	1	$\frac{1}{p}$	8	$e^{at}\sin\omega t$	$\frac{\omega}{(p-a)^2-\omega^2}$
2	e^{at}	$\frac{1}{p-a}$	9	$e^{at}\cos\omega t$	$\frac{p-a}{(p-a)^2-\omega^2}$
3	t^n	$\frac{n!}{p^{n+1}}$	10	$t^n e^{at}$	$\frac{n!}{(p-a)^{n+1}}$
4	$\sin\omega t$	$\frac{\omega}{p^2+\omega^2}$	11	$t\sin\omega t$	$\frac{2\omega p}{(p^2+\omega^2)^2}$
5	$\cos\omega t$	$\frac{p}{p^2+\omega^2}$	10	$t\cos\omega t$	$\frac{p^2-\omega^2}{(p^2+\omega^2)^2}$
6	$sh\omega t$	$\frac{\omega}{p^2-\omega^2}$	11	$tsh\omega t$	$\frac{2\omega p}{(p^2-\omega^2)^2}$
7	$ch\omega t$	$\frac{p}{p^2-\omega^2}$	12	$tch\omega t$	$\frac{p^2+\omega^2}{(p^2-\omega^2)^2}$

Calculation-graphical work №2 1-st and 2-nd types curvilinear integrals. 1-st and 2-nd types surface integrals. Elements of the field theory.

Calculated tasks.

Task 1. Calculate the 1-st type curvilinear integral on the line AB, where where AB - a line segment concluded between points A and B (task in part a).

№	Individual task		
	a)	b)	
1.1	$\int_L xy^2 dl$	$\int_L xdx - 2y^2 dy$	A(-1;0); B(2;1)
1.2	$\int_L y^2 xdl$	$\int_L x^2 dy + ydx$	A(1;3); B(2;-4)
1.3	$\int_L xydl$	$\int_L x^3 dx + 2ydy$	A(1;1); B(2;4)
1.4	$\int_L (x+1)dl$	$\int_L 3xdy - y^2 dx$	A(1;-1); B(2;3)
1.5	$\int_L (x^2+2)dl$	$\int_L 5xdx - 4y^2 dy$	A(-2;0); B(0;3)
1.6	$\int_L x^3 ydl$	$\int_L (x+y)dx - dy$	A(-2;-1); B(1;2)
1.7	$\int_L (x^3+y)dl$	$\int_L x^2 ydx + dy$	A(-3;0); B(0;2)
1.8	$\int_L x^2 y^2 dl$	$\int_L 4xdy _ ydx$	A(1;3); B(3;4)
1.9	$\int_L (4+x^2)dl$	$\int_L (3x+1)dy + dx$	A(-3;2); B(1;-2)
1.10	$\int_L (x+1)ydl$	$\int_L (4x+y)dx + ydy$	A(-4;-1); B(2;-3)
1.11	$\int (x^2+2)ydl$	$\int_L x^2 dx - 3xydy$	A(-2;-1); B(3;-4)
1.12	$\int_L x(y+1)dl$	$\int_{L:} (x _ y)dx + x^2 ydy$	A(3;4); B(2;-3)
1.13	$\int_L (x+1)y^2 dl$	$\int_L x^2 dy - ydx$	A(-1;0); B(0;3)
1.14	$\int_L x(y+2)dl$	$\int_L \sqrt{x}dx + 2y^2 dy$	A(-1;2); B(2;3)
1.15	$\int_L x^2 y^2 dl$	$\int_L \sqrt[3]{x}dy - y^3 dx$	A(0;1); B(-2;3)
1.16	$\int_L x^3 y^2 dl$	$\int_L 5xdx - 4(y+1)dy$	A(-3;-1); B(1;2)
1.17	$\int_L (3+x^2)ydl$	$\int_L 3xdy - (2x+1)ydx$	A(-3;4); B(0;1)
1.18	$\int_L 4xyydl$	$\int_L y^2 xdy$	A(-2;-2); B(2;2)
1.19	$\int_L 5x^2 ydl$	$\int_L (x^2+y)dx$	A(0;-3); B(2;-4)

1.20	$\int_L 3xy^2 dl$	$\int_L 4x^2 dy - (y+1)dx$	A(1;-1); B(3;4)
1.21	$\int_L (x^3 + y)dl$	$\int_L x^3 dx - \sqrt{x}ydy$	A(-3;-1); B(4;2)
1.22	$\int_L x^2 y^2 dl$	$\int_L \sqrt[3]{x}dy + \sqrt{x}ydx$	A(0;-3); B(2;2)
1.23	$\int_L (4 + x^2)dl$	$\int_L x^2 dy - y^2 dx$	A(-3;0); B(4;5)
1.24	$\int_L (x+1)ydl$	$\int_L (2x+1)dx - dy$	A(0;2); B(3;3)
1.25	$\int_L (x^2 + 2)ydl$	$\int_L (4y+1)dx - dy$	A(-4;0); B(2;4)

Task 2. Calculate the 2-nd type curvilinear integral on the line AB, where where AB - a line segment concluded between points A and B (see the previous task, part b).

Task 3. Calculate the 2-nd type curvilinear integral to the closed contour $\oint_L P(x,y)dx + Q(x,y)dy,$ where L- circle $x^2 + y^2 = R^2$, using Green's formula.

3.1 $\oint_L (2x - 3y^2 + 1)dx + (2 - 6xy + 5x)dy$, L: $x^2 + y^2 = 12.$

3.2 $\oint_L \left(\frac{2xy^2}{1+x^2y^2} - 3y\right)dx + \left(\frac{2x^2y}{1+x^2y^2}\right)dy$, L: $x^2 + y^2 = 36.$

3.3 $\oint_L -\left(\frac{1}{2}\cos 2y + y\sin 2x\right)dx + \left(x\sin 2y + \cos^2 x + x\right)dy$, L: $x^2 + y^2 = 4.$

3.4 $\oint_L \left(y^2 e^{xy^2} + 3y\right)dx + \left(2xye^{xy^2} - 1\right)dy$, L: $x^2 + y^2 = 36.$

3.5 $\oint_L \left(\frac{1}{x+y} - \cos x\cos y - 3x^2\right)dx + \left(\frac{1}{x+y} - \sin x\sin y - 4x\right)dy$, L: $x^2 + y^2 = 44.$

3.6 $\oint_L \left(\frac{y}{x} + \ln y + 2y\right)dx + \left(\ln x + \frac{x}{y} + 1\right)dy$, L: $x^2 + y^2 = 9.$

3.7 $\oint_L \left(e^{x+y} - \cos x\right)dx + \left(e^{x+y} + \sin y + 2x\right)dy$, L: $x^2 + y^2 = 25.$

3.8 $\oint_L \left(\frac{y}{\sqrt{1-x^2y^2}} + 2x - 4y\right)dx + \left(\frac{x}{\sqrt{1-x^2y^2}} + 6y\right)dy$, L: $x^2 + y^2 = 2.$

3.9 $\oint_L \left(e^{xy} + xye^{xy} + 2\right)dx + \left(x^2 e^{xy} + x\right)dy$, L: $x^2 + y^2 = 3.$

3.10 $\oint_L \left(ye^{xy} + y^2 - 7y\right)dx + \left(xe^{xy} + 2xy\right)dy$, L: $x^2 + y^2 = 9.$

3.11 $\oint_L \left(y\cos(xy) + 2x - 3y\right)dx + \left(x\cos(xy) - 6x + 4y\right)dy$, L: $x^2 + y^2 = 4.$

3.12 $\oint_L (3x-2xy+y^2)dx+(2xy-x^2-3y^2)dy$, L: $x^2+y^2=8.$

3.13 $\oint_L (5y+\cos x+6xy^2)dx+(7x+6x^2y)dy$, L: $x^2+y^2=7.$

3.14 $\oint_L (y^2e^{xy}-3y)dx+e^{xy}(1+xy)dy$, L: $x^2+y^2=12.$

3.15 $\oint_L (2+\cos(xy))ydx+(1+\cos(xy))xdy$, L: $x^2+y^2=25.$

3.16 $\oint_L (3y-\sin x)dx+(x-2y\cos y^2)dy$, L: $x^2+y^2=16.$

3.17 $\oint_L \left(2x-\frac{1}{x^2y}\right)dx-\frac{1}{xy^2}dy$, L: $x^2+y^2=6.$

3.18 $\oint_L \frac{x+y}{xy}dx+\left(\frac{y-x}{y^2}+2x\right)dy$, L: $x^2+y^2=5.$

3.19 $\oint_L (20x^3-21x^2y_2y)dx+(3+4x-7x^3)dy$, L: $x^2+y^2=2.$

3.20 $\oint_L (ye^{xy}-2\sin x)dx+(xe^{xy}+5x)dy$, L: $x^2+y^2=36.$

3.21 $\oint_L y(e^{xy}+6)dx+x(e^{xy}+5)dy$, L: $x^2+y^2=16.$

3.22 $\oint_L \left(x-\frac{y}{x^2-y^2}\right)dx+\left(\frac{x}{x^2-y^2}-2x\right)dy$, L: $x^2+y^2=9.$

3.23 $\oint_L \frac{x\ln y+y}{x}dx+\left(\frac{y\ln x+x}{x}+4x\right)dy$, L: $x^2+y^2=25.$

3.24 $\oint_L (e^{x-y}(1+x+y)+8y)dx+e^{x-y}(1-x-y)dy$, L: $x^2+y^2=16.$

3.25 $\oint_L (3x^2-2xy+7y)dx+(x-x^2-3y^2-4y)dy$, L: $x^2+y^2=5.$

Task 4. Calculate the 1-st type surface integral on the surface S, where S- part of the plane (*p*), be cut off coordinate planes.

№	Individual task	№	Individual task
4.1	$\iint_S (2x+3e+2z)ds$, $(p): x+3y+z=3$	4.14	$\iint_S (9x+2y+z)ds$, $(p): 2x+y+z=4$
4.2	$\iint_S (2+y-7x+9z)ds,$ $(p): 2x-y-2z=-2$	4.15	$\iint_S (3x+8y+8z)ds$, $(p): x+4y+2z=8$
4.3	$\iint_S (6x+y+4z)ds,$ $(p): 3x+3y+z=3$	4.16	$\iint_S (4y-x+4z)ds$, $(p): x-2y+2z=2$

4.4	$\iint\limits_S (x+2y+3z)ds,$ $(p): x+y+z=2$	4.17	$\iint\limits_S (7x+y+2z)ds$, $(p): 3x-2y+2z=6$
4.5	$\iint\limits_S (3x-2y+6z)ds,$ $(p): 2x+y+2z=2$	4.18	$\iint\limits_S (2x+3y+z)ds$, $(p): 2x+3y+z=6$
4.6	$\iint\limits_S (2x+5y-z)ds,$ $(p): x+2y+z=2$	4.19	$\iint\limits_S (4x-y+z)ds$, $(p): x-y+z=2$
4.7	$\iint\limits_S (5x-8y-z)ds,$ $(p): 2x-3y+z=6$	4.20	$\iint\limits_S (6x-y+8z)ds$, $(p): x+y+2z=2$
4.8	$\iint\limits_S (3y-x-z)ds,$ $(p): x-y+z=2$	4.21	$\iint\limits_S (4x-4y-z)ds$, $(p): x+2y+2z=4$
4.9	$\iint\limits_S (3y-2x-2z)ds,$ $(p): 2x-y-2z=-2$	4.22	$\iint\limits_S (2x+5y+z)ds$, $(p): x+y+2z=2$
4.10	$\iint\limits_S (2x-3y+z)ds,$ $(p): x+2y+z=2$	4.23	$\iint\limits_S (4x-y+4z)ds$, $(p): 2x+2y+z=4$
4.11	$\iint\limits_S (5x+y-z)ds,$ $(p): x+2y+2z=2$	4.24	$\iint\limits_S (5x+2y+2z)ds$, $(p): x+2y+z=2$
4.12	$\iint\limits_S (3x+2y+2z)ds,$ $(p): 3x+2y+2z=6$	4.25	$\iint\limits_S (2x+5y+10z)ds$, $(p): 2x+y+3z=6$
4.13	$\iint\limits_S (2x+3y-z)ds,$ $(p): 2x+y+z=2$		

Task 5. Given the function $u(M)=u(x,y,z)$ and points M_1, M_2. Calculate the derivative of this function at the point M_1 in the direction of the vector $\vec{a}=\overrightarrow{M_1M_2}$.

№	Задания		
	$u(M)$	M_1	M_2
5.1	$x^2y+y^2z+z^2x$	(1,-1,2)	(3,4,-1)
5.2	$5xy^3z^2$	(2,1,-1)	(4,-3,0)
5.3	$\ln\left(x^2+y^2+z^2\right)$	(-1,2,1)	(3,1,-1)
5.4	$z\cdot e^{x^2+y^2+z^2}$	(0,0,0)	(3,-4,2)
5.5	$\ln(xy+yz+xz)$	(-2,3,-1)	(2,1,-3)
5.6	$\sqrt{1+x^2+y^2+z^2}$	(1,1,1)	(3,2,1)
5.7	x^2y+xz^2-2	(1,1,1)	(2,-1,3)
5.8	$xe^y+ye^x-z^2$	(3,0,-2)	(4,1,3)
5.9	$3xy^2+z^2-xyz$	(1,2,2)	(3,-1,4)
5.10	$5x^2yz+xy^2z+yz^2$	(1,1,1)	(9,-3,9)
5.11	$\frac{x}{x^2+y^2+z^2}$	(1,2,2)	(-3,2,-1)
5.12	$y^2z-2xyz+z^2$	(3,1,-1)	(-2,1,4)
5.13	$x^2+y^2+z^2-2xyz$	(1,-1,2)	(5,-1,4)
5.14	$\ln\left(1+x+y^2+z^2\right)$	(1,1,1)	(3,-5,1)
5.15	$x^2+2y^2-4z^2-5$	(1,2,1)	(-3,-2,6)
5.16	$\ln\left(x^3+y^3+z+1\right)$	(1,3,0)	(-4,1,3)
5.17	$x-2y+e^x$	(-4,-5,0)	(2,3,4)
5.18	x^4-3xyz	(2,2,-4)	(1,0,-3)
5.19	$3x^2y^3z$	(-2,-3,1)	(5,-2,0)
5.20	e^{xy+z^2}	(-5,0,2)	(2,4,-3)
5.21	x^{yz}	(3,1,4)	(1,-1,-1)
5.22	$\left(x^2+y^2+z^2\right)^3$	(1,2,-1)	(0,-1,3)
5.23	$(x+y)^z$	(1,5,0)	(3,7,-2)
5.24	x^2y+y^2z-3z	(0,-2,-1)	(12,-5,0)
5.25	$\frac{10}{x^2+y^2+z^2+1}$	(-1,2,-2)	(2,0,1)

Task 6. Find the value and direction of the greatest changes of the functions $u(M)=u(x,y,z)$ at the point $M_0(x_0,y_0,z_0)$.

№	$u(x,y,z)$	$M_0(x_0,y_0,z_0)$.
6.1	$xyz+3xy^2-5yz^2$	(0,1,-2)

6.2	$x^2yz-2xyz^2+3xy^2z$	(2,0,2)
6.3	$2x^2y^2z-xyz^3+x^2yz^2$	(1,-1,0)
6.4	$x^2(y+z)-yz^2+2xyz$	(3,0,1)
6.5	$(xy-yz)z^2+xy^2z$	(-1,0,3)
6.6	$2xy^2-3xyz+2xz^2$	(2,1,0)
6.7	$(y+z)xyz-(x+y-z)yz^2$	(0,1,1)
6.8	$3xyz^2-4xy^2z+x^2y^2$	(0,-2,1)
6.9	$-x^2y+2xyz-3xz^2$	(0,1,2)
6.10	$xyz-z^3+2xy^2$	(-1,0,1)
6.11	$zx(y-x)-2xyz+z^3$	(1,2,0)
6.12	$x(y^2+z^2)+3xyz-z^3$	(-3,1,0)
6.13	$(y+z)x^2-2xy^2+xyz$	(1,0,4)
6.14	$xy(z-y)-2x^3+3xy^2$	(0,-1,2)
6.15	$(x+y)zx^2+2xy^2z-xyz^2$	(-1,0,1)
6.16	$xy^2-z+xyz(z-x)$	(2,1,0)
6.17	$(x+y)z^2+x(y^2+z^2)$	(0,1,1)
6.18	$x-yz_xyz-xz^2$	(-1,1,0)
6.19	$x^3y+y^3z+z^3x$	(2,1,0)
6.20	$xy^2+yz^2-zx^2$	(-1,0,1)
6.21	$xyz-x^2y-2xy^2$	(0,2,-1)
6.22	$(x-z)y^2+(y-x)z^2$	(1,2,0)
6.23	$xy^2z+x^2yz+xyz^2$	(1,1,1)
6.24	$(x-yz)x^2-2xyz^2$	(-2,0,1)
6.25	$(x+y)z^2+(y+z)x^2$	(1,-1,0)

Task 7. Find the greatest density of circulation of the vector field $\vec{a}(M)=P(x,y,z)\vec{i}+Q(x,y,z)\vec{j}+R(x,y,z)\vec{k}$ at the point $M_0(x_0,y_0,z_0)$.

№	$\vec{a}(M)$	$M_0(x_0,y_0,z_0)$
7.1	$x^2\vec{i}-xy^2\vec{j}+z^2\vec{k}$	(0,1,-2)
7.2	$xy\vec{i}+yz\vec{j}+xz\vec{j}+xz\vec{k}$	(2,0,3)
7.3	$xy^2\vec{i}+yz^2\vec{j}-x^2\vec{k}$	(1,-2,0)
7.4	$xz\vec{i}+z\vec{j}+yz\vec{k}$	(3,0,1)
7.5	$xy\vec{i}+xyz\vec{j}-x\vec{k}$	(-1,0,3)
7.6	$yz\vec{i}-z^2\vec{j}+xyz\vec{k}$	(2,1,-1)
7.7	$y^2\vec{i}-xy\vec{j}+z^2\vec{k}$	(-2,1,1)
7.8	$xz\vec{i}-xyz\vec{j}+x^2z\vec{k}$	(0,1,1)
7.9	$xy\vec{i}-y^2z\vec{j}-xz\vec{k}$	(0,-2,1)

7.10	$xz\vec{i} - y\vec{j} - zy\vec{k}$	(0,1,2)
7.11	$y^2\vec{i} - xy^2\vec{j} + z^2\vec{k}$	(-1,2,1)
7.12	$xy\vec{i} - xy^2\vec{j} - xy^2\vec{j} + z^2\vec{k}$	(1,-1,1)
7.13	$(x+y)\vec{i} + yz\vec{j} + xz\vec{k}$	(2,1,0)
7.14	$xy\vec{i} - (y+z)\vec{j} + xz\vec{k}$	(4,0,1)
7.15	$x\vec{i} - zy\vec{j} + x^2z\vec{k}$	(-3,0,2)
7.16	$(x+y^2)\vec{i} + yz\vec{j} - x^2\vec{k}$	(1,0,4)
7.17	$xz\vec{i} - y\vec{j} + yz\vec{k}$	(0,-1,4)
7.18	$xy\vec{i} - x\vec{j} + yz\vec{k}$	(2,2,2)
7.19	$(x+y)\vec{i} + xyz\vec{j} - x\vec{k}$	(4,1,-3)
7.20	$(x-y)\vec{i} + yz\vec{j} - y\vec{k}$	(-4,1,0)
7.21	$(y-z)\vec{i} - z^2\vec{j} + xyz\vec{k}$	(3,0,1)
7.22	$yz\vec{i} - z^2\vec{j} + (x+y)z\vec{k}$	(1,3,0)
7.23	$z^2\vec{i} - xz\vec{j} + z^2\vec{k}$	(1,-2,1)
7.24	$xy\vec{i} + (x-z)\vec{j} + (y-x)\vec{k}$	(0,0,1)
7.25	$xz\vec{i} + (x-y)\vec{j} + x^2z\vec{k}$	(1,1,-2)

Task 8. Determine whether the vector field $\vec{a}(M) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$ solenoidal, potential or harmonic.

№	$\vec{a}(M)$
8.1	$(\alpha-\beta)x\vec{i} + (\gamma-\alpha)y\vec{j} + (\beta-\gamma)z\vec{k}$
8.2	$x^2y\vec{i} - 2xy^2\vec{j} + 2xyz\vec{k}$
8.3	$(yz-2x)\vec{i} + (xz+2y)\vec{j} + xy\vec{k}$
8.4	$(x^2-z^2)\vec{i} - 3xy\vec{j} + (y^2+z^2)\vec{k}$
8.5	$2xyz\vec{i} - y(yz+1)\vec{j} + z\vec{k}$
8.6	$(2x-3y)\vec{i} + 2xy\vec{j} - z^2k$
8.7	$(x^2-y^2)\vec{i} + (y^2-z^2)\vec{j} + (z^2-x^2)\vec{k}$
8.8	$yz\vec{i} + (x-y)\vec{j} + z^2\vec{k}$
8.9	$(y+z)\vec{i} + (x+z)\vec{j} + (x+y)\vec{k}$
8.10	$3x^2y\vec{i} - 2xy^2\vec{j} - 2xyz\vec{k}$
8.11	$(x+y)\vec{i} - 2(y+z)\vec{j} + (z-x)\vec{k}$
8.12	$(yz-2x)\vec{i} + (xz+zy)\vec{j} + xy\vec{k}$
8.13	$yz\vec{i} + xz\vec{j} + xy\vec{k}$
8.14	$6xy\vec{i} + (3x^2-2y)\vec{j} + z\vec{k}$
8.15	$(2x-yz)\vec{i} + (2x-xy)\vec{j} + yz\vec{k}$
8.16	$(y-z)\vec{i} + 3xyz\vec{j} + (z-x)\vec{k}$
8.17	$(y-z)\vec{i} + (x+z)\vec{j} + (x^2-y^2)\vec{k}$

8.18	$(x+y)\bar{i}-2xz\bar{j}-3(y+z)\bar{k}$
8.19	$z^2\bar{i}+(xz+y)\bar{j}+x^2y\bar{k}$
8.20	$xy(3x-4y)\bar{i}+x^2(x-4y)\bar{j}+3z^2\bar{k}$
8.21	$6x^2i+3\cos(3x+2z)\bar{j}+\cos(3y+2z)\bar{k}$
8.22	$(x+y)\bar{i}+(z-y)\bar{j}+2(x+z)\bar{k}$
8.23	$3(x-z)\bar{i}+(x^2-y^2)\bar{j}+3z\bar{k}$
8.24	$(2x-yz)\bar{i}+(xz-2y)\bar{j}+2xyz\bar{k}$
8.25	$3x^2\bar{i}+4(x-y)\bar{j}+(x-z)\bar{k}$

Task 9. Using Stokes' formula to calculate the circulation of a vector field $\vec{a}(M)$ along the contour of the triangle, formed by the intersection of the plane (p) with the coordinate plane (the positive direction of the contour relative to the outer surface normal) Make a drawing.

№	$\vec{a}(M)$	(p)
9.1	$(2x-3y)\bar{i}+(x+4y)\bar{j}-6z\bar{k}$	$x+2y-z=2$
9.2	$3x\bar{i}-y^2z\bar{j}+yz^2\bar{k}$	$x-y+3z=3$
9.3	$(4x+z)\bar{i}+(2z-y)\bar{j}+(3x+2y)\bar{k}$	$x-2y+z=4$
9.4	$-x^2y\bar{i}+xy^2\bar{j}+3z^2\bar{k}$	$x+2y+z=2$
9.5	$(3y-2z)\bar{j}$	$x+y-2z=4$
9.6	$-3x^2y\bar{i}+3xy^2\bar{j}+z^3\bar{k}$	$x+2y-z=2$
9.7	$(2y-x)\bar{i}+(2x+5y)\bar{j}+(4y-3z)\bar{k}$	$x-y+z=2$
9.8	$-x^2z\bar{i}+3y\bar{j}+xz^2\bar{k}$	$2x-y+2z=4$
9.9	$(5x+2y)\bar{i}+(2x+3y)\bar{j}+(z+2y)\bar{k}$	$x-y-z=1$
9.10	$4x^2y\bar{i}-4xy^2\bar{j}+(5z+3)\bar{k}$	$2x-3y+2z=6$
9.11	$(x-3z)\bar{i}+4y\bar{j}+(2x+z)\bar{k}$	$2x+y+z=4$
9.12	$x^2z\bar{i}+3y\bar{j}-xz^2\bar{k}$	$x+2y+z=2$
9.13	$(x-3y)\bar{i}+(2x-y)\bar{j}+4z\bar{k}$	$x+3y-z=3$
9.14	$-3x^2y\bar{i}+3xy^2\bar{j}+2z\bar{k}$	$2x+y+3z=6$
9.15	$(4x-1)\bar{i}+(2y+z)\bar{j}(5y-z)\bar{k}$	$4x+y+2z=4$
9.16	$(x+2y)\bar{i}+(2x+y)\bar{j}+(2y+z)\bar{k}$	$x-2y+z=4$
9.17	$(x-2z)\bar{i}+3y\bar{j}+(2x+3z)\bar{k}$	$x-2y+3z=6$
9.18	$(3x+5)\bar{i}+(y-2z)\bar{j}+(2y+z)\bar{k}$	$-2x+y+2z=4$
9.19	$3y\bar{i}+(2y+3z)\bar{j}+(3y-2z)\bar{k}$	$x-2y-2z=2$
9.20	$(2x-3z)\bar{i}+5y\bar{j}+(x+2z)\bar{k}$	$x-y+2z=2$
9.21	$(5x-3)\bar{i}+(2y-3z)\bar{j}+(3y+2z)\bar{k}$	$x+2y+z=6$
9.22	$(x-2y)\bar{i}+(3y+2z)\bar{j}+(2y-5z)\bar{k}$	$x+y-2z=4$
9.23	$(x+2z)\bar{i}+(y+4z)\bar{j}+(4y-3z)\bar{k}$	$x+2y-3z=6$
9.24	$(x+4z)\bar{i}+(y+2z)\bar{j}+(4x+3z)\bar{k}$	$-2x+y+z=4$

9.25	$(x-2y)\bar{i}+(3z+2y)\bar{j}+(3y-4z)\bar{k}$	$x+y+3z=3$

Task 10. Calculate the work of a vector field $\vec{a}(M)$ along the curve L, given parametrically $\begin{cases} x=x(t), \\ y=y(t), \\ z=z(t), \\ 0\le t\le 1. \end{cases}$

№	$\vec{a}(M)$	$x(t)$	$y(t)$	$z(t)$
10.1	$(x^2+2)\bar{i}+(y+1)\bar{j}-z\bar{k}$	$2t$	t^2	$t-1$
10.2	$(x-y)\bar{i}-(z+x^2)\bar{j}+xy\bar{k}$	t^3	2	$3t$
10.3	$-(z+x^2)\bar{i}+(y-2)\bar{j}-xyz\bar{k}$	$t+1$	t^2	4
10.4	$(z^2-1)\bar{i}-(xy-z)\bar{j}+x^2\bar{k}$	3	$t-4$	t^2
10.5	$(xy+2)\bar{i}+(yz-x)\bar{j}-z^2\bar{k}$	$t+1$	t^2	2
10.6	$-x\bar{i}+(xyz-1)\bar{j}+(z+x)\bar{k}$	$2\,t^2$	1	$3t-1$
10.7	$(y^2+1)\bar{i}-(xy+z)\bar{j}-z\bar{k}$	$4t$	2	t^3
10.8	$(xyz-1)\bar{i}+(x+2)\bar{j}-xy\bar{k}$	5	t^2+1	t
10.9	$(z-x)\bar{i}-(y+z^2)\bar{j}-z\bar{k}$	$t+4$	1	$2t^2-1$
10.10	$(-y+zx)\bar{i}+(x+2)\bar{j}-z^3\bar{k}$	2	t^2-1	$3t$
10.11	$(x^2+1)\bar{i}-(x-z^2)\bar{j}+xy\bar{k}$	$t-1$	5	t^3
10.12	$(yz-1)\bar{i}+xyz\bar{j}-(z-1)\bar{k}$	2	$3t$	$4\,t^2$
10.13	$(xy-1)\bar{i}+xy\bar{j}+(z^2+1)\bar{k}$	$t-1$	$t+1$	2
10.14	$z^2x\bar{i}-(xy+1)\bar{j}-x^2\bar{k}$	4	t^2-1	t^3
10.15	$xz\bar{i}-xy\bar{j}+(z+1)\bar{k}$	$t+2$	t^2	2
10.16	$(z+2)\bar{i}+(z-3)\bar{j}+xy\bar{k}$	t^2-1	$2t$	$t-1$
10.17	$(x+y+z)\bar{i}-xy\bar{j}+(z^3+1)\bar{k}$	$3t-1$	t^2	7
10.18	$(y-z)\bar{i}+xz\bar{j}-(z-1)\bar{k}$	t^2+1	1	$t-2$
10.19	$(z-x)\bar{i}-(y+z^2)\bar{j}+x^2\bar{k}$	$2t$	3	$3t^2-1$
10.20	$yz\bar{i}+(x+yz)\bar{j}-y^2\bar{k}$	5	$t+1$	t^2-1
10.21	$xyz\bar{i}+(x+1)\bar{j}-zx\bar{k}$	$2t+2$	$3t^2$	2
10.22	$(z^2-4)\bar{i}-xz\bar{j}+xy^2\bar{k}$	$2t$	$3t^3-2$	2
10.23	$(x-1)\bar{i}-xy^2\bar{j}+xyz\bar{k}$	$t+1$	2	t^2
10.24	$(y+zx)\bar{i}-yz\bar{j}+(x^2+1)\bar{k}$	2	$t+1$	t^2
10.25	$(x+zy)\bar{i}+(y+1)\bar{j}-zx\bar{k}$	t^3-1	$t-1$	2

Task 11. Find the flux of a vector field $\vec{a}(M)$ through a part of the plane (p), located in the first octant (normal plane forms an acute angle with the axis Oz).

№	$\vec{a}(M)$	(p)
11.1	$x\bar{i}+y\bar{j}+z\bar{k}$	$x+y+z=1$
11.2	$y\bar{j}+z\bar{k}$	$x+y+z=1$
11.3	$2x\bar{i}+y\bar{j}+z\bar{k}$	$x+y+z=1$
11.4	$x\bar{i}+3y\bar{j}+2z\bar{k}$	$x+y+z=1$
11.5	$2x\bar{i}+3y\bar{j}$	$x+y+z=1$
11.6	$x\bar{i}+y\bar{j}+z\bar{k}$	$x+2y+2z=2$
11.7	$x\bar{i}+2y\bar{j}+z\bar{k}$	$x+2y+2z=2$
11.8	$y\bar{j}+3z\bar{k}$	$x+2y+2z=2$
11.9	$x\bar{i}+y\bar{j}+z\bar{k}$	$6x+3y+2z=6$
11.10	$2x\bar{i}+y\bar{j}+z\bar{k}$	$6x+3y+2z=6$
11.11	$3x\bar{i}+2z\bar{k}$	$6x+3y+2z=6$
11.12	$2x\bar{i}+3y\bar{j}+z\bar{k}$	$2x+6y+3z=6$
11.13	$x\bar{i}+3y\bar{j}-z\bar{k}$	$2x+6y+3z=6$
11.14	$-2x\bar{i}+y\bar{j}+4z\bar{k}$	$2x+6y+3z=6$
11.15	$x\bar{i}-y\bar{j}+6z\bar{k}$	$3x+2y+6z=6$
11.16	$2x\bar{i}+5y\bar{j}+5z\bar{k}$	$3x+2y+6z=6$
11.17	$x\bar{i}+y\bar{j}+z\bar{k}$	$4x+y+2z=2$
11.18	$2x\bar{i}+y\bar{j}-2z\bar{k}$	$4x+y+2z=2$
11.19	$x\bar{i}+y\bar{j}+2z\bar{k}$	$4x+y+2z=2$
11.20	$-x\bar{i}+y\bar{j}+12z\bar{k}$	$4x+y+2z=2$
11.21	$x\bar{i}+3y\bar{j}+8z\bar{k}$	$2x+4y+z=2$
11.22	$x\bar{i}-y\bar{j}+6z\bar{k}$	$2x+4y+z=2$
11.23	$x\bar{i}+2y\bar{j}+5z\bar{k}$	$2x+4y+z=2$
11.24	$x\bar{i}+4y\bar{j}+5z\bar{k}$	$2x+4y+z=2$
11.25	$x\bar{i}+y\bar{j}+z\bar{k}$	$2x+3y+z=1$

Task 12. Calculate flux of the vector field $\vec{a}(M)$ through the outer surface of the pyramid formed by the plane (P) and the coordinate plane, using the formula Ostrogradskii-Gauss.

№	$\vec{a}(M)$	(p)
12.1	$(x+\cos zy)\bar{i}+2y\bar{j}+(3z-e^{x})\bar{k}$	$x+y+2z=4$
12.2	$(2x+e^{y})\bar{i}-(y+x^{z})\bar{j}+(4z+xy)\bar{k}$	$x+2y+z=6$
12.3	$(x-e^{zy})\bar{i}+(x^{z+1}+2y)\bar{j}-(z+x)\bar{k}$	$2x+3y+z=6$
12.4	$(3x-\sin y^{2})\bar{i}+(2y-z^{x})\bar{j}+(z-tgx)\bar{k}$	$-x+2y+3z=6$
12.5	$(2x+\cos yz)\bar{i}-(y-tgz)\bar{j}+2z\bar{k}$	$x+2y+2z=4$
12.6	$(x-\arcsin y)\bar{i}+(2y+\sin z)\bar{j}-(z+e^{x})\bar{k}$	$x+y+z=1$
12.7	$(4x+\sin y^{z})\bar{i}-(2y-ctgxz)\bar{j}-(z+xy)\bar{k}$	$-x+y+3z=3$

12.8	$(2x-\ln yz)\bar{i}+(y+\cos x)\bar{j}-(2z+x^y)\bar{k}$	$3x+y+z=6$
12.9	$(x+tgy)\bar{i}+(2y+e^x)\bar{j}-(z-\ln x)\bar{k}$	$2x+y+z=4$
12.10	$(3x-e^{yz})\bar{i}-(2y+\cos z)\bar{j}+(z-\sin y)\bar{k}$	$x-y+2z=4$
12.11	$(2x+\ln(y+1))\bar{i}+(2y-\cos z^x)\bar{j}+(3-2z)\bar{k}$	$-2x+3y+z=6$
12.12	$(thz+x)\bar{i}-(\sin x+y)\bar{j}+(2z-y)\bar{k}$	$x+y-z=1$
12.13	$(\ln yz+2x)\bar{i}+(\cos z+y)\bar{j}+(y^x-z)\bar{k}$	$x+2y-3z=6$
12.14	$(3x-\ln y)\bar{i}+(e^x-2y)\bar{j}+(z-x\ln y)\bar{k}$	$2x-y-z=4$
12.15	$(tgz-x)\bar{i}+(e^{xz}+3y)\bar{j}+(\sin x-z)\bar{k}$	$x+y-2z=2$
12.16	$(x+ye^z)\bar{i}+(x\ln x-y)\bar{j}+(2z+e^x)\bar{k}$	$-x+y+2z=3$
12.17	$(\arcsin y-2x)\bar{i}+(3y+\sin z)\bar{j}+(z+1)\bar{k}$	$x+y+3z=6$
12.18	$(e^{yz}+3x)\bar{i}-(2y+\cos zx)\bar{j}+(tgx+z)\bar{k}$	$3x-2y+z=6$
12.19	$(x-y\ln z)\bar{i}-(3\sin x-2y)\bar{j}+(e^x-z)\bar{k}$	$2x+2y-z=4$
12.20	$(ye^z-2x)\bar{i}+(3y-\sin z)\bar{j}+(tgxy+z)\bar{k}$	$-3x+2y-z=6$
12.21	$(e^z+x)\bar{i}-(x\ln z-2y)\bar{j}+(z-\sin x)\bar{k}$	$2x-2y+z=4$
12.22	$(\ln z+2x)\bar{i}+(e^x-y)\bar{j}+(tgy+z)\bar{k}$	$x-y+3z=3$
12.23	$(e^y+2x)\bar{i}-(x\ln z+2y)\bar{j}+(z+x\cos y)\bar{k}$	$-2x+y+3z=6$
12.24	$(arctgy+3x)\bar{i}-(2y+e^x)\bar{j}+(z-tgx)\bar{k}$	$x+2y+z=4$
12.25	$(2x-y\ln z)\bar{i}+(2y-e^x)\bar{j}+(z-\cos x)\bar{k}$	$x-2y+3z=6$

Task 13. Show that the vector field $\vec{a}(M)$ is a potential, and to find the potential of this field.

№	$\vec{a}(M)$
13.1	$3x^2e^y\bar{i}+(x^3e^y-1)\bar{j}$
13.2	$(3x^2-2xy+y)\bar{i}+(x-x^2-3y^2-4y)\bar{j}$
13.3	$(3x^2-2xy+y^2)\bar{i}+(2xy-x^2-3y^2)\bar{j}$
13.4	$(2xe^{x^2-y^2}-\sin x)\bar{i}+(\sin y-2ye^{x^2-y^2})\bar{j}$
13.5	$(20x^2-21x^2y+2y)\bar{i}+(3+2x-7x^3)\bar{j}$
13.6	$(y-\sin x)\bar{i}+(x-2y\cos y^2)\bar{j}$
13.7	$(5y+\cos x+6xy^2)\bar{i}+(5x+6x^2y)\bar{j}$
13.8	$(e^{x+y}-\cos x)\bar{i}+(e^{x+y}+\sin y)\bar{j}$
13.9	$(2x-3y^2+1)\bar{i}+(2-6xy)\bar{j}$
13.10	$(x^2-2y)\bar{i}+(y^2-2x)\bar{j}$
13.11	$(3x^2+6x^2y+3xy^2)\bar{i}+(2x^3+3x^2y)\bar{j}$
13.12	$xy^2\bar{i}+y(x^2+y^2)\bar{j}$
13.13	$2(3xy^2+2x^3)\bar{i}+3(2x^2y+y^2)\bar{j}$
13.14	$(x^2-4xy-2y^2)\bar{i}+(y^2-4xy-2x^2)\bar{j}$
13.15	$(\cos(x^2+y^2)+\sin x)\bar{i}+2y\cos(x+y^2)\bar{j}$

13.16	$\left(5xy^2 - x^3\right)\bar{i} + \left(5x^2y - y\right)\bar{j}$
13.17	$xe^{y^2}\bar{i} + \left(x^2ye^{y^2} + tg^2y\right)\bar{j}$
13.18	$\left(y^3 + \cos x\right)\bar{i} + \left(3xy^2 + e^y\right)\bar{j}$
13.19	$e^y\bar{i} + \left(\cos y + xe^y\right)\bar{j}$
13.20	$\left(3x^2 + 4y^2\right)\bar{i} + \left(8xy + e^y\right)\bar{j}$
13.21	$\left(y^2 + y\sec^2 x\right)\bar{i} + \left(2xy + tgx\right)\bar{j}$
13.22	$\left(3x^2y + 2y + 3\right)\bar{i} + \left(x^3 + 2x + 3y^2\right)\bar{j}$
13.23	$\left(\sin 2x - 2\cos(x+y)\right)\bar{i} - 2\cos(x+y)\bar{j}$
13.24	$\left(2x - 1 - \frac{y}{x^2}\right)\bar{i} - \left(2y - \frac{1}{x}\right)\bar{j}$
13.25	$\left(\frac{1}{x^2} + \frac{3y^2}{x^4}\right)\bar{i} - \frac{2y}{x^3}\bar{j}$

Decision of an exemplary embodiment.

Task 1. Calculate the 1-st type curvilinear integral $\int_L x \cdot arctg\frac{y}{x}dl$, where L - segment of the line $y = 4x$ from point A(0; 0) to point B(1; 4).

Solution.

For the calculation of the 1-st type curvilinear integral using a formula, which converts curvilinear integral in a definite integral (see reference material):

$$\int_L x \cdot arctg\frac{y}{x}dl = \int_0^1 x \cdot arctg\frac{4x}{x}\sqrt{1+16}\,dx = \sqrt{17}arctg\,4 \cdot \frac{x^2}{2}\Big|_0^1 = \frac{\sqrt{17}}{2}arctg\,4.$$

Answer: $\frac{\sqrt{17}}{2}arctg\,4$.

Task 2. Calculate the 2-nd type curvilinear integral $\int_L 2x \cdot (y-1)dx + x^2dy$, where L – the contour of the figure bounded by the parabola $y = x^2$ and straight line $y = 9$ at the positive direction of the contour.

Solution.

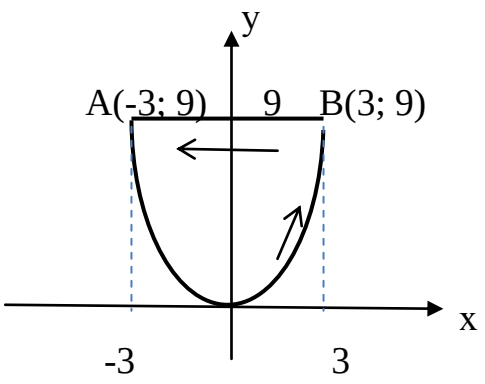

Figure 2

Contour L consists of two different lines, therefore split into two portions L_1 and L_2:

$$\int_{L_1} 2x\cdot(y-1)dx + x^2dy + \int_{L_2} 2x\cdot(y-1)dx + x^2dy =$$

$$= \left| \begin{matrix} L_1:\ y = x^2\ , dy = 2xdx,\ -3 \le x \le 3, \\ L_2: y = 9,\ \mathrm{dy} = 0,\ \ -3 \le x \le 3 \end{matrix} \right| =$$

$$= \int_{-3}^{3} 2x\left(x^2-1\right)dx + x^2 2xdx + \int_{-3}^{3} 2x(9-1)dx + x^2\cdot 0 = \int_{-3}^{3}\left(4x^2-2x\right)dx + 16\int_{-3}^{3} xdx = 0.$$

Answer: 0.

Task 3. Calculate the 2-nd type curvilinear integral by the Green formula in a closed contour

$$I = \oint_L \left(e^{x^2} - 5y^2 - 7\cdot\sin x^2\right)dx + \left(\sin y^2 + 2x^2 - \sqrt[3]{1+2y^2}\right)dy\ ,$$

where L – runnable in the positive direction of the contour, bounding area D, for which $0 < x < 1$ and $0 < y < x^2$.

Solution.

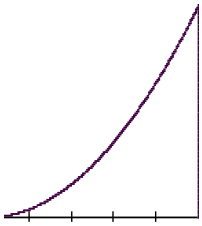

Figure 3

In the given integral functions $P = e^{x^2} - 5y^2 - 7\sin x^2$ and $Q = \sin y^2 + 2x^2 - \sqrt[3]{1+2y^2}$ are continuous together with its partial derivatives of the first order, therefore we use the Green formula.

$$I = \oint_L \left(e^{x^2} - 5y^2 - 7\cdot\sin x^2\right)dx + \left(\sin y^2 + 2x^2 - \sqrt[3]{1+2y^2}\right)dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy =$$

$$= \iint_D \left(4x - (-10y)\right)dxdy = 2\int_0^1 dx\int_0^{x^2}(2x+5y)dy = 2\int_0^1\left(2x^3 + 2{,}5x^4\right)dx = 2\ .$$

Answer: 2.

Task 4. Calculate the 1-st type surface integral $\iint_S (3x - y + z)ds$ on the surface S, where S – part of the plane (p): x-2y+z =2, separat- ed off by coordinate planes.

Solution.

From the equation of the plane:

$$z = 2 - x + 2y, \quad z_x' = -1, \quad z_y' = 2, \; ds = \sqrt{1 + z_x'^2 + z_y'^2}\,dxdy = \sqrt{6}dxdy.$$

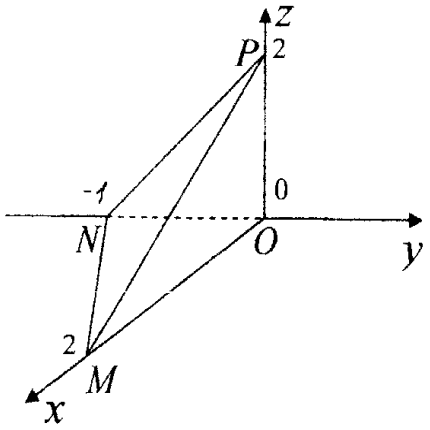

Figure 4

Reduces the calculation of surface integral to the calculation of the double integral over the region D, where D triangle MON is the projection of S on the plane XOY :

$$\iint\limits_S (3x - y + z)\,ds = \iint\limits_D (3x - y + 2 - x + 2y)\sqrt{6}dxdy = \iint\limits_D (2x + y + 2)\sqrt{6}dxdy =$$

$$= \sqrt{6}\int\limits_{-1}^{0} dy \int\limits_{0}^{2+2y} (2x + y + 2)\,dx = \sqrt{6}\int\limits_{0}^{2+2y} dy\left(x^2 + (y+2)x\right)\Big|_0^{2+2y} =$$

$$= \sqrt{6}\int\limits_{-1}^{0}\left(4 + 8y + 4y^2 + 2y + 2y^2 + 4 + 4y\right)dy = \sqrt{6}\int\limits_{-1}^{0}\left(6y^2 + 14y + 8\right)dy =$$

$$= \sqrt{6}\left(2y^3 + 7y^2 + 8y\right)\Big|_{-1}^{0} = 3\sqrt{6}.$$

Answer: $3\sqrt{6}$.

Task 5. Given the function (scalar field) $u(M) = \sqrt{x^2 + y^2 + z^2}$ and points M_1(-2; 3; 6), M_2(-1; 1; 4). Calculate the derivative of this function at the point M_1 in the direction of the vector $\vec{a} = \overrightarrow{M_1M_2}$.

Solution.

The derivative function *u(M)* in the direction of the vector $\vec{a}$ found by the formula

$$\frac{\partial u(M)}{\partial a} = u_x'(M)\cdot\cos\alpha + u_y'(M)\cdot\cos\beta + u_z'(M)\cdot\cos\gamma,$$

where $\cos\alpha, \cos\beta, \cos\gamma$ - the direction cosines of the vector $\vec{a} = \overrightarrow{M_1M_2}$, which are the coordinates of the unit vector $\overrightarrow{a^0} = \dfrac{\overrightarrow{M_1M_2}}{\left|\overrightarrow{M_1M_2}\right|} = \{\cos\alpha; \cos\beta; \cos\gamma\}$.

We find the components of the formula.

$$u'_x(M_1)=\left.\frac{x}{\sqrt{x^2+y^2+z^2}}\right|_{M_1}=\frac{-2}{\sqrt{4+9+36}}=-\frac{2}{7}\ ,$$

$$u'_y(M_1)=\left.\frac{y}{\sqrt{x^2+y^2+z^2}}\right|_{M_1}=\frac{3}{\sqrt{4+9+36}}=\frac{3}{7},$$

$$u'_z(M_1)=\left.\frac{z}{\sqrt{x^2+y^2+z^2}}\right|_{M_1}=\frac{6}{\sqrt{4+9+36}}=\frac{6}{7}.$$

$\overrightarrow{M_1M_2}=\{-1-(-2);1-3;4-6\}=\{1;-2;-2\}$, $\left|\overrightarrow{M_1M_2}\right|=\sqrt{1+4+4}=3$.

$\{\cos\alpha;\cos\beta;\cos\gamma\}=\left\{\frac{1}{3};\frac{-2}{3};\frac{-2}{3}\right\}$.

So, $u'_{\vec{a}}(M_1)=-\frac{2}{3}\cdot\frac{1}{3}+\frac{3}{7}\cdot\left(\frac{-2}{3}\right)+\frac{6}{7}\cdot\left(\frac{-2}{3}\right)=-\frac{20}{21}$.

Answer: $-\frac{20}{21}$.

Task 6. Find the direction and value of the greatest changes of the function (scalar field) $u(M)=x^2-2xy+3z^3$ at the point $M_0(0;2;1)$.

Solution.

The direction of the greatest changes of the function shows the gradient of this function (the field), and the magnitude of the greatest changes equal to the modulus of the gradient.

$\overrightarrow{grad}u(M_0)=u'_x(M_0)\vec{i}+u'_y(M_0)\vec{j}+u'_z(M_1)\vec{k}$.

$u'_x(M_0)=(2x-2y)\big|_{M_0}=-4$,

$u'_y(M_0)=-2x\big|_{M_0}=0$, $u'_z(M_0)=9z^2\big|_{M_0}=9$,

$\overrightarrow{grad}u(M_0)=-4\vec{i}+9\vec{k}$, $\left|\overrightarrow{grad}u(M_0)\right|=\sqrt{16+81}=\sqrt{97}$.

Answer: $\overrightarrow{grad}u(M_0)=-4\vec{i}+9\vec{k}$ - direction of the greatest changes of the function, $\left|\overrightarrow{grad}u(M_0)\right|=\sqrt{97}$ - the magnitude of the greatest changes.

Task 7. Find greatest density of circulation of the vector field $\vec{a}(M)=xy^2z^2\vec{i}+x^2yz^2\vec{j}+xyz\vec{k}$ at the point $M_0(1;2;1)$.

Solution.

Find greatest density of circulation of the vector field $\vec{a}(M)$ at the point M_0 it is achieved in the direction of the curl and is numerically equal to its module.

$$curl\,\vec{a}(M)=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ xy^2z^2 & x^2yz^2 & xyz\end{vmatrix}=\left(\frac{\partial(xyz)}{\partial y}-\frac{\partial(x^2yz^2)}{\partial z}\right)\vec{i}-$$

$$\left(\frac{\partial(xyz)}{\partial x}-\frac{\partial(xy^2z^2)}{\partial z}\right)\vec{j}+\left(\frac{\partial(x^2yz^2)}{\partial x}-\frac{\partial(xy^2z^2)}{\partial y}\right)\vec{k}=(xz-2x^2yz)\vec{i}-$$

$$(yz-2xy^2z)\vec{j}+(2xyz^2-2xyz^2)\vec{k}=(xz-2x^2yz)\vec{i}+(2xy^2z-yz)\vec{j}$$

$$curl\,\vec{a}(M_0)=-3\vec{i}+6\vec{j}\ ,$$

$$\left|curl\,\vec{a}(M_0)\right|=\sqrt{9+36}=3\sqrt{5}\ .$$

Answer: the greatest density of circulation of a vector field is achieved in the direction $curl\,\vec{a}(M_0)=-3\vec{i}+6\vec{j}$ and equals $3\sqrt{5}$.

Task 8. Determine whether the vector field $\vec{a}(M)=(2xy+z)\vec{i}+(x^2-2y)\vec{j}+x\vec{k}$ is solenoidal, potential or a harmonic.

Solution.

The vector field $\vec{a}(M)$ called solenoidal, if at every point $div\,\vec{a}(M)=0$. The field $\vec{a}(M)$ called potential, if at every point $curl\,\vec{a}(M)=0$.The field called harmonic, if field are solenoidal and potential.

$$div\,\vec{a}(M)=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}=2y-2+0=2y-2\neq 0,$$

therefore, field $\vec{a}(M)$ is not solenoidal.

$$curl\,\vec{a}(M)=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ 2xy+z & x^2-2y & x\end{vmatrix}=\left(\frac{\partial x}{\partial y}-\frac{\partial(x^2-2y)}{\partial z}\right)\vec{i}-$$

$$\left(\frac{\partial(x)}{\partial x}-\frac{\partial(2x+z)}{\partial z}\right)\vec{j}+\left(\frac{\partial(x^2-2y)}{\partial x}-\frac{\partial(2xy+z)}{\partial y}\right)\vec{k}=(0-0)\vec{i}-$$

$$(1-1)\vec{j}+(2x-2x)\vec{k}=0-\text{field potential.}$$

Answer: field $\vec{a}(M)$ potential, but not solenoidal and not harmonic.

Task 9. Using Stokes' formula to find the circulation of a vector field $\vec{a}(M)=(x+3y+2z)\vec{i}+(2x+z)\vec{j}+(x-y)\vec{k}$ along the contour of the triangle, formed by the intersection of the plane (p): 3x + 2y + 6z = 6 with the coordinate plane with the positive direction relatively the contour of the vector (the outer normal) to the plane (p). Make a drawing.

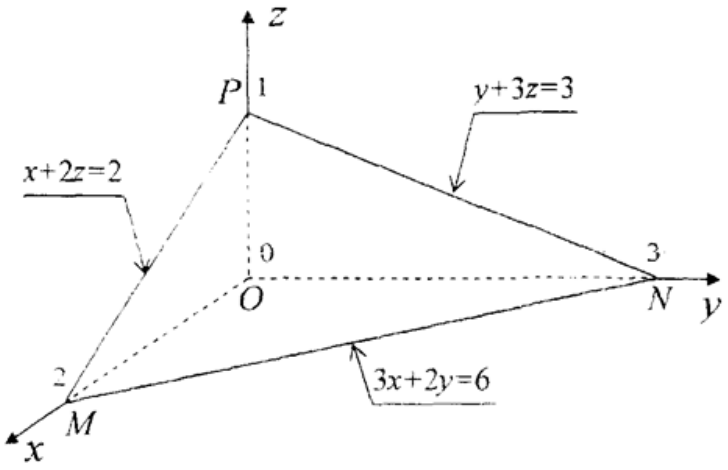

Figure 5

$S - \triangle MNP$,

$S_{YZ} - \triangle NOP$,

$S_{XZ} - \triangle POM$,

$S_{XY} - \triangle MON$.

Solution.

Stokes' formula in vector form: $C = \oint_L \vec{a} \cdot d\vec{r} = \iint_S curl\,\vec{a} \cdot d\vec{s}$, where $\iint_S curl\ \vec{a} \cdot d\vec{s}$ - the 2-nd type surface integral, $curl\,\vec{a} \cdot d\vec{s}$ - scalar product of the vectors $curl\,\vec{a}$ and $\vec{ds}$. Normal vector $\vec{n}$ (Figure) outside of the plane (P) forms an acute angle with the coordinate plane. Therefore, the sign in front of the surface integral in the beginning of calculations not changing.

We find the vector $curl\,\vec{a}$.

$$curl\,\vec{a}(M) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x+3y+2z & 2x+z & x-y \end{vmatrix} = \left|\text{see instructions to task 8}\right| = (-1-1)\vec{i} -$$

$(1-2)\vec{j} + (2-3)\vec{k} = -2\vec{i} + \vec{j} - \vec{k}$.

By definition, $\vec{ds} = dy\,d\vec{z} + i \quad d\vec{x}\,dz + j \quad \vec{c}$ - differential area in vector form.

Consequently,

$$C = \iint\limits_S curl\,\vec{a}\cdot d\vec{s} = \iint\limits_S \left(-2\vec{i} + \vec{j} - \vec{k}\right)\cdot\left(dydz\cdot\vec{i} + dxdz\cdot\vec{j} + dxdy\cdot\vec{k}\right) =$$

$$= \iint\limits_S (-2dydz + dxdz - dxdy = \left|\text{transform the surface integral to a double}\right| =$$

$$= -2\iint\limits_{S_{yz}} dydz + \iint\limits_{S_{xz}} dxdz - \iint\limits_{S_{xy}} dxdy = -2\int\limits_0^3 dy \int\limits_0^{1-\frac{y}{3}} dz + \int\limits_0^1 dx \int\limits_0^{2-2z} dz - \int\limits_0^2 dx \int\limits_0^{3-\frac{3x}{2}} dy =$$

$$= -2\left(y - \frac{y^2}{6}\right)\Bigg|_0^3 + \left(2z - z^2\right)\Big|_0^1 - \left(3x - \frac{3}{4}x^2\right)\Bigg|_0^2 = -2\left(3 - \frac{9}{6}\right) + (2-1) -$$

$$-\left(6 - \frac{3}{4}\cdot 4\right) = -5.$$

Answer: C = -5.

Task 10. Calculate the work of a vector field $\vec{a} = (x+y)\vec{i} - \left(y^2+2\right)\vec{j} + z\vec{k}$ along the curve L, given parametrically $L: \begin{cases} x = t^2, \\ y = 2, \\ z = t+1, \end{cases} \quad 0 \le t \le 1.$

Solution.

The work $A = \int\limits_L Pdx + Qdy + Rdz$ of the vector field along the line L calculate with the 2-nd type curvilinear integral (written in coordinate form). Thus,

$$A = \int\limits_L (x+y)dx - \left(y^2+2\right)dy + zdz = \begin{vmatrix} x = t^2, & dx = 2tdt \\ y = 2, & dy = 0 \\ z = t+1, & dz = dt \\ 0 \le t \le 1 \end{vmatrix} =$$

$$= \int\limits_0^1 \left(t^2+2\right)2tdt - (4+2)\cdot 0 + (t+1)dt = \int\limits_0^1 \left(2t^2 + 5t + 1\right)dt =$$

$$= \left(\frac{2t^4}{4} + \frac{5t^2}{2} + t\right)\Bigg|_0^1 = 4.$$

Answer: A = 4.

Task 11. Find the flux of the vector field $\vec{a} = (2z - x)\vec{i} + (x + 2z)\vec{j} + 3z\vec{k}$ through the part of the plane (p) given by the equation x + 4y + z = 4 and located in the first octant (normal forms an acute angle with the axis Oz).

Solution.

The equation of the plane (p) we give to the equation "in segments" and con

struct corresponding drawing. (p): $\frac{x}{4}+y+\frac{z}{4}=1.$ Part of the plane through which the calculated flux of the field is denoted by S (ΔMNP), $M(4;0;0)$, $N(0;1;0)$, $P(0;0;4)$ - coordinates of the vertices of the triangle.

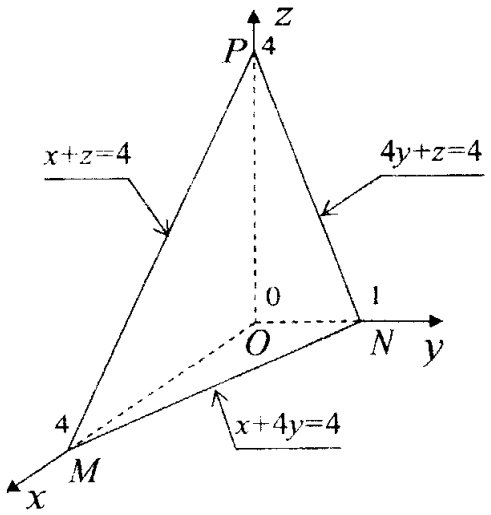

Figure 6

- S - ▯MNP
  S_{yz} –▯NOP
  S_{xz} –▯POM
  S_{xy} –▯*M*ON
  } projection of S on coordinate planes.

The unit vector $\vec{n}$ normal (not shown in figure) to the plane x + 4y + z = 4 has coordinates $\vec{n}(\cos\alpha;\ \cos\beta;\ \cos\gamma)=\vec{n}\left(\frac{1}{\sqrt{17}};\ \frac{4}{\sqrt{17}};\ \frac{1}{\sqrt{17}}\right)$, $\cos\gamma=\frac{1}{\sqrt{17}}>0$, consequently, the angle γ– acute angle with axis Oz, i.e. flux of the field passes through the outside of the ΔMNP.

By definition of the flux we get:

$$F=\oiint_S P(x,y,z)dydz+Q(x,y,z)dxdz+R(x,y,z)dxdy=$$

$$=\left|\text{the 2 - nd type surface integral in coordinate form}\right|=$$

$$\oiint_S (2z-x)dydz+(x+2z)dxdz+(3z)dxdy=$$

$$\left|\text{surface integrals change in a double}\right|=$$

$$= \iint\limits_S \big(2z - \big(4 - 4y - z\big)\big)dydz + \big(x + 2z\big)dxdz + 3\big(4 - x - 4y\big)dxdy =$$

$$= \iint\limits_{S_{yz}} \big(3z + 4y - 4\big)dydz + \iint\limits_{S_{xz}} \big(x + 2z\big)dxdz + 3\iint\limits_{S_{xy}} \big(4 - x - 4y\big)dxdy =$$

$$= \int\limits_0^1 dy \int\limits_0^{4-4y} \big(3z + 4y - 4\big)dz + \int\limits_0^4 dz \int\limits_0^{4-z} \big(x + 2z\big)dx + 3\int\limits_0^4 dx \int\limits_0^{4-z} \big(4 - x - 4y\big)dy =$$

$$= \int\limits_0^1 \left[\frac{3}{2}\cdot 16\big(1-y\big)^2 - 4^2\big(1-y\big)^2\right]dy + \int\limits_0^4 \left[\frac{\big(4-z\big)^2}{2} + 2z\big(4-z\big)\right]dz +$$

$$+3\int\limits_0^4 \left[\frac{1}{4}\big(4-x\big)^2 - \frac{\big(4-x\big)^2}{8}\right]dx = 8\int\limits_0^1 \big(1-y\big)^2\,dy + \frac{1}{2}\int\limits_0^4 \big(16 + 8z - 3z^2\big)dz + 3\int\limits_0^4 \frac{\big(4-x\big)^2}{8}dx =$$

$$= \frac{8}{3}\big(y-1\big)^3\Big|_0^1 + \left(8z + 2z^2 - \frac{z^3}{2}\right)\Big|_0^4 + \frac{3}{8}\frac{\big(x-4\big)^3}{3}\Big|_0^4 = 42\frac{2}{3}.$$

Answer: $F = 42\frac{2}{3}$.

Task 12. Calculate the flux of the vector field $\vec{a} = \big(x + z\big)\vec{i} + \big(2y - x\big)\vec{j} + z\vec{k}$ through the outer surface of the pyramid formed by the plane (p): x-2y + 2z = 4 and the coordinate planes, using the formula Ostrogradskii-Gauss.

Solution.

(p): x-2y+2z=4 $\rightarrow$ $\frac{x}{4} - \frac{y}{2} + \frac{z}{2} = 1$, coordinates of the vertices of the pyramid: $M\big(4;0;0\big)$, $N\big(0;-2;0\big)$, $P\big(0;0;2\big)$.

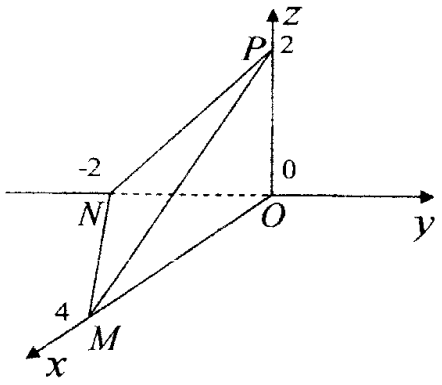

Figure 7

The total surface of the pyramid is a closed surface, then apply the formula Ostrogradskii-Gauss:

$$F = \iiint\limits_V \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right)dxdydz$$

We find:

$$\frac{\partial P}{\partial x}=\frac{\partial\left(x+z\right)}{\partial x}=1,$$

$$\frac{\partial Q}{\partial y}=\frac{\partial\left(2y-x\right)}{\partial y}=2,$$

$$\frac{\partial R}{\partial x}=\frac{\partial\left(z\right)}{\partial x}=1.$$

Thus, $F=\iiint\limits_V(1+2+1)dxdydz=4\iiint\limits_V dxdydz$.

Since the integral $\iiint\limits_V dxdydz$ equal to the volume of a rectangular pyramid MNPO, then for its calculation we use the formula of elementary mathematics: $F=4\iiint\limits_V dxdydz=4\cdot\frac{1}{6}\cdot2\cdot2\cdot4=\frac{32}{3}$.

Answer: $F=\frac{32}{3}$.

Task 13. Show that the vector field $\vec{a}=\left(2xy+3y^2+9y\right)\vec{i}+\left(x^2+6xy+9x\right)\vec{j}$ is a potential and find the potential of this field.

Solution.

Potential field has zero curl. We check it.

$$curl\,\vec{a}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 2xy+3y^2+9y & x^2+6xy+9x & 0\end{vmatrix}==(0-0)\vec{i}-$$

$$(0-0)\vec{j}+(2x+6y-(2x+6y))\vec{k}=0.$$

Consequently, field $\vec{a}$ is potential field. Potential $u=u(x,y)$ calculated by the formula $u\left(x;y\right)=\int\limits_{x_0}^{x}P\left(x;y_0\right)dx+\int\limits_{y_0}^{y}Q\left(x;y\right)dy+C.$ As the initial point $\left(x_0;\mathrm{y}_0\right)$ select the point $\left(0;0\right)$.

$$u\left(x;y\right)=\int\limits_0^x(2x\cdot0+3\cdot0+9\cdot0)dx+\int\limits_0^y\left(x^2+6xy+9x\right)dy+C=x^2y+3xy^2+9xy+\mathrm{C}.$$

Answer: $u\left(x;y\right)=x^2y+3xy^2+9xy+\mathrm{C}$.

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