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ALMATY UNIVERSITY OF POWER ENGINEERING AND TELECOMMUNICATIONS

Department of Physics

PHYSICS 2

Synopsis of lectures for students of specialty
5B070200 – Automation and control

Almaty 2016

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We present the synopsis of lectures on the discipline "Physics 2" for students of specialty 5B070200 - Automation and control.

Lectures "Physics 2" are another element of the system of methodological maintenance of educational process on the discipline and can be used as a handout for the lectures, as well as in the CGW on theoretical material while preparing for practical, laboratory classes and examinations.

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Reviewer: senior teacher Bukhina S.B.

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Introduction

Studying the physics course creates a fundamental base of engineering - technical knowledge and skills of graduates of higher technical school and forms their scientific outlook.

The main objectives of the course are:

1) formation of students' skills in using fundamental laws, classical physics theories and methods of physical research as the basis of their professional activity;

2) formation of students' creative thinking and scientific outlook, skills of independent cognitive activity, the ability to modulate physical situations. The course "Physics 2" studies such areas as "Maxwell's equations", "Physics of oscillations and waves," "Quantum physics and physics of atom", "Solid-state physics, nuclear and elementary particles".

Knowledge and skills acquired in the study of physics constitute the foundation needed in the study of technical disciplines.

1 Lecture №1. Electromagnetic induction. Maxwell's theory bases

Lecture's content: phenomena and law of electromagnetic induction, a basis of Maxwell's theory for an electromagnetic field are briefly described in the lecture.

1.1 Phenomenon and law of electromagnetic induction

Emergence of electromotive forces under the influence of magnetic fields is called *electromagnetic induction* (Faraday, 1831). Cause of electric current at the movement of the conductor in a magnetic field is explained by the action of Lorentz force arising at conductor's movement. Let's consider the scheme in figure 1.1 when two parallel wires of AB and CD are placed in a homogeneous magnetic field, perpendicular to the plane of the drawing and directed to us. The left wires AB and CD are closed, the right ones are opened. The conducting rod BC can freely slide along wires. When the rod moves the speed $\vec{\upsilon}$ to the right, electrons move with it. Each moving charge e^- in a magnetic field is affected by Lorentz force

$$\vec{F} = e\left[\vec{\upsilon}\,\vec{B}\right]. \qquad (1.1)$$

As a result electrons will start moving in the rod up, i.e. the current will go down. *It is induction current*. Lorentz force in the described experience plays a role of a foreign force exciting electric current.

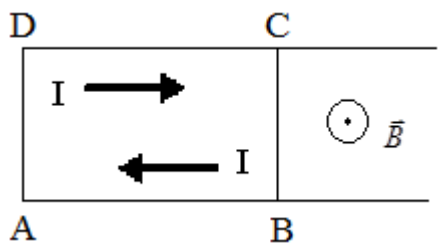


Figure 1.1

In this case a foreign electric field E is equal to:

$$\vec{E}^{\,f} = \left[\vec{\upsilon}\,\vec{B}\right]. \qquad (1.2)$$

The electromotive force created by this field is called the electromotive force of induction and is marked ε_i. In this case

$$\varepsilon_i = -\upsilon Bl\,. \qquad (1.3)$$

The minus sign is put because the foreign field is directed against a positive round of the contour determined by a vector $\vec{B}$ by the rule of the right screw. Thus,

$$\varepsilon_i = -\frac{d\Phi}{dt}. \tag{1.4}$$

So, this is Faradey's law. Faradey's law states: *that the inducted EMF in a circuit is directly proportional to the time rate of change of the magnetic flux Φ through the circuit.* Unit of measure EMF induction – volt (V).

Lenz rule defines the sign «minus» in the law of electromagnetic induction and is a consequence of the law of energy conservation: *induction current is always directed so that to counteract the reason causing it.*

1.2 Vortical electric field

If the contour is based in a variable magnetic field, the emergence of induction current testifies that the magnetic field changing in time causes emergence of foreign forces in a contour. By the nature they cannot be magnetic as they cannot actuate based charges in motion. The role of foreign forces, according to Maxwell's hypothesis, plays vortical electric field $\vec{E}$, which is generated by a variable magnetic field (appendix 1). Then circulation of a vector $\vec{E}$ by a motionless contour is equal to:

$$\oint_L \vec{E} d\vec{\ell} = -\frac{\partial \Phi}{\partial t}. \tag{1.5}$$

or

$$\oint_L \vec{E} d\vec{\ell} = -\int \frac{\partial \vec{B}}{\partial t} d\vec{S}. \tag{1.6}$$

In a differential form we will write it as

$$\left[\nabla \vec{E}\right] = -\frac{\partial \vec{B}}{\partial t}. \tag{1.7}$$

This electric field, as well as a magnetic field is vortical. It found application in the induction accelerator of electrons – the betatron.

1.3 Self-induction. Inductance

The electric current flowing in the closed contour creates round itself a magnetic field, which induction, by Biot-Savart-Laplace's law is proportional to the current. The magnetic flux Φ is linked to a contour, therefore it is proportional to the current of I in the contour:

$$\Phi = LI\ , \tag{1.8}$$

where the coefficient of proportionality L is called *inductance* of a contour.

If the force of current in a contour changes the magnetic flux linked to it changes as well; therefore, EMF will be induced in a contour. The emergence of EMF induction in the conducting contour at current's changing is called a self-induction.

Inductance unit of measurement is *henry* (Hn): 1 Hn=1 Wb/A.

Using formula (1.8), it is possible to receive expression for inductance of the solenoid which depends on number of turns of the solenoid *N*, its length ℓ, areas of *S* and magnetic permeability μ of substance the solenoid core is made of

$$L = \mu_0 \mu \frac{N^2 S}{\ell}\ . \tag{1.9}$$

If to apply to the self-induction phenomenon Faraday's law in case that the contour is not deformed and magnetic permeability does not change, we will receive

$$\varepsilon_i = -L\frac{dI}{dt}\ . \tag{1.10}$$

Characteristic manifestations of a self-induction take place at circuit's closing and disconnection of current in a circuit (appendix 1).

1.4 Mutual induction

Let's consider two motionless contours (1 and 2) are located rather close to each other. At change of current in one of the contours in the other contour arises EMF induction. This phenomenon is called as *mutual induction*. EMF, arising in contours 1 and 2, are equal to:

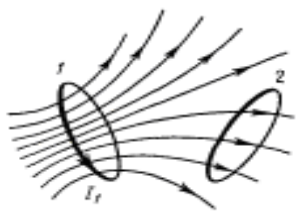


Figure 1.2

$$\varepsilon_1 = -\frac{d\Phi_1}{dt} = -L_{12}\frac{dI_2}{dt}; \quad \varepsilon_2 = -\frac{d\Phi_2}{dt} = -L_{21}\frac{dI_1}{dt}\ . \tag{1.11}$$

Coefficients of proportionality L_{21} and L_{12} are called *mutual inductance* of contours. The calculations confirmed by the experience show that for lack of ferromagnetic materials:

$$L_{21} = L_{12}\ . \tag{1.12}$$

It is possible to show that mutual inductance of two coils with the number of turns N_1, N_2, reeled up on the general toroidal core equals to:

$$L_{12} = L_{21} = \mu_0 \mu \frac{N_1 N_2}{\ell} S . \qquad (1.13)$$

Transformers operating is based on the phenomenon of mutual induction.

1.5 Energy of a magnetic field

The magnetic field is the carrier of energy. It is received through work spent by current for creation of a magnetic field. To change a magnetic flux at a size of $d\Phi$ it is necessary to make work $dA = Id\Phi = LIdI$. Then the work on creating of a magnetic flux will be equal to:

$$A = \int_0^I LIdI = LI^2/2 .$$

Therefore, magnetic energy of current can be defined as:

$$W = LI^2/2 . \qquad (1.14)$$

The energy of a magnetic field can be presented as a function of the sizes characterizing this field in surrounding space. It is possible to show that the formula (1.14) can be transformed into

$$W = \frac{\mu_0 \mu H^2}{2} V , \qquad (1.15)$$

where V is the volume of the space occupied by a magnetic field. Then the volumetric energy density equals:

$$w = \frac{W}{V} = \frac{\mu_0 \mu H^2}{2} . \qquad (1.16)$$

Formula (1.16) is valid both for homogeneous and non-uniform fields. It is true for variable fields as well. Mind that this expression is only for the environments where the dependence of vector $\vec{B}$ from $\vec{H}$ is linear, i.e. it concerns only para - and diamagnetics.

1.6 Displacement current

The theorem of circulation of a magnetic field intensity

$$\oint_L \vec{H} d\vec{\ell} = \Sigma I = \int_S \vec{j} d\vec{S} , \qquad (1.17)$$

becomes incorrect in case of alternating fields. For eliminating this discrepancy Maxwell entered the concept of displacement current (appendix 3). The density of the current of displacement is equal to $\vec{j}_S = \partial\vec{D}/dt,$ and the equation is:

$$\oint_L \vec{H}d\vec{\ell} = \int_S (\vec{j} + \partial\vec{D}/\partial t)d\vec{S}\,. \qquad (1.18)$$

1.7 System of Maxwell's equations

Having added the basic facts from electromagnetism area with establishment of magnetic actions of displacement currents, Maxwell wrote the system of the fundamental equations of electrodynamics. There are four equations in integrated and differential forms. They are:

Table 1.1

№	Integrated form	Differential form
1	$\oint_L \vec{E}d\vec{\ell} = -\int_S \frac{\partial\vec{B}}{\partial t}d\vec{S}$	$rot\,\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
2	$\oint_L \vec{H}\,d\,\vec{\ell} = \int_S \left(\vec{j} + \frac{\partial\vec{D}}{\partial t}\right)d\,\vec{S}$	$rot\,\vec{H} = \vec{j} + \frac{\partial\vec{D}}{\partial t}$
3	$\oint_S \vec{B}d\vec{S} = 0$	$di\upsilon\,\vec{B} = 0$
4	$\oint_S \vec{D}\,d\,\vec{S} = \int_V \rho\,d\,V$	$di\upsilon\,\vec{D} = \rho$
5 6 7	Material equations $\vec{D} = \varepsilon_0\varepsilon\,\vec{E}$ $\vec{B} = \mu_0\mu\,\vec{H}$ $\vec{j} = \gamma\,\vec{E}$	

The important conclusion follows from the first two equations: *variable electric and magnetic fields are inseparably linked with each other, forming a uniform electromagnetic field.*

The third and the fourth equations testify that *electric field has sources – electric charges, but magnetic charges are absent*; therefore Maxwell's equations are not symmetric with regard to electric and magnetic fields. This three equations (5, 6, 7) in table 1.1 are called material equations as they show individual properties of the environment.

The four equations must be supplemented by three equations reflected the relationship between the main characteristics of electromagnetic fields and currents. Here *D* is electric displacement, *E* - electric field, *B* - magnetic field, *H* - intensity of magnetic field, *j* - the vector of electric current density , *E** - electric field of foreign forces, γ -conductivity.

Let's note that Maxwell's equations cannot be removed. It is necessary to consider them as the main axioms of classical electrodynamics received by generalization of the experimental facts.

2 Lecture №2. Oscillatory processes

Lecture's content: mechanical and electromagnetic oscillations are briefly given in the lecture.

Periodic motion or fluctuation is a process periodically repeated in time. There are following fluctuations: free, compelled, self-oscillations, parametrical (appendix 5).

Harmonic oscillations are called periodic ones taking place according to cosine (or sine) law.

2.1 Free harmonic oscillations

Free harmonic oscillations can be presented as:

$$S(t) = A\cos(\omega_0 t + \varphi_0), \tag{2.1}$$

where A - *amplitude* of fluctuations (oscillations);

ω_0 - *own circular* frequency;

$(\omega_0 t + \varphi_0)$ - *phase of fluctuations;*

φ_0 - *an initial phase,* when $t = 0$.

Free harmonic oscillations are described by the uniform differential equation of the second order:

$$\ddot{S} + \omega_0^2 S = 0 \quad (\ddot{S} = d^2 S / dt^2). \tag{2.2}$$

The solution of equation (2.2) is the equation of harmonious periodic motion (2.1). Graphs of value change S, $\dot{S}$, $\ddot{S}$ from t are given in figure 2.1.

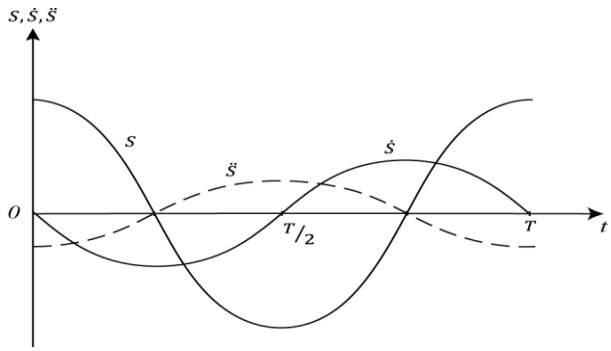


Figure 2.1 – Diagrams of value change S, $\dot{S}$, $\ddot{S}$ from *t*.

Oscillatory system is called an oscillator, and the system making harmonic oscillations is called *harmonious oscillator*. The differential equations and characteristics of various oscillators are given in appendix 5.

2.2 Energy of harmonic oscillations

Total energy W of mechanical oscillations is the sum of kinetic W_k and potential W_p energy. Using the formulae from the table of Item 1 (appendix 5) it is possible to write down:

$$W_k = \frac{m\upsilon^2}{2} = \frac{m\omega_0^2 A^2}{2}\sin^2(\omega_0 t + \varphi) = \frac{m\omega_0^2 A^2}{4}[1 - \cos(2\omega_0 t + 2\varphi)]; \qquad (2.3)$$

$$W_p = \frac{kx^2}{2} = \frac{kA^2}{2}\cos^2(\omega_0 t + \varphi) = \frac{m\omega_0^2 A^2}{4}[1 + \cos(2\omega_0 t + 2\varphi)] \qquad (2.4)$$

and

$$W = W_k + W_p = \frac{kA^2}{2} = \frac{m\omega^2 A^2}{2} = W_{k\max} = W_{p\max} = const. \qquad (2.5)$$

Graphs of dependence W_k, W_p and W from time are given in appendix 6. The total energy of an electromagnetic field W of an oscillatory contour is $W = W_e + W_m$. An electric field with energy W_e appears between the capacitor plates while it charges. There is a magnetic field with energy W_m (appendix 7) in the coil of inductance when the capacitor discharges. So, total energy W is experienced:

$$W = W_e + W_m = \frac{q_m^2}{2C} = \frac{LI_m^2}{2} = const. \qquad (2.6)$$

2.3 Addition of oscillations of identical and mutually perpendicular directions

Finding the law of the resulting oscillations of the system as *addition* of periodic motions is a process when this system simultaneously participates in several oscillatory processes.

A harmonic oscillation can be presented by means of *the rotating amplitude vector* (appendix 8).

If the system participates simultaneously in two oscillations the equations of which expressed:

$$x_1 = A_1\cos(\omega_0 t + \varphi_{01}),\ x_2 = A_2\cos(\omega_0 t + \varphi_{02}), \qquad (2.7)$$

The addition can be made by the *method of vector diagrams*, using the rotating amplitude vector (appendix 9). A projection of the resulting vector $\vec{A}$ on axis *x* is equal to the sum of projections of composed vectors

$$x = x_1 + x_2 .$$

The resulting amplitude, as seen in the drawing in appendix 9 is determined by the theorem of cosines:

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_{02} - \varphi_{01}), \tag{2.8}$$

and the initial phase φ_0 of the resulting fluctuation by a tangent:

$$tg\varphi_0 = \frac{A_1\sin\varphi_{01} + A_2\sin\varphi_{02}}{A_1\cos\varphi_{01} + A_2\cos\varphi_{02}} . \tag{2.9}$$

The equation of the resulting harmonic fluctuation is

$$x = A\cos(\omega_0 t + \varphi_0).$$

Let's consider the analysis of the equations in the appendix 10.

If periodic motions take place simultaneously along axis *x* and axis *y*, their equations can be written down in such a way:

$$x = A\cos\omega t, \; y = B\cos(\omega t + \varphi_0), \tag{2.10}$$

where φ_0 - difference of phases of two oscillations (shift of phases).

Such harmonic oscillations can take place if periodic harmonious signals are sent on to operating horizontal and vertical plates of an oscillograph. To define resulting oscillation trajectory it is necessary to exclude time from the equations (2.10). Then we will have the trajectory equation:

$$\frac{x^2}{A^2} - \frac{2xy}{AB}\cos\varphi_0 + \frac{y^2}{B^2} = \sin^2\varphi_0 . \tag{2.11}$$

Special cases resulting from this equation are given in appendix 11. Trajectories of the resulting movement are the complex curves called *by Lissazhu's figures* if frequencies of mutually perpendicular periodic oscillations are not identical.

2.4 Free damping and forced electromagnetic oscillations

2.4.1 Free damping oscillations

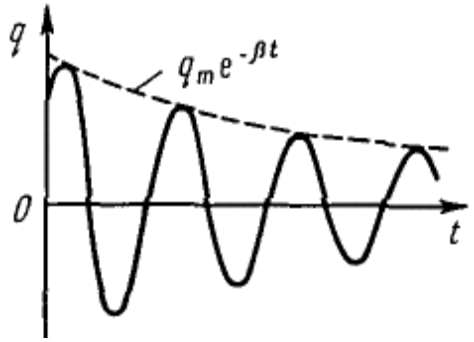


Figure 2.2

The periodic oscillations of ideal systems are given in appendix 5, table 5.1 where the energy stored by the system is not transformed into an other types of energy, i.e. there is no energy *dissipation* in it.

The real oscillatory contour unlike ideal one, (appendix 12) contains the resistor of resistance R connected in series with the condenser and the coil of inductance. In this case the differential equation of damping oscillation is expressed:

$$\ddot{q} + 2\beta\dot{q} + \omega_0^2 q = 0, \qquad (2.12)$$

where β - *coefficient of damping* $\beta = \dfrac{R}{2L}$.

The solution of the equation (2.12) is the equation of damping oscillations

$$q = q_{m0} e^{-\beta t} \cos(\omega t + \varphi_0), \qquad (2.13)$$

where constants q_{m0} (initial amplitude) and φ_0 (an initial phase) depend on initial conditions, i.e. from values q and $\dot{q}$ in a primary time point. The graphs of $q(t)$ is shown in figure 2.2.

The fluctuation period is equal to:

$$T = 2\pi / \sqrt{\omega_0^2 - \beta^2}$$

and frequency

$$\omega = \sqrt{\omega_0^2 - \beta^2}\,. \qquad (2.14)$$

We can write a time period and frequency of electromagnetic damping oscillations using the earlier entered values:

$$T = 2\pi / \sqrt{1/(LC)^2 - R^2/(4L^2)} \quad \text{and} \quad \omega = \sqrt{1/(LC)^2 - R^2/(4L^2)}\,. \qquad (2.15)$$

Relaxation time is called the time period while damping oscillation amplitude decreases in e times and is expressed: $\tau = 1/\beta$.

There is a logarithmic damping decrement as a quantitative characteristic of decreasing amplitude speed. *Logarithmic damping decrement is a natural logarithm of the ratio of amplitude values corresponding to points of time different for the period*.

$$\lambda = \ln \frac{A(t)}{A(t+T)} = \beta T = \frac{T}{\tau} = \frac{1}{N_e}, \qquad (2.16)$$

where N_e - a number of fluctuations while amplitude decreases in e time.

A real oscillatory contour is characterized *by quality* factor Q, equal to 2π multiplied by the energy ratio $W(t)$ of system fluctuations in any time point t to decreasing of the energy within the conditional period of damping oscillations

$$Q = 2\pi \frac{W(t)}{W(t) - W(t+T)}.$$

So, the contour quality factor is equal to:

$$Q = \frac{\pi}{\lambda} = \pi N_e. \qquad (2.17)$$

2.4.2 Forced electromagnetic oscillations. Resonance.

Forced electromagnetic oscillations take place if variable EMF is connected in series in contour with elements $R - L - C$:

$$\varepsilon = \varepsilon_m \cos\omega t.$$

In this case the equation of an oscillatory contour is:

$$L\frac{dI}{dt} + RI + \frac{q}{C} = \varepsilon_m \cos\omega t$$

or

$$\ddot{q} + 2\beta\dot{q} + \omega_0^2 q = (\varepsilon_m / L)\cos\omega t. \qquad (2.18)$$

The particular solution of this equation is equal to:

$$q = q_m \cos(\omega t - \psi), \qquad (2.19)$$

where q_m - charge amplitude on the condenser;

ψ - difference of phases between charge oscillations and external EMF.

Differentiating (2.19) on t we can receive the current in a contour:

$$I = \dot{q} = \omega q_m \sin(\omega t - \psi) = \omega q_m \cos(\omega t - \psi + \pi/2). \qquad (2.20)$$

We can show from vector diagram for real oscillatory contour (appendix 12) that:

$$q_m = \frac{\varepsilon_m}{\omega\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

We can see the charge amplitude q_m (and phase φ) of the forced periodic oscillations is defined by EMF frequency at the given values of ε_m, R, L, C. The less is the difference of own frequency ω_0 and frequency of the variable EMF the more amplitude q_m is. *Resonance is the phenomenon of sharp increase of forced periodic oscillations amplitude at a certain value of external frequency influence*. External influence (EMF) frequency is a *resonant frequency* when the resonance takes place (in detail in appendix 13).

3 Lecture №3. Wave processes

Lecture's content: the process of periodic oscillations propagation, elastic waves and dispersion of waves are described in the lecture.

Lecture objective: to study wave processes.

3.1 Elastic wave and equation of a wave

Extending periodic motion is called wave process. *An elastic wave is the propagating mechanical process of indignation in an elastic medium*. Periodic motion can spread in elastic medium from a particle to a particle with some speed due to interaction between them if fluctuation in the environment is excited. That doesn't make particles move but only fluctuate at balance position. Therefore, the main property of waves is to transfer energy without transferring a substance.

There are *longitudinal and transverse waves* depending on direction of particles movements near their spread balance positions (along or across the direction of wave propagation).

Transverse waves extend in media resisting shift (in solids). Longitudinal waves propagate in environments resisting to compression and stretching (in liquids, gases and solids).

Locus fluctuating in an identical phase is called *a wave surface*. *It reached by indignation at the given time point is called the front of a wave*. There are many wave surfaces but the wave front is one. Wave surfaces are not mobile, and the wave front moves. A wave can be flat or spherical depending on a form of wave surface (the front of a wave).

The wave is characterized by the following parameters: λ - wave*length* of *wave*, the distance passed per a period of fluctuation time; T - *period*, time of one fluctuation; ν - *frequency*, number of fluctuations per a unit of time. There is dependence between these parameters expressed by the formulas:

$$\lambda = \upsilon \cdot T, \ \upsilon = \lambda \nu .$$

In general, the wave equation represents the function of time and three spatial coordinates. The shift of the environment particle from balance position ξ is a function of coordinate *x* and time t, i.e. $\xi = f(x,t)$ at distribution of indignation along an *x* axis.

At some distance x from the source periodic oscillations of particles will lag behind in time $\tau = x/\upsilon$ (υ - speed of wave propagation) if fluctuations of the points lying on the plane *x*=0 are described by the function $\xi(0,t) = A\cos(\omega t + \varphi_0)$. So, *the equation of fluctuations* of the particles lying on the plane *x* is expressed:

$$\xi(x,t) = A\cos[\omega(t - x/\upsilon) + \varphi_0].$$

The wave number is used for the characteristic of waves*:*

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\upsilon T} = \frac{\omega}{\upsilon}, \tag{3.1}$$

it shows how many lengths of waves there are within a piece of length 2π.

Therefore, it is possible to write down:

$$\xi(x,t) = A\cos(\omega t - kx + \varphi_0), \tag{3.2}$$

where φ_0 is an initial phase of a wave;

$(\omega t - kx + \varphi_0)$ - phase of a flat wave.

The equation (3.2) is the equation of the flat running wave propagating along axes *x* in not absorbing energy medium.

Running waves are called the waves transferring energy in space. The speed υ in the equation (3.1) is *the phase speed* of a wave, i.e. wave propagation speed with a certain value of a wave phase.

If the energy is absorbed by environment we'll have the following equation

$$\xi = A_0 e^{-\gamma x} \cos(\omega t - kx + \varphi_0),$$

where γ is coefficient of damping of a wave.

A wave vector $\vec{k}$ is introduced when a flat can travel wave in any direction characterized by a single vector $\vec{n}$, perpendicular to the front of a wave:

$$\vec{k} = k\vec{n} = \frac{2\pi}{\lambda}\vec{n}.$$

In this case the equation of a flat wave is expressed:

$$\xi(\vec{r},t) = A\cos(\omega t - \vec{k}\vec{r} + \varphi_0),$$

where $\vec{k}\vec{r} = k_x \cdot x + k_y \cdot y + k_z \cdot z$.

3.2 Wave equation

There are equations of wave processes as generalized expressions of wave similar to the main equation of dynamics describing all possible movements of a material point. They are differential equations in the particular derivatives connecting changes of functions characterizing a wave in time and space.

They are called *the wave equations*. An example of a wave equation of the flat running wave along axis *x* is:

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{\upsilon^2}\frac{\partial^2 \xi}{\partial t^2}. \tag{3.3}$$

The equation of a flat wave (3.2) is the solution of the wave equation (3.3). Generally it look like (3.4) when the shift is the function of four variables:

$$\nabla^2 \xi = \frac{1}{\upsilon^2}\frac{\partial^2 \xi}{\partial t^2}, \tag{3.4}$$

where

$$\nabla^2 \xi = \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2}.$$

3.3 Energy of a wave. Umov's vector

The kinetic energy of medium volume ΔV where an elastic wave travels is equal to (appendix 14):

$$\Delta W_k = \frac{1}{2}\rho A^2 \omega^2 \sin^2(\omega t - kx)\Delta V,$$

where *A*– amplitude of the wave propagating along axes *x*;
ρ – environment density;
ω – frequency.
The considered volume also has potential energy (appendix 14):

$$\Delta W_p = \frac{1}{2}\rho A^2 \omega^2 \sin^2(\omega t - kx)\Delta V.$$

The total energy is equal to the sum ΔW_k and ΔW_p

$$\Delta W = \Delta W_k + \Delta W_p = \rho \omega^2 A^2 \sin^2(\omega t - kx)\Delta V. \tag{3.5}$$

Energy density can be calculated if this energy is divided by the contained volume:

$$w = \frac{\Delta W}{\Delta V} = \rho\omega^2 A^2 \sin^2(\omega t - kx).$$

Density of energy changes in each point of the environment according to the law of square sine, therefore the average density of energy is equal to:

$$\langle w \rangle = \frac{1}{2}\rho\omega^2 A^2. \tag{3.6}$$

Energy dW, transferred by a wave through some surface by unit of time dt, is called *energy stream* $d\Phi$ through this surface:

$$d\Phi = \frac{dW}{dt}.$$

The energy stream can be various in different points. That's why the concept of energy stream density is introduced. It is an energy stream through a single platform, perpendicular to the direction of energy transfer:

$$j = \frac{d\Phi}{dS_\perp} = \frac{dW}{dt \cdot dS_\perp}. \tag{3.7}$$

Energy transfer speed by a wave for harmonious (sinusoidal) waves is equal to phase speed υ. Energy dW inside a slant cylinder (Figure 3.1) with the basis of area dS and the forming length $\upsilon\, dt$,

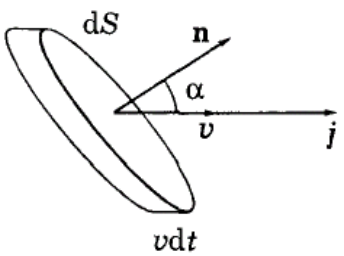


Figure 3.1

$$dW = w\upsilon dt dS\cos\alpha = w\upsilon dt dS_\perp.$$

We will receive the formula for energy stream density if this formula is put in (3.7)

$$j = w \cdot \upsilon.$$

We can use *Umov's vector* $\vec{j}$ for determination of stream density and its direction:

$$\vec{j} = w \cdot \vec{\upsilon}, \tag{3.8}$$

where $\vec{\upsilon} = \frac{\omega}{k}\vec{n}$ is a speed vector. It is normal to a wave surface in this place and its module is equal to the phase speed of a wave.

The average time value of energy stream density is called wave *intensity*:

$$I = \left|\left\langle \vec{j} \right\rangle\right| = \langle w \rangle \upsilon = \frac{1}{2}\rho\omega^2 A^2 \upsilon .$$

3.4 Wave package. Group speed. Dispersion of waves

Let's consider the simplest group of waves - quasisinusoidal wave. It travels in the linear medium as a superposition of two flat waves with close frequencies:

$$\xi_1 = A_0 \cos(\omega t - kx) \text{ and } \xi_2 = A_0 \cos[(\omega + d\omega)t - (k + dk)x],$$

where $k = \dfrac{\omega}{\upsilon_1}$, $(k + dk) = (\omega + d\omega)/\upsilon_2$, $d\omega \langle\langle \omega$, $dk \langle\langle k$.

The result of addition of fluctuations is:

$$\xi = 2A_0 \cos[(td\omega - xdk)/2]\cos(\omega t - kx).$$

This wave is different in amplitude from a sinusoidal wave:

$$A = 2A_0 \cos[(td\omega - xdk)/2]. \tag{3.9}$$

Its amplitude is slowly changing function of coordinate *x* and time *t*. So, the speed of a spreading package is the speed of a point where amplitude A has any maximum value (center of a wave package) $A = 2A_0$. As in this point the energy density is maximum so, *group speed is the speed of wave energy transferring*.

The center of a wave package moves according to the law $td\omega - xdk = const$, therefore group speed is equal to:

$$u = \frac{dx}{dt} = \frac{d\omega}{dk}.$$

As $\omega = \upsilon k$, $k = 2\pi/\lambda$, $dk = 2\pi d\lambda/\lambda^2$ (λ - wavelength),

so,
$$u = \frac{d\omega}{dk} = \upsilon + k\frac{d\upsilon}{dk} = \upsilon - \lambda\frac{d\upsilon}{d\lambda}. \tag{3.10}$$

Equation (3.10) shows that group speed can be both less, and more than phase speed υ that is connected with dependence of phase speed on wavelength (frequency), i.e. on medium properties.

The dependence of phase speed of monochromatic waves on frequency (or wavelength) is called dispersion.

In the absence of dispersion a wave conserves its form.

4 Lecture №4. Electromagnetic waves

Lecture's content: differential equation, energy and intensity of electromagnetic waves are briefly given in this lecture.

Lecture objective: to study electromagnetic waves.

Analysis of Maxwell's theory shows that the time-varying magnetic field generates a varying electric field and time-varying electric field produces a magnetic field. If a vortex electric field is initiated in a point of space a sequence of mutual transformations of electric and magnetic fields occurs in surrounding space of charges, i.e a variable magnetic field spreading in time and space is emerged. This process is periodic and represents *an electromagnetic wave*.

4.1 Differential equation of an electromagnetic wave and its properties

Maxwell's equations for an electromagnetic field far from free electric charges generating it $(\rho = 0)$ and macroscopic currents $(j = 0)$ are expressed:

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t},\ rot\vec{H} = \frac{\partial \vec{D}}{\partial t};$$

$$div\vec{D} = 0, \qquad div\vec{B} = 0.$$

Taking into account $\vec{D} = \varepsilon_0 \varepsilon \vec{E}$ and $\vec{B} = \mu_0 \mu \vec{H}$, these equations can be written in the following form:

$$rot\vec{E} = -\mu_0 \mu \frac{\partial \vec{H}}{\partial t};\ rot\vec{H} = \varepsilon_0 \varepsilon \frac{\partial \vec{E}}{\partial t};\ div\vec{E} = 0,\ div\vec{H} = 0, \tag{4.1}$$

where μ and ε - constants of permeability of the medium.

Vectors $\vec{E}$ and $\vec{H}$ as well as their components on coordinate axes will not depend on coordinates y and z in the case of flat wave travelling along the positive direction of axis x. In this case it is possible to have, two independent groups of the equations from equations (4.1) presented in projections on coordinate axis:

$$\frac{\partial E_y}{\partial x} = -\mu_0 \mu \frac{\partial H_z}{\partial t}, \quad \frac{\partial H_z}{\partial x} = -\varepsilon_0 \varepsilon \frac{\partial E_y}{\partial t}; \tag{4.2}$$

$$\frac{\partial E_z}{\partial x} = \mu_0 \mu \frac{\partial H_y}{\partial t}, \quad \frac{\partial H_y}{\partial x} = \varepsilon_0 \varepsilon \frac{\partial E_z}{\partial t} \tag{4.3}$$

and equations

$$\mu_0\mu\frac{\partial H_x}{\partial t}=0\,,\ \varepsilon_0\varepsilon\frac{\partial E_x}{\partial t}=0\,. \tag{4.4}$$

Equations (4.2) can be given as:

$$\frac{\partial^2 E_y}{\partial x^2}=\varepsilon_0\varepsilon\mu_0\mu\frac{\partial^2 E_y}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 H_z}{\partial x^2}=\varepsilon_0\varepsilon\mu_0\mu\frac{\partial^2 H_z}{\partial t^2}\,. \tag{4.5}$$

The comparison of equations (4.5) with elastic wave equation (3.3) shows that equations (4.5) are the electromagnetic wave equations.

Solutions of these equations are the functions:

$$E_y=E_m\cos(\omega t-kx+\varphi_1) \text{ and } H_z=H_m\cos(\omega t-kx+\varphi_2)\,. \tag{4.6}$$

The main properties of electromagnetic waves follow from equations (4.2) - (4.6).

4.1.1 Equations (4.4) show that E_x and H_x depend neither on *x*, nor *t*. Therefore $E_x=H_x=0$ for a variable field of a flat wave and vectors $\vec{E}$ and $\vec{H}$ are perpendicular to the direction of wave propagation, i.e. electromagnetic wave are *transverse.*

4.1.2 The comparison of equations (4.5) with ones (3.3) shows that *the phase speed* of an electromagnetic wave depends on medium properties:

$$\upsilon=\frac{1}{\sqrt{\varepsilon_0\mu_0\varepsilon\mu}}\,. \tag{4.7}$$

Speed of an electromagnetic wave in vacuum $(\varepsilon=\mu=1)$,

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}=3\cdot10^8\,m/s\,.$$

4.1.3 Equations (4.5) testify that vectors $\vec{E}$ and $\vec{H}$ of electromagnetic wave fields are mutually perpendicular: $\vec{\upsilon},\vec{E},\vec{H}$ form a right screw system (figure 4.1).

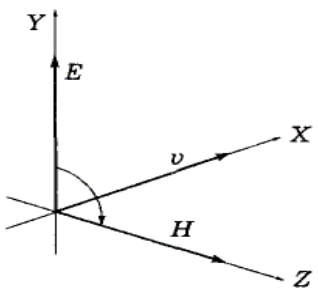


Figure 4.1

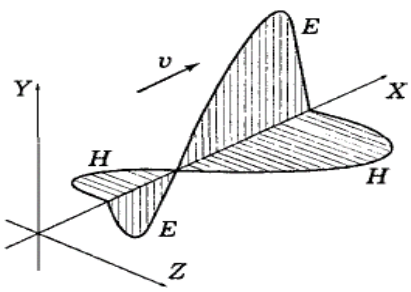


Figure 4.2

4.1.4 The initial phases in equations (4.6) are equal: $\varphi_1 = \varphi_2$ and $\varepsilon_0 \varepsilon E_m^2 = \mu_0 \mu H_m^2$.

Therefore, fluctuations of vectors $\vec{E}$ and $\vec{H}$ occur *sinphase* (in one phase) (figure 4.2) and their instant values are connected by a ratio:

$$\sqrt{\varepsilon_0 \varepsilon} E = \sqrt{\mu_0 \mu} H \ . \tag{4.8}$$

The equations of flat wave propagating in homogeneous isotropic medium in a vector form are expressed:

$$\vec{E} = \vec{E}_m \cos(\omega t - kx + \varphi), \quad \vec{H} = \vec{H}_m \cos(\omega t - kx + \varphi).$$

4.1.5 In each point of an electromagnetic field vectors $\vec{E}$ and $\vec{H}$ make harmonic oscillations of identical frequency (wave frequency), therefore an electromagnetic wave is a *monochromatic* wave.

4.2 Energy of an electromagnetic wave. Poynting's vector

Energy transfer is connected with an electromagnetic wave. Energy density of an electromagnetic field is equal to the sum of energy density of electric and magnetic fields in an isotropic medium:

$$w = \frac{\varepsilon_0 \varepsilon E^2}{2} + \frac{\mu_0 \mu H^2}{2} \ .$$

The ratio between field vectors $\vec{E}$ and $\vec{H}$ of an electromagnetic wave (4.8) shows that the energy volume density of electromagnetic waves is expressed:

$$w = \varepsilon_0 \varepsilon E^2 = \mu_0 \mu H^2 = \sqrt{\varepsilon_0 \varepsilon \mu_0 \mu} EH = \frac{\sqrt{\varepsilon \mu}}{c} EH = \frac{EH}{\upsilon}, \tag{4.9}$$

where υ - speed of a wave (4.7).

We will receive energy stream density S if expression (4.9) is multiplied by υ

$$S = w\upsilon = EH \ . \tag{4.10}$$

As vectors $\vec{E}$ and $\vec{H}$ are mutually perpendicular and form a right screw system (figure 4.1) with the direction of propagation, it is possible to express formula (4.10) as:

$$\vec{S} = \left[\vec{E}\vec{H}\right]. \tag{4.11}$$

Vector $\vec{S}$ is called *Poynting's vector*. It is directed towards the propagation of an electromagnetic wave and its module is equal to energy transferred by an electromagnetic wave per a unit of time through a single platform, perpendicular to the direction of wave propagation.

The energy density of a travelling harmonious electromagnetic wave is:

$$S = \sqrt{\varepsilon\varepsilon_0 / \mu_0\mu} E_m^2 \cos^2(\omega t - kx).$$

Wave intensity I is equal to average value of energy stream density:

$$I = \left|\left\langle \vec{S} \right\rangle\right| = \sqrt{\varepsilon\varepsilon_0 / \mu_0\mu} / E_m^2 / 2, \tag{4.12}$$

i.e average value of a square cosine is 1/2.

4.3 Radiation of electromagnetic waves

Process of excitement of electromagnetic waves by any system in a space is called *wave radiation* and the system is called *radiating system*.

According to classical electrodynamics electromagnetic waves are excited by moving electric charges with acceleration. An elementary radiating system is an electric dipole and its moment $\vec{p}$ changes in time. Such dipole is called *an elementary vibrator*. If a radiating system is electric neutral and its sizes are small in comparison with radiated wave length λ we have *a wave zone* ($r \rangle\rangle \lambda$, where r is the distance from the system). In this case a radiation field is close to a radiation field of an oscillator with the electric moment similar to the radiating system.

Moment $\vec{p}$ of *a linear harmonious oscillator* changes in time according to the law

$$\vec{p} = \vec{p}_m \cos\omega t. \tag{4.13}$$

In homogeneous isotropic medium duration of wave passing to the remote points from a dipole in distance r and the phase of fluctuations are identical. Therefore the wave front is spherical in a wave zone. Wave amplitude decreases with the increase of distance r from the dipole:

$$E_m \sim H_m \sim \frac{1}{r} \sin\theta, \tag{4.14}$$

where θ - an angle between an axis of a dipole and a radius of vector $\vec{r}$ (figure 4.3).

Figure 4.3 shows that vector $\vec{E}$ is directed to the meridian in each point of the wave surface and vector $\vec{H}$ - tangentially to the parallel, forming the right screw rule with Poynting's vector $\vec{S}$. Intensity of a wave is

$$I \sim \frac{1}{r^2}\sin^2\theta . \qquad (4.15)$$

This dependence is represented by means of *a directivity diagram* of dipole radiation (figure 4.4). Expression (4.14) and the given diagram show that the dipole radiates as much as possible in the equatorial plane $\left(\theta = \frac{\pi}{2}\right)$ but along the axis $(\theta = 0)$ it does not radiate. Power of radiation depends on the frequency of oscillations and is proportional ω^4.

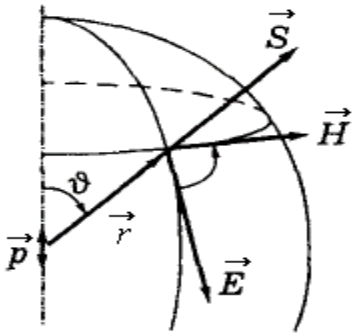


Figure 4.3

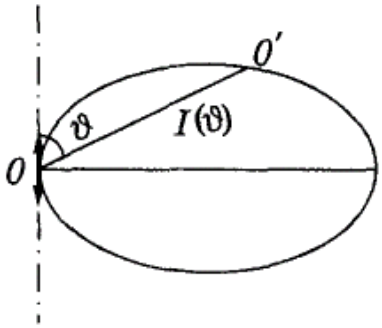


Figure 4.4

5 Lecture №5. Wave optics

Lecture's content: basic concepts of wave optics are given in the lecture.
Lecture objective: to study basic concepts of wave optics.

5.1 Light wave

Coincidence of electromagnetic waves speed value in vacuum

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \cdot 10^8 \, m/s ,$$

with the speed of light measured by astronomical methods served as the basis for the conclusion that *light is an electromagnetic wave*. All properties of electromagnetic waves are the same for the light.

$$\sqrt{\varepsilon\mu} = n \qquad (5.1)$$

is called a refraction index. The speed of electromagnetic waves or light in a medium is:

$$\upsilon = \frac{c}{n}. \tag{5.2}$$

Permeability constant μ does not differ practically from one unit for the majority of transparent substances, therefore

$$n = \sqrt{\varepsilon}. \tag{5.3}$$

The length of a light wave in a medium is equal to:

$$\lambda = \frac{\lambda_0}{n},$$

where λ_0 - wavelength in vacuum.

Light intensity I is defined by Poynting's vector $\vec{S}$ (4.11), therefore

$$I \sim nE_m^2 = nA^2, \tag{5.4}$$

i.e. it is proportional to the medium refraction index and a square of a light wave amplitude.

The vector of electric field intensity is used as a light vector since physiological, photochemical, electric and other actions of light are caused by oscillations of the electric vector.

Light waves are transverse, but in *natural* light (emitted by usual sources) oscillations occur in the most various directions, perpendicular *to a ray* – a line along which light energy extends. Existence of these oscillations is explained by radiation of a shining body consisting from the waves emitted by its atoms. Process of radiation of a separate atom lasts near 10^{-8} s. During this time *wave trains* manage to be formed (discontinuous radiation of atoms in the form of separate impulses). Wave trains impose on each other and form the light wave emitted by a body. The plane of oscillations for each train is oriented in a random way, therefore in the resulting wave the oscillations of various directions are equally probable.

5.2 Light interference. Coherence

The phenomenon *of light interference* consists of mutual amplification of light waves in one point of space and attenuation – in others while imposing. A necessary condition of waves interference is their *coherence*.

Coherence is coordinated passing of several oscillatory or wave processes in time and space.

Monochromatic waves satisfy the condition *of specified frequency* and *constant amplitude*. The real source does not give strictly monochromatic light (5.1) since radiations of separate atoms are not in harmony with each other, because the

phases of their waves are shifted on random values. Therefore, coherence of waves in time and length is necessary for a steady picture of interference.

The coherence is that the difference of phases of two oscillations remains invariable in one point of space; it is called temporary coherence. The period of time during which the initial phase will take the value different from the primary value on π is called *time of coherence*.

The coherence is called spatial when the difference of oscillation phases in different points of a wave surface is constant. The distance is called *coherence length*, if the reached values of phase difference is equal to π.

Oscillations created by a wave are not coherent outside of coherence time and coherence length.

Therefore, the necessary condition of wave interference is the equality of frequencies and constant difference of phases (coherence of waves) in time. It is possible to create coherent light oscillations by means of usual light sources only by one way – by "splitting" of the same light wave on two ones and then to combine them.

The superposition of light waves conforms to the superposition principle in each point of space, therefore resulting intensity is equal to $\vec{E} = \vec{E}_1 + \vec{E}_2$. It is possible to apply the method of vector diagrams if vectors $\vec{E}_1$ and $\vec{E}_2$ fluctuate in one direction (appendix 12). Using expressions (2.5) and (5.4) we can find the intensity of the resulting wave:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\varphi_2 - \varphi_1). \qquad (5.5)$$

Intensity $I \rangle I_1 + I_2$ in space points where $\cos(\varphi_2 - \varphi_1) \rangle 0$, and intensity $I \langle I_1 + I_2$ where $\cos(\varphi_2 - \varphi_1) \langle 0$.

The difference of oscillations phases in the point of supervision of an interferential picture is:

$$\Delta\varphi = \varphi_2 - \varphi_1 = \omega\left(\frac{S_2}{\upsilon_2} - \frac{S_1}{\upsilon_1}\right) = \frac{2\pi}{\lambda_0}(S_2 n_2 - S_1 n_1) = \frac{2\pi}{\lambda_0}(L_2 - L_1) = \frac{2\pi}{\lambda_0}\Delta,$$

where S_1, S_2 - the ways passed by two coherent waves from the point of their division to the point of supervision of an interferential picture, υ_1 and υ_2 - phase speeds of these waves in media with refraction index n_1 and n_2, λ_0 - wavelength in vacuum.

The product of geometrical length S of light wave path by the refraction index of a medium is called *optical path length* L, and $\Delta = L_2 - L_1$- *optical path difference*.

Difference of phases $\Delta\varphi$ and optical path length Δ are connected by the ratio:

$$\Delta = \frac{\lambda}{2\pi}\Delta\varphi. \qquad (5.6)$$

Using expression (5.6) it is possible to receive conditions of maximum and minimum for the resulting oscillation intensity:

$I_{\max} = I_1 + I_2$ at $\Delta\varphi = 2m\pi$, where $m = 0,1,2,...$ and $\Delta = 2m\frac{\lambda}{2} = k\lambda$, $I_{\min} = |I_1 - I_2|$ at $\Delta\varphi = (2m+1)\pi$, where $m = 0,1,2,...$ and $\Delta = (2m+1)\frac{\lambda}{2}$.

5.3 Dispersion of light

Dispersion of light is called dependence of refraction index on frequency. Dispersion of light is a result of electromagnetic waves interaction with optical electrons as a part of a substance. Optical electrons are the shared electrons weakly connected with atoms and molecules. That is why Maxwell's electromagnetic theory could not explain this phenomenon. According to Lorentz's classical electronic theory dispersion is the result of interaction of electromagnetic waves with a substance. Optical electrons (harmonious oscillators) make compelled fluctuations radiating their own electromagnetic waves forming in total a secondary wave.

The electron is affected by three forces: quasi-elastic, returning force $\vec{F} = -m\omega_0^2\vec{r}$, caused by its interaction with a nucleus and other electrons; resistance force $\vec{F} = -2m\beta\vec{\upsilon}$, expressing loss of energy due to the radiation and a transition of a part of electron oscillatory energy into atom translational; compelling force caused by the action of electric field of the falling electromagnetic wave.

In such conditions the differential equation of compelled oscillations can be presented in the form:

$$\ddot{\vec{r}} + 2\beta\dot{\vec{r}} + \omega_0^2\vec{r} = -\frac{e}{m}\vec{E}_m\cos\omega t,$$

where $\vec{r}$ - shift of an optical electron;

m and ω_0 - its mass and own frequency of oscillations;

β - coefficient of attenuation of free oscillations of an electron;

$\vec{E}_m$ and ω - vector of intensity amplitude and frequency of a variable field.

If a medium does not absorb light $(\beta = 0)$, the amplitude of compelled oscillations is

$$\vec{r}_m = \frac{e\vec{E}_m}{m(\omega_0^2 - \omega^2)}.$$

Polarization of a medium at shifting optical electrons is expressed:

$$\vec{P} = -n_0 e\vec{r} = \frac{n_0 e^2\vec{E}}{m(\omega_0^2 - \omega^2)} = \varepsilon_0\chi\vec{E},$$

where χ - electric susceptibility of a substance;

n_0 - concentration of atoms (medium molecules).

So,

$$\chi = \frac{n_0 e^2}{\varepsilon_0 m\left(\omega_0^2 - \omega^2\right)},$$

$$n^2 = \varepsilon = 1 + \chi = 1 + \frac{n_0 e^2}{\varepsilon_0 m\left(\omega_0^2 - \omega^2\right)}. \qquad (5.7)$$

The dependence curve $n(\omega)$ is presented in figure 5.1 corresponding to equation (5.7). Value n monotonously increases from the value close to 1, to $+\infty$ process where ω increases from 0 to ω_0. Value n changes abruptly from $+\infty$ to $-\infty$ when $\omega = \omega_0$ and then in the process of further increase from ω_0 to ∞ n increases monotonously again from $-\infty$ to 1. Unlimited increase of n at $\omega \to \omega_0$ is physically senseless. Such result is received because energy losses connected with the radiation of secondary waves as well as impacts between atoms and other reasons are not considered. A continuous curve in figure 5.1 is drawn with regard to losses. *Abnormal dispersion* takes place in the field of the frequency close to the own one, where $dn/d\omega \langle 0$ and we have normal dispersion in the others, where $dn/d\omega \rangle 0$.

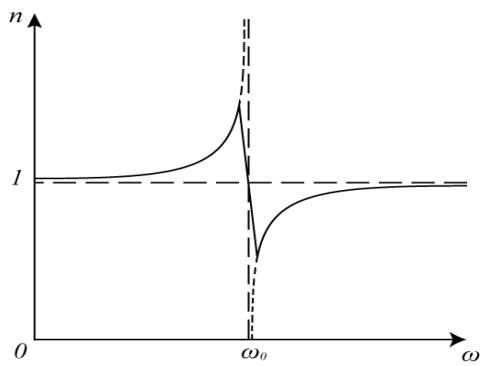


Figure 5.1

6 Lecture №6. Thermal radiation. Corpuscular properties of electromagnetic radiation

Lecture's content: quantum nature of radiation is briefly given in the lecture.

Lecture objectives:

- to study the phenomena of radiation of a black body, photoeffect and Compton's effect;
- to study the role of quantum mechanics in modern physics and learn basic principles of quantum mechanics.

6.1 Properties and characteristics of thermal radiation

The electromagnetic radiation is called thermal radiation emitted by a substance and occurring due to changing its internal energy (energy of thermal motion of atoms and molecules).

Thermal radiation is the only radiation capable to be in thermodynamic balance with a substance. The statistical balance will establish in time if a heated body is placed in a cavity impermeable for radiation. In this case the body will absorb energy of radiation in a unit of time as much as it will radiate itself. Thus, the distribution of energy between the body and radiation will stay invariable for each wavelength, but radiation density in space between the body and walls will reach some certain value, corresponding a given temperature. Established radiation in this cavity being in statistical balance with the heated body is the *equilibrium thermal radiation*. *Thermal radiation has a continuous range of frequencies (lengths of waves) with maximum intensity at a certain frequency (wavelength).*

Let's consider the main characteristics of thermal radiation emitted by any body.

Power luminosity R_T is energy equal to a full energy stream Φ emitted inside out by surface unit S of a radiating body in all directions (within a space angle 2π):

$$R_T = \frac{d\Phi}{dS}. \tag{6.1}$$

Power luminosity is connected with a power luminosity spectral density of a body $r_{\omega,T}$ (emissive ability):

$$R_T = \int_0^\infty r_{\omega,T} d\omega. \tag{6.2}$$

Let's consider a platform *dS* of a body surface on which the radiation stream $d\Phi_{\omega,T}$ falls. A part of this stream $d\Phi'_{\omega,T}$ is absorbed by the body, and $d\Phi''_{\omega,T}$ is reflected. The shares of absorbed and reflected energy are characterized by dimensionless values: *absorptive ability* $a_{\omega,T}$ *and reflective ability* $b_{\omega,T}$ of a body

$$в_{\omega,T} = \frac{d\Phi''_{\omega,T}}{d\Phi_{\omega,T}}. \tag{6.3}$$

It is obvious that

$$a_{\omega,T} + в_{\omega,T} = 1. \tag{6.4}$$

The body is called absolutely black if it completely absorbs the radiation of all frequencies falling on it: $a_{\omega,T} = 1$, $b_{\omega,T} = 0$. A good model of absolutely black body is the small hole in an opaque closed cavity. There is an interrelation between

reflective and absorptive abilities of opaque bodies established by G. Kirchhoff in 1859 called *Kirchhoff's law*: *the relation of power luminosity spectral density of a body to its absorptive ability does not depend on a body material and is the function of temperature and frequencies* $f(\omega,T)$ *for all bodies.*

$$\left(\frac{r_{\omega,T}}{a_{\omega,T}}\right)_1 = \left(\frac{r_{\omega,T}}{a_{\omega,T}}\right)_2 = ... = f(\omega,T). \qquad (6.5)$$

Function $f(\omega,T)$ is called Kirchhoff's function. The formula (6.5) shows that emissive ability of any body can't exceed emissive ability of a black body at the same temperature values.

6.2 Laws of thermal radiation of a black body and Reyleigh-Jeans' formula

The experimental investigation of thermal radiation of an absolutely black body established the dependence $f(\omega,T)$ at various temperatures (figure 6.1). As see from figure power luminosity R_T of an absolutely black body is equal to the area under a curve $f(\omega,T)$ and it increases with temperature. The maximum emissive ability is displaced towards big frequencies (shorter waves) while temperature increases: $\omega_{m1} < \omega_{m2} < \omega_{m3}$. It is established experimentally that:

$$R_T = \sigma T^4, \qquad (6.6)$$

$$\omega_m = bT, \qquad (6.7)$$

where σ – Stephan-Boltzmann constant;
b – Wien constant.

Ratio (6.6) is called *Stephane-Boltzmanm's law*, and a ratio (6.7) – *Wien's displacement law*. These two laws are important in the practical relation and played the essential role in the development of thermal equilibrium radiation theory.

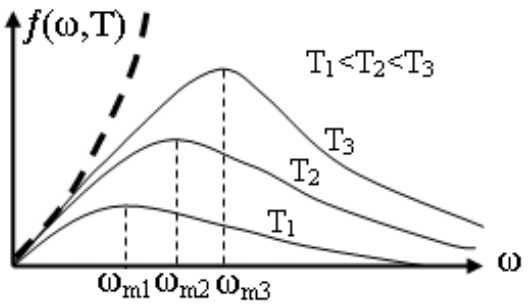


Figure 6.1

An attempt to explain theoretically regularities of thermal radiation was made by Rayleigh and Jeans using the classical statistics theorem of equal distribution of energy according to freedom degrees. In this connection equilibrium thermal radiation was considered in a closed cavity. For Kirchhoff's function they received:

$$f(\omega,T)=\frac{\omega^2}{4\pi^2c^2}kT. \qquad (6.8)$$

Comparing function graph (6.8) with the experiment we can see that coincidence is observed only in the field of small frequencies. Fundamental difference is observed in the field of big frequencies: at $\omega\to 0$, $f(\omega,T)\to\infty$ (a dotted curve in figure 6.1). Power luminosity R_T of an absolutely black body in Rayleigh-Jeans' theory is also infinite that is physically senseless.

The set of all facts showed that *the classical physics incorrectly describes thermal radiation in the field of high frequencies. Developed situation in the theory of radiation, known in the history of physics as "ultra-violet catastrophe", led to the necessity of fundamental physics revision.*

6.3 Plank's hypothesis and formula

The correct expression for Kirchhoff's function and the theoretical basis for spectral regularities of black radiation were given first by the German physicist M. Planck. He suggested that energy of a harmonious oscillator, fluctuating with a certain frequency ω, can accepts only certain discrete values equal to the whole number of elementary portions – energy quantum:

$$W=n\hbar\omega, \qquad (6.9)$$

where $\hbar=h/2\pi$ – Plank's universal constant, $n=1,2,3...$ - a whole number.

On the basis of this assumption Planck received a formula for emissive ability of an absolutely black body:

$$f(\omega,T)=\frac{\omega^3}{4\pi^2c^2}\frac{1}{\exp(\hbar\omega/kT)-1}. \qquad (6.10)$$

The Plank's formula is completely coordinated with experimental data in all interval of frequencies from 0 to ∞. All experimentally discovered laws of thermal radiation were explained on its basis; Stephan-Bolzsman and Wien's constants are calculated too. This formula transforms into Rayleigh-Jean's formula in the field of small frequencies.

All these facts say that *Planck's hypothesis of corpuscular character of electromagnetic radiation is right*. It was Planck's idea that became the first impulse for development *of quantum physics.*

6.4 Photons. Compton's effect

Developing M. Planck's idea, A. Einstein assumed that light *is not only emitted and absorbed, but it also spreads in quantum,* i.e. discretion is the main property of light. It consists of separate particles – *photons.* Photon energy W (light quantum), according to Einstein's hypothesis, equals:

$$W = \hbar\omega, \tag{6.11}$$

where ω – cyclic frequency of a light wave.

The photon always moves with the speed $c = 3 \cdot 10^8$ *m/s*. Photon impulse is:

$$p = \hbar\frac{\omega}{c} = \hbar k, \tag{6.12}$$

where $k = \omega / c = 2\pi / \lambda$ – module of a wave vector $\vec{k}$ directed along the vector of light wave distribution speed . In a vector form this formula is expressed:

$$\vec{p} = \hbar\vec{k}. \tag{6.13}$$

The relation between energy and photon impulse is:

$$W = cp. \tag{6.14}$$

The relation between the mass and energy shows that a photon has a mass:

$$m = \frac{W}{c^2} = \frac{\hbar\omega}{c^2}, \tag{6.15}$$

but unlike other particles the photon hasn't a rest mass $m_0 = 0$.

So, *the photon is a quantum of electromagnetic radiation. Like any particle it has energy, an impulse and mass. These corpuscular characteristics of a photon are connected with wave characteristics of a wave, frequency and a wave vector.* Corpuscular properties of electromagnetic radiation were confirmed in a number of physical experiments.

In 1922, A. Compton showed experimentally that at dispersion of X-rays by free electrons their frequency (wavelength) changes according to the law of collision of two particles – a photon and an electron. The schematic diagram of Compton's experience is given in figure 6.2.

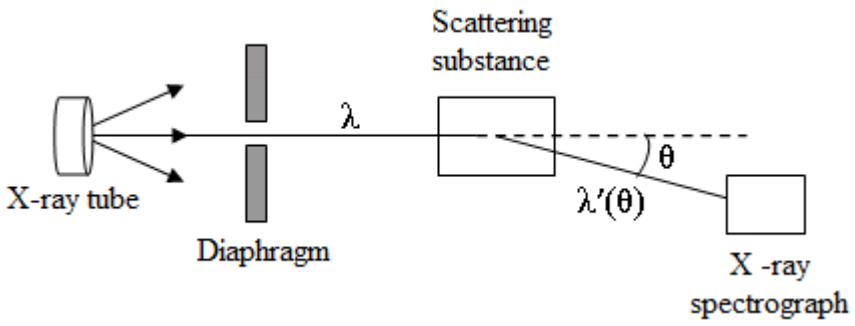


Figure 6.2

Characteristic property of Compton's effect is changing of wavelength $\Delta\lambda = \lambda' - \lambda$ that depends neither on the falling radiation wavelength, nor on substance, where a dispersion occurs, but it is defined only by the dispersion angle θ:

$$\Delta\lambda = \lambda'(\theta) - \lambda = \lambda_C(1 - \cos\theta), \qquad (6.16)$$

where λ_C – Compton's wavelength of an electron, it is equal to $2{,}43 \cdot 10^{-12} m$, (explanation of Compton's effect is presented in appendix 15).

7 Lecture №7. Wave properties of substance

Lecture content: some elements of quantum mechanics are briefly stated in the lecture.

Lecture objective: to study wave properties of substance.

In classical physics there was an idea of *basic distinction of the nature of particles and waves*. The particle is discrete, concentrated in a very small volume, as to the wave, it always occupies final part of space (maybe very big). Waves partially take place in the second medium at meeting with a barrier. They partially reflect themselves and can interfere between one another. A particle always proves to be a whole unit, incapable to interfere. However, in the 20s of the XXth century the notions of waves and particles were united in the conformity with experimental facts in physics of microcosm. The fundamental law named corpuscular – wave dualism of a substance was discovered.

7.1 De Broglie's hypothesis and formula

The dualism of waves and particles was formulated for the first time in 1924 by Louis de Broglie. De-Broglie's idea meant that the dualism is not a feature of only optical phenomena, but it has a universal value. He supposed that substance particles together with corpuscular properties have wave properties as well. De Broglie transferred the rules of transition from one physical picture to another both

true for light (electromagnetic radiation) and particles. *De-Broglie's hypothesis is a hypothesis of universalism of corpuscular - wave dualism according to which not only photons, but also electrons and other micro particles of matter have both corpuscular and wave properties.*

Let's write down the energy and impulse of a photon and a material particle, using De-Broglie's hypothesis.

Table 7.1

Photon	$W = h\nu = \hbar\omega$ $p = \frac{h}{\lambda}$ $\vec{p} = \hbar\vec{k}$
Material particle	$W = mc^2$ $\vec{p} = m\vec{\upsilon}$
Particle energy and particle impulse through wave characteristics	$W = \hbar\omega = h\nu$ $p = \frac{h}{\lambda} = \frac{h\nu}{c}$ $\vec{p} = \hbar\vec{k}$

Therefore, if a particle has corpuscular characteristics - energy W and impulse p, the corresponding wave characteristics of the particle, frequency ω and wavelength λ, are connected with corpuscular ratios:

$$\omega = \frac{W}{\hbar}, \ \lambda = \frac{2\pi\hbar}{p}. \tag{7.1}$$

The waves associated with freely moving particles are called *waves of De Broglie. So,* to a moving electron (or another particle with not really high energy ($\upsilon \ll c$) a wave process corresponds. The wavelength of this process is:

$$\lambda = \frac{h}{m\upsilon}, \tag{7.2}$$

where m and υ – mass and speed of a particle.

7.2 Experimental confirmation of de Broglie's hypothesis

American physicists K. Devison and L. Dzhermer established that selective reflection of electron bunches from a metal surface on certain angles is observed while reflecting.

The parallel mono energetic bunch from an electron tube T was directed to target M and the reflected electrons were caught by collector K connected to a galvanometer.

The accelerating voltage changed continuously at the fixed angle of the bunch falling from the crystal surface and thus indications of the galvanometer were registered. The intensity was maximum at certain angle θ.

The experience with selective reflection of electrons from a crystal surface represents analogue diffraction reflection of X-rays from a crystal by Bragg's method.

Calculations by Bragg's formula were completely coordinated with calculations λ by de Broglie's formula.

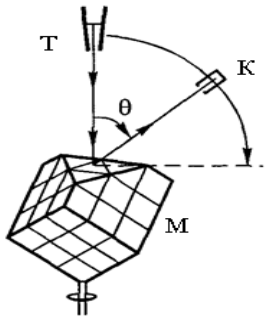


Figure 7.1

A large number of experiments (diffraction picture of electrons, atoms, molecules) passing through thin metal plates confirmed de Broglie's hypothesis as well.

7.3 Some properties of de Broglie's waves

According to de-Broglie's hypothesis the movement of a free electron can be compared with a monochromatic wave length

$$\lambda = \frac{h}{m\upsilon}.$$

In the case of dispersion, we must differ two speeds of waves - phase and group speed which reflect wave properties of De-Broglie's waves. Calculations show that electronic waves have big dispersion, group speed is equal to the speed of particle υ.

Thus, it is possible to present the link between corpuscular and wave properties of free particles having the mass and speed as:

Table 7.2

Corpuscular properties	Wave properties
Particle speed υ Particle impulse $p=m\upsilon$ Energy of a free particle $W = mc^2 = \frac{p^2}{2m}$	De-Broglie's wavelength $\lambda = \frac{h}{m\upsilon} = \frac{h}{p}$ De-Broglie's wave frequency $\nu = \frac{W}{h}$ De-Broglie's waves group speed $u = \upsilon$ Phase speed $\upsilon_{ph} = \frac{c^2}{v}$

7.4 Heisenberg's uncertainty principle

In classical mechanics, the particle condition at the moment is defined by the value of coordinates and an impulse. They are so called dynamic variables. That microparticles have wave properties leads inevitably to the fact that concepts characterizing a particle in classic mechanics must be limited.

However, we receive some information on microparticles, observing their interaction with devices. Therefore the results of measurements are expressed in the terms accepted for macrobodies, i.e. using values of dynamic variables.

The originality of properties of microparticles is reflected in the fact that coordinates and impulse of a particle *cannot have certain values at the same time.* These restrictions are not defined by perfection of measuring equipment, and express fundamental properties of matter. So, for example, an electron (as well as any other microparticle) cannot have exact values of coordinate x and impulse components p_x at the same time. Uncertainties Δx and Δp_x satisfy the expression:

$$\Delta x \cdot \Delta p_x \geq \hbar . \tag{7.3}$$

The expression similar to (7.3), takes place for y and p_y, z and p_z, and also for energy and time

$$\Delta W \cdot \Delta t \geq \hbar . \tag{7.4}$$

Expressions (7.3) and (7.4) are called *uncertainty relations.* For the first time they were established by W. Heisenberg in 1927.

According to (7.4) it is necessary to have time on less than $\Delta t \approx \hbar / \Delta W$ to measure energy with error ΔW. "Degradation" of power levels of hydrogen-like atoms can be an example (except the main state). It is explained by the fact that electron life time in all excited conditions is approximately 10^{-8} s. Degradation of levels leads to broadening of spectral lines *(natural broadening). If a system is not stable energy always has non removable statistic error no less than:*

$$\Delta W \approx \hbar / t , \tag{7.5}$$

where *t – life time of a system.*

The statement that the product of uncertainties of values of two interfaced variables cannot be in the order of value less than Planck's constant ħ, is called Heisenberg's uncertainties principle.

7.5 Statistical interpretation of De Broglie waves. Wave function

Heisenberg's uncertainties lead to probabilistic description of microparticles movement that allows to combine organically their corpuscular properties with

wave ones. According to statistical interpretation, *intensity of de Broglie's waves of in any place of space is proportional to the probability to find a particle in this place.*

The major elements in the fundamental physical theory are *the concept of a state* and *the equation describing dynamics (change) of the state*.

In classical mechanics the state of a particle at the present time moment is set by coordinates *x, y, z* and impulse p_x, p_y, p_z, and the main equation is Newton's second law. In a microcosm physics the state of microparticles can be described only by a function having wave properties.

In quantum mechanics the state of a microparticle is set by wave function $\Psi(x, y, z, t)$, *which is the function of spatial coordinates and time. The change of this state in time is described by Schrödinger's equation in the nonrelativistic case. It is the main equation of the quantum theory*.

The wave function is a field in mathematical sense (as it is complex, the waves described by function Ψ, are not observable). Interpretation of physical sense of wave function was given for the first time by M. Bourne. It is as follows:

Square module $\Psi(x, y, z, t)^2$ *of complex function* Ψ *is the density of probability to find a particle in volume* dV *near the point with coordinates x, y, z.* The probability of finding a microparticle point at time in this volume is given by expression:

$$dP(x, y, z, t)^2 dV . \tag{7.6}$$

It is already observable value.

Function Ψ must satisfy some natural conditions. It has to be continuous and single-valued everywhere. It is necessary to normalize this function so that the probability of a reliable event should be equal 1:

$$\int_V |\Psi(x, y, z, t)|^2 dV = 1. \tag{7.7}$$

7.6 Schrödinger's equation

Change of a state in time, i.e. in the nonrelativistic case dynamics of a microparticle is described by the non-stationary equation of Schrödinger. It is the main equation of the quantum theory.

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + U(x, y, z, t)\Psi , \tag{7.8}$$

where $i = \sqrt{-1}$ - imaginary unit;
m - mass of a particle;
Δ - Laplace's operator;

U - potential energy of a microparticle (in the case, when U does not depend on t).

Schrödinger's equation plays the role in nonrelativistic quantum mechanics the, as the equation of Newton's second law does in classical mechanics.

If a microparticle is in a stationary force field and its potential energy *does not depend obviously on time*, in this case the stationary equation of Schrödinger takes place:

$$\Delta\Psi + \frac{2m}{\hbar^2}(W - U)\Psi = 0\,. \tag{7.9}$$

Parameter W in this equation has meaning to total energy of a particle, and the solution of this equation $\Psi(x, y, z)$ is function of spatial coordinates.

Functions $\Psi(x, y, z)$ satisfying to the equation (7.9) at this $U(x, y, z)$ are called own functions. Values of energy W are called own values. They are solutions of the Schrödinger's equation.

8 Lecture №8. Solution of Schrödinger's equations. Quantum numbers. Principle of Bohr's conformity

Lecture content: application of the equation of Schrödinger for various quantum mechanical tasks, and the principle of Bohr's conformity are briefly described in the lecture.

Lecture objectives:

– learn to apply Schrödinger's equation for various quantum mechanical tasks solution;

– study the principle of Bohr's conformity.

8.1 Examples of Schrödinger equation solutions. Principle of Bohr's conformity

8.1.1 A microparticle is in one-dimensional potential pit of infinite depth.

Let's suppose that a particle with mass m can move only along Ox axis. This movement is limited by impermeable walls for the particle, which coordinates are x=0 and x=L (this task is similar to a task about free electrons in metal in the classical electronic theory).

Potential energy of the particle in such a field can be seen in (Figure 8.1). As the psi-function of the particle depends only on the coordinate x, the stationary Schrödinger's equation (7.9) in this task is expressed:

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(W - U)\Psi = 0\,. \tag{8.1}$$

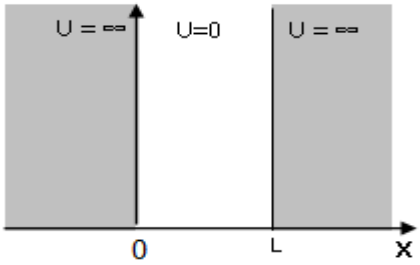


Figure 8.1

The particle can not get out of the limits of the pit therefore let's consider $\Psi(x)=0$ in areas $x\langle 0$ and $x\rangle L$. It has to be equal to zero on the pit borders too that follows from a condition of a continuity of psi-function:

$$\Psi(0)=\Psi(L)=0\,. \tag{8.2}$$

Equalities (8.2) are the boundary conditions added to equation (8.1). Within a pit (in this area $U=0$) equation (8.1) is expressed:

$$\frac{d^2\Psi}{dx^2}+\frac{2m}{\hbar^2}W\Psi=0\,. \tag{8.3}$$

The solution of this equation consists in finding possible values of total energy of particle W (a power range) and wave functions energy corresponding to these values of energy $\Psi(x)$.

Equation (8.3) is known from the theory of oscillations. It answers condition (8.2) only at certain values of energy of a particle

$$W_n=\frac{\pi^2\hbar^2}{2mL^2}n^2\,, \tag{8.4}$$

where $n=1,2,3...$ - whole numbers.

This result shows that the power range of a microparticle in a potential pit is discrete, i.e. energy of a particle is quantized. Quantized values of energy W_n are called energy levels, and n is the main quantum number.

Own functions of a particle corresponding (8.4) are:

$$\Psi_n(x)=A\sin\left(\frac{n\pi}{L}x\right),\ o\le x\le L\,. \tag{8.5}$$

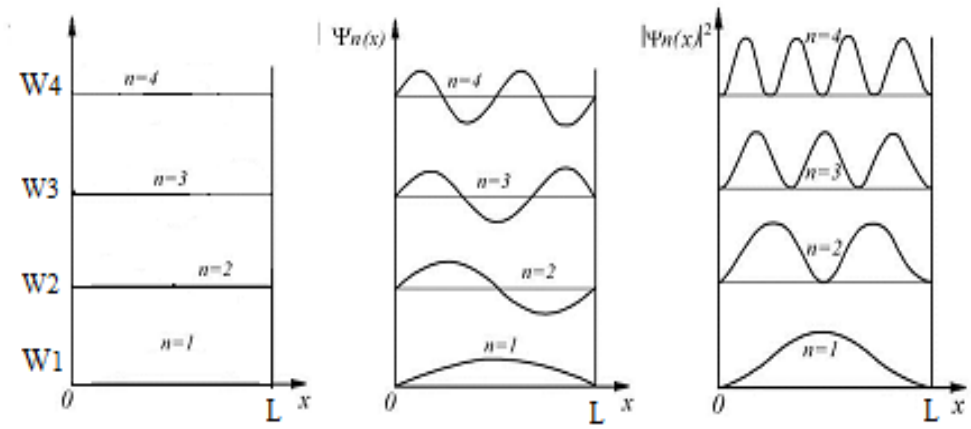
Coefficient A is found from a normalization condition (7.7).

Finally we have

$$\Psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}. \qquad (8.6)$$

In figure (8.2) the scheme is drawn for a particle power levels in a potential pit (a). Function graphs $\Psi_n(x)$ (b) and graphs of probability density dP/dx (c) of particle detection are presented in the vicinity of points with coordinates x.

Figure 8.2 illustrates fundamental difference in behavior of quantum and classical particles. So, a classical particle can possesses any energy in a pit, including $W_{\min} = 0$ corresponding a particle placed on the bottom of a pit. The energy range of a quantum particle is discrete, and its minimum energy corresponds to value n=1 about is not equal to zero. A quantum particle cannot be on the bottom of this pit. A classical particle can be found with equal probability in any point of a pit. A quantum particle being in the lowest power state $(n = 1)$ for example, can be found with the greatest probability in the center of a pit. But the probability density of finding a quantum particle at the edges of a pit is equal to zero for any state. Using Schrödinger's equation in relation to various cases is given in appendix 16: tunnel photoeffect, harmonious oscillator.



a b c

Figure 8.2

8.1.2 Principle of Bohr's conformity

The results of quantum mechanics at large quantum numbers must correspond to classical results. It is Bohr's principle.

Equation (8.4) shows that energy difference of two nearby levels is:

$$\Delta W_n = W_{n+1} - W_n = \frac{\pi^2 \hbar^2}{2mL^2}(2n+1). \qquad (8.7)$$

Expression (8.7) shows that the increase of mass m of a particle or value of area of its localization L reduces an interval between nearby levels. The interval value ΔW increases linearly with the increase of quantum number n.

Let's find the relation $\Delta W / W$, using formulae (8.4) and (8.7):

$$\frac{\Delta W}{W} = \frac{2n+1}{n^2}, \text{ when n>>1, } \frac{\Delta W}{W} \approx \frac{2}{n}. \tag{8.8}$$

The received result shows that with growth of number n (8.8) the distance between the nearby levels of energy ΔW becomes small in comparison with the energy of a particle. In this case it is possible to neglect discreteness of a power range, i.e. the quantum description becomes close to the classical ones. Amplitude values of probability density equal to $2/L$ and are identical for all n. The number of function knots (zero) $\Psi_n(x)$ increases with growth of n. At big $n \rangle\rangle 1$ maxima and minima of a curve follow one another so closely that the curves and we have a classical result.

8.2 Orbital and magnetic quantum numbers

The electron energy in a hydrogen atom depends only on the main quantum number n. But Schrödinger's equation shows that own functions (defining a condition of an electron) contain three integer parameters: n, ℓ, m. All quantum numbers are defined from function properties ψ.

Orbital (or azimuthal) quantum number ℓ defines the impulse moment L of an electron (classical analogue is physical quantity $\vec{L} = m[\vec{r}\vec{\upsilon}]$, playing an important role in studying particle movement in the field of central forces).

$$L = \sqrt{\ell/(\ell+1)} \cdot \hbar. \tag{8.9}$$

At a given quantum number n the number ℓ can accept values $\ell = 0, 1, 2 ... (n-1)$.

Impulse moment L has specific properties in quantum mechanics: impulse moment value L and only one of its projections can be given at once (for example, L_z), two other projections are not defined (a ratio of Heisenberg's uncertainties).

Magnetic quantum number m defines a projection L_z of an impulse orbital moment in the direction marked out in space (for example, let's mark out axis z creating a magnetic field along it)

$$L_z = m\hbar. \tag{8.10}$$

At given value ℓ it can have values $m = 0, \pm 1, \pm 2, ... \pm \ell$.

Integer (in units $\hbar$) projections of an impulse moment can be interpreted as *quantization of impulse moment orientation relation to* the direction marked out in space (Figure 8.3).

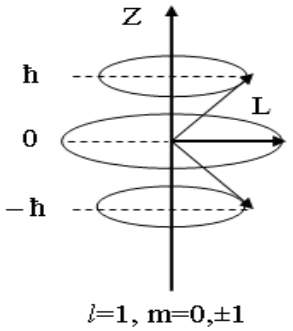


Figure 8.3

Table 8.1

W_n	$\Psi_{n\ell m}$	n	ℓ	m	A multiplicity of degeneration
W_1	Ψ_{100}	1	0	0	1
W_2	Ψ_{200}	2	0	0	4
	Ψ_{211}		1	+1	
	Ψ_{210}		1	0	
	Ψ_{21-1}		1	- 1	

Thus, in Schrödinger's theory the electron state in a hydrogen atom is defined by three quantum numbers n, ℓ, m. Each one value W_n (except W_1) has some own functions differing on values of the orbital ℓ and magnetic m quantum numbers. It means that a hydrogen atom can have the same energy value being in several various states. The states of first two power levels are given in table 8.1.

States with identical energy are called *degenerate*, and the number of various states with a value of energy is called *a multiplicity of degeneration* of an appropriate power level. It is easy to calculate a degeneration multyplicity, proceeding from possible values for ℓ and m:

$$\sum_{\ell=0}^{n-1}(2\ell+1)=n^2 . \qquad (8.11)$$

8.3 Electron spin

Spin is significantly quantum value, it doesn't have a classical analogue. Spin is an internal property of a quantum particle characterizing it equally with a mass or a charge. An electron spin is a quantum and relativistic property at the same time.

The module of its own moment of an electron impulse is determined *by spin quantum number* s, equal to 1/2.

$$L_S=\hbar\sqrt{S(S+1)}=\hbar\sqrt{(1/2)\cdot(3/2)}=(1/2)\hbar\sqrt{3} . \qquad (8.12)$$

An important difference of the spin from the orbital moment is conservation *of a spin absolute value*, it can change only its projection L_{SZ} in a given direction:

$$L_{SZ}=m_S\hbar ,\ m_S=\pm s=\pm 1/2 . \qquad (8.13)$$

9 Lecture №9. Quantum statistics and their application

Lecture content: elements of modern physics of atoms and molecules are briefly described in the lecture.

Lecture objective: to study quantum statistics, Pauli's principle.

9.1 Indiscernibility of identical quantum particles. Pauli's principle

According to the quantum theory all micro particles are divided into two classes corresponding to two quantum statistics:

- particles with *half-integer spin* are called *fermions* and they correspond *to Fermi-Dirac's* statistics;

- particles with *the whole spin are called bosons and* conformed to *Bose-Einstein's* statistics.

Both quantum statistics under certain conditions transfer into *Boltzmann's* classical statistics being an approximate limit case.

The admissible microstates of particles are considered *equiprobable* in all three statistics. The difference of these statistics is determined by ways of definition of microstates and statistical scales. The movement of a separate particle is possible to track in a system, because even identical particles are fundamentally different distinguishable in classical statistics. *There is a principle of identity* (indiscernibility) of identical particles in the quantum theory of particles systems: *all identical particles forming this quantum-mechanical system are identical (completely indiscernible).* The physical nature of difference of two quantum statistics follows from this principle: there are two types of wave ψ - functions describing the state of identical particles – *symmetric* and *antisymmetric.*

The type of symmetry of wave function is a property of particles themselves and depends neither on interaction between them, nor on existence of external fields. Their spin is a criterion that differs types of particles.

Fermions have an important feature. Particles of this type correspond to Paulie's principle: there is not more than one fermion in the same state in any quantum-mechanical system of identical fermions.

While there is any number of particles in each quantum state according to Bose-Einstein’s statistics.

Differences of the statistics explain figure 9.1 where the placement of two identical particles is shown in two quantum states (squars).

It is seen that there are four microstates in Boltzmann's statistics, but the probability of each of them is equal to 1/4. In both quantum statistics the first two states are indiscernible in the same state. Moreover the last two states are impossible (Paulie's principle) in Fermi-Dirac's statistics. There is only one microstate which probability of implementation is equal to 1.

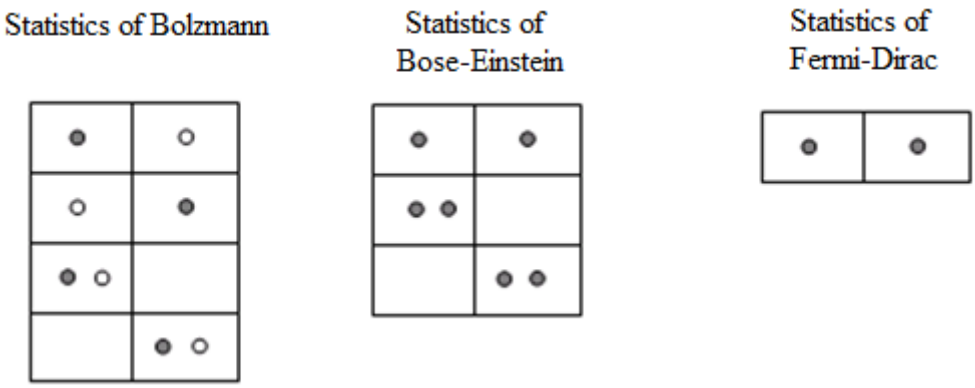


Figure 9.1

9.2 Quantum distributions

The main objective of quantum statistics is finding functions of particles distribution corresponding to these or those parameters (for example, on energy), and also determining average values of these parameters characterizing the most probable condition of the whole system of particles. Let's consider quantum distributions of particles on energy W. These distributions represent functions $f(W)$ defining average numbers of particles *in the same state with energy* W,

for fermions

$$f(W) = \frac{1}{e^{(W-\mu)/kT} + 1}; \tag{9.1}$$

for bosons

$$f(W) = \frac{1}{e^{(W-\mu)/kT} - 1}; \tag{9.2}$$

where μ is called a *chemical potential (some typical energy*, which value is possible to find from a definite condition. The total number of particles is equal to full number N of macro system of particles in all states).

Features of these distributions are:

– value of function $f(W)$ for fermions cannot be more than one unit, and for bosons it can be any;

– value μ for bosons cannot be positive in (9.2);

– the formula turns into Boltzmann's distribution if $f(W) \ll 1$ because we can neglect one unit in denominators of both distributions and the distribution is expressed:

$$f(W) = e^{\frac{\mu}{kT}} \cdot e^{-\frac{W}{kT}} = Ae^{-\frac{W}{kT}}, \tag{9.3}$$

where A – *normalizing coefficient. It is called parameter of extinct*. In this case we can tell about coincidence of formulas, but not about change of behavior of particles (fermions remain fermions, and bosons will be bosons).

9.3 Fermi-Dirac's distribution for electrons in metal

In the classical electronic theory many properties of metals are explained by means of a free electrons model. In the quantum theory free electrons can be considered as an ideal gas containing fermions in a rectangular potential pit. The energy spectrum of electrons is discrete, but energy levels are located so densely that it can be considered *quasicontinuous*.

9.3.1 Let's consider behavior of electronic gas at temperature $T = 0K$. In this case $f(W) = 1$, if $W \le \mu$ and $f(W) = 0$, if $W \rangle \mu$.

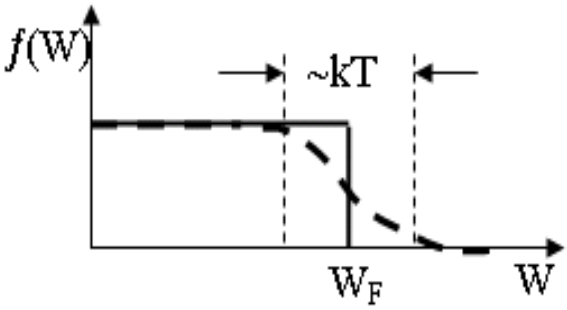


Figure 9.2

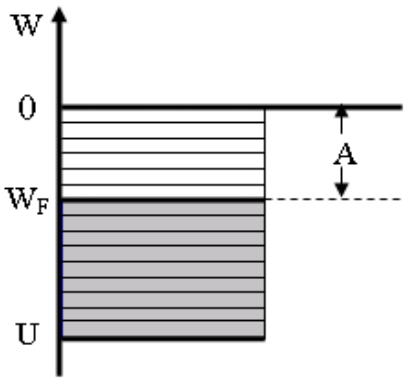


Figure 9.3

The function graph *f(W)* is shown by a continuous line. We can see that all states with energy $W \langle \mu$ are filled, but the states with $W \rangle \mu$ are unoccupied. In this case value μ is called *energy* or *Fermi's level* $W_F = \mu$. This is a maximum value of free electrons energy in metal at $T = 0\,K$

$$W_F(0) = \frac{\hbar^2}{2m}\left(3\pi^2 n_e\right)^{2/3}, \tag{9.4}$$

where m is mass of an electron;

n_e is the concentration of electrons in metal.

The evaluation of value W_F gives value ≈ *5 eV*. Average value of energy of free electrons according to the corresponding calculation is equal to:

$$\langle W \rangle \cong \frac{3}{5} W_F . \tag{9.5}$$

Such average energy could correspond to temperature $T \sim 5 \cdot 10^4 K$ for a classical gas. This temperature exceeds in many times the melting temperature of any metal. The speed of electrons being at the level of Fermi is about $10^6 m/s$.

Such a state of an electronic gas (a continuous curve on graphic $f(W)$ (Figure 9.2) is called *completely degenerated*. Distribution of particles on energy differs a lot from classical theory, it is only a quantum effect.

9.3.2 At $T \rangle 0$ Fermi-Dirac's distribution is blurred away a little bit in the vicinity of Fermi's level (a dotted curve on function graph $f(W)$) owing to the interaction of free electrons with thermal movement of atoms. The area of blurring has an energy order of the thermal movement *kT*. Therefore, only those electrons which are at the highest levels close to Fermi's level (owing to the thermal movement) can change the energy. This fact explains the difficulties of the classical theory in the explanation of lack of an electronic component of thermal capacity of metals.

The electronic gas is in a potential pit, therefore distribution of electrons can be presented as follows (Figure 9.3), where U - depth of a potential pit. The energetic levels occupied with free electrons are tinted. Energy W_F is Fermi's level, the amount of work of an electron exit from a metal is designated by arrows.

As seen in figure 9.3 the *work of electron exit A (the smallest energy which should be given to an electron for its removal from a metal) should be counted not from the bottom of a potential pit as it is done in the classical theory, but from the electron top of the power levels occupied by electrons.*

Fermi's energy depends on temperature a little bit, therefore the work of electron exit will also depend on temperature. Kinetic energy of electrons is usually counted from the bottom of a potential pit.

9.3.3 *The quantum theory of conductivity of metals* leads to the Ohm's law, it is similar to formula which was received in the classical electronic theory

$$\vec{j} = \gamma \vec{E} .$$

Formula for conductivity is expressed:

$$\gamma = \frac{ne^2 \lambda(\mu)}{m \upsilon_0(\mu)}. \qquad (9.6)$$

Electrons speed υ_0 (corresponding to its value at the top of an occupied energy level) is in the denominator instead of average thermal speed $\langle \upsilon \rangle$. This speed *does not* practically *depend on* metal *temperature*. Value λ – average distance, which can be passed by a wave without dispersion on the knots of a crystal lattice. It can make hundreds of periods of a lattice. The dispersion of electronic waves grows with temperature increasing on thermal oscillations of a lattice and therefore $\lambda(\mu)$ decreases. It is the average length of free run of an electron having Fermi's energy.

At the usual room temperature, the value λ is inversely proportional to the first degree of temperature. It results in experimental data: $\gamma \sim 1/T$. Therefore, it was succeeded in eliminating one more difficulty in the classical theory, according to which $\gamma \sim (1/T)^{1/2}$.

The density of electronic gas is so great $\left(n = 10^{28} - 10^{29} m^{-3}\right)$ in metals, that the gas is in extinct state even at usual temperatures.

10 Lecture №10. Zonal theory of solid bodies

Lecture content: elements of physics of a solid body are briefly described in this lecture.

Lecture objectives:

- to study structure of semiconductors, distribution of electrons on levels;
- to consider types of energy levels and the principle of action of *p-n* transition.

10.1 Zonal structure of an energy spectrum of electrons in crystals

All variety of solid bodies properties is caused by existence of two subsystems: electrons and atomic nuclei forming a solid body.

The quantum model of free electrons in metal (zero approach) explains well metal conductivity and many other properties of a metal, but it does not answer the question «why do not all solids have similar properties?»

The strict approach considers that electrons move in a periodic field of a crystal lattice (in the field with periodically changing potential). This leads to the fact that *the spectrum of possible values of electron energy is split into a series of permitted and forbidden zones.*

The origin of energy zones is connected with splitting discrete atomic levels owing to interaction of atoms in a crystal lattice: the conformity of electrons with Pauli's principle leads to impossibility of identical power states of interacting atoms.

Each permitted zone consists of N close located levels the number of which is equal to the amount of atoms in a crystal. Energy zones of permitted energy are divided by forbidden zones – intervals where energy levels are absent.

According to Pauli's principle, electrons are located depending on different states, consistently filling the permitted power zones, starting with the lowest zone. The filled energy levels are shown by toned drawing.

Thus, the energy spectrum of electrons has zonal structure in crystals. The width of zones does not depend on the crystal dimensions. A crystal contains the more atoms the closer levels are located in a zone. The permitted zones have the width about several electron-volt. Therefore, if the crystal contains 10^{23} atoms, the distance between levels in a zone is approximately $10^{-23} eV$. There may be two electrons having opposite directed spins at each energy level.

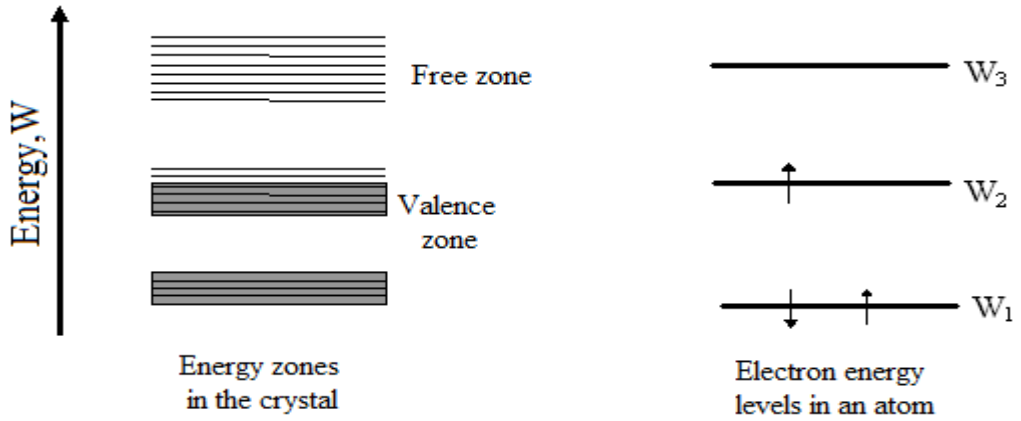


Figure 10.1

10.2 Power zones in metals, dielectrics and semiconductors

Depending on particular atom properties the equilibrium distance between atoms supposes the existence of forbidden zone ΔW or nearby zones that can be overlapped (figure 10.2). The forbidden zone is located between permitted zones arising from nearby energy atom levels. It depends of the concrete properties of atoms. Thus, there are two opportunities for valence electrons of atoms: they fill either partially, or up to the end one of the permitted zones. This zone is called *valence zone*. Empty zones are located on the top of it.

The crystal conductivity depends on the structure of a zonal energy spectrum of electrons in a crystal and on filling this spectrum with electrons at temperature $T = 0$. This defines whether a crystal belongs to metals, dielectrics or semiconductors.

Electrons in the filled zones fully or partially have different properties. If the zone is partially filled with electrons, even weak electric field will transfer electrons to unoccupied states within the same zone. Average speed of the movement of electrons will become greater than zero and an electric current will appear in a crystal. Therefore any *partially* filled energy zone is *a zone of conductivity*.

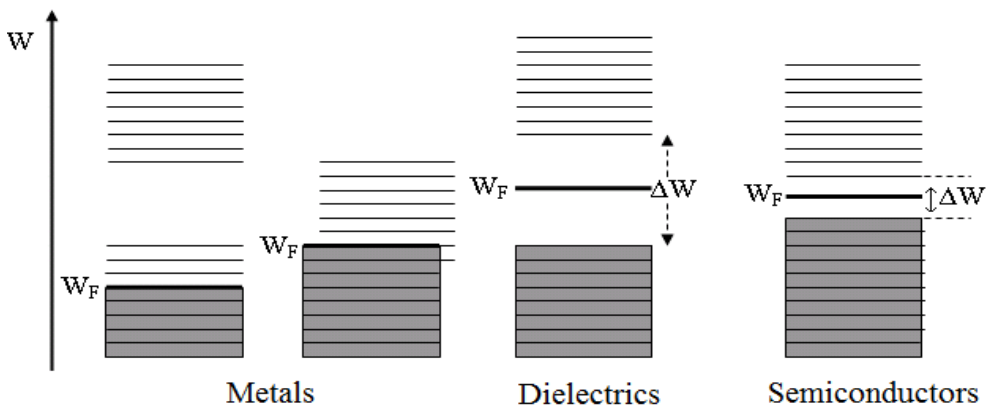


Figure 10.2

If the valence zone is completely filled at $T = 0$ (is not a conductivity zone), the crystal will be an insulator or a semiconductor. Heating of such a crystal leads to that a part of electrons of a valence zone passes into the nearby empty zone because of thermal fluctuations . As a result *both* zones become *conductive zones.* If a width of forbidden zone ΔW is about several electron-volt, a number of conductive electrons is insignificant. Therefore, crystals with such width of the forbidden zone are *dielectrics*. If the crystal has width of forbidden zone $\Delta W \leq 1\,eV$, at $T \rangle 0$ it is *a semiconductor.*

10.3 Conductivity of semiconductors

A semiconductor qualitatively differs from a metal in that a semiconductor has *two sorts of current carriers.* First, there are *electrons* passed from a valent zone into a free zone of the semiconductor, and second, these "holes" in a valent zone are free states at its top levels. Movement of "holes" is not a motion of a real particle, but it is the fact that its movement reflects the character of the movement of all set of electrons in top levels of a valence zone.

Moreover, there are two types of conductivity in semiconductors: *own conductivity* (it is typical for pure semiconductors) and *impure* one (it appears when in a pure semiconductor some atoms are replaced by impure atoms with a valence bigger or smaller by on a unit).

Distribution of electrons in free and valence zones is described by Fermi-Dirac's function. Calculation shows that Fermi's level is located in the middle of the forbidden zone, and levels of a free zone (with the electrons which passed there) are on the "tail" of distribution $f(W)$. It means that $W - W_F \approx \Delta W / 2$. Regarding the last ratio and that $\Delta W \rangle\rangle kT$, the probability of filling levels in a free zone (i.e. formula 9.1) can be written:

$$f(W) \approx e^{-\Delta W / 2kT} . \qquad (10.1)$$

The number of the electrons which passed into a free zone and number of the formed holes are proportional to $f(W)$. These electrons and holes are the current carriers; the free zone is a zone of electrons conductivity, and valence zone is a zone of holes conductivity.

As conductivity σ is proportional to concentration of carriers, own conductivity of the semiconductor is:

$$\sigma = \sigma_0 e^{-\frac{\Delta W}{2kT}} , \qquad (10.2)$$

where $\sigma_0 \approx const$.

As seen from (10.2) own conductivity of a semiconductor grows quickly with the increase of temperature. We know that own conductivity of metal decreases

with the increase of temperature according classical electrodynamics. Thus temperature dependences of a semiconductor and a metal are opposite to each other.

Own conductivity of a semiconductor is very small because the width of forbidden zone ΔW considerably exceeds thermal energy kT.

Conductivity of a semiconductor can be considerably increased (in hundreds of times and more) by introduction of a small amount of additives (about 10^{-4} %). Additional levels arise in the forbidden zone depending on impurity valence. They are located close the bottom of a free zone (donor) or to the ceiling of a valence zone (acceptor). It helps to increase significantly the conductivity of the semiconductor.

Semiconductors with impurity conductivity are widely used in modern electronics.

11 Lecture №11. Nuclear physics

Lecture's content: elements of physics of an atomic nucleus and elementary particles are briefly described in this lecture.

Lecture objective: to study structure of matter.

The structure of substance is investigated in nuclear physics where characteristic dimensions of matter are not only negligible in comparison with macroscopic distances, but they are very small in comparison with atom. The scale of various distances of nuclear physics is given in logarithmic scale in figure 11.1.

Time periods up to 10^{-9} s are directly measured in modern nuclear physics. *However, much smaller intervals,* up to $10^{-22} \div 10^{-24}$ s, are indirectly measured by means of uncertainty relations "energy-time".

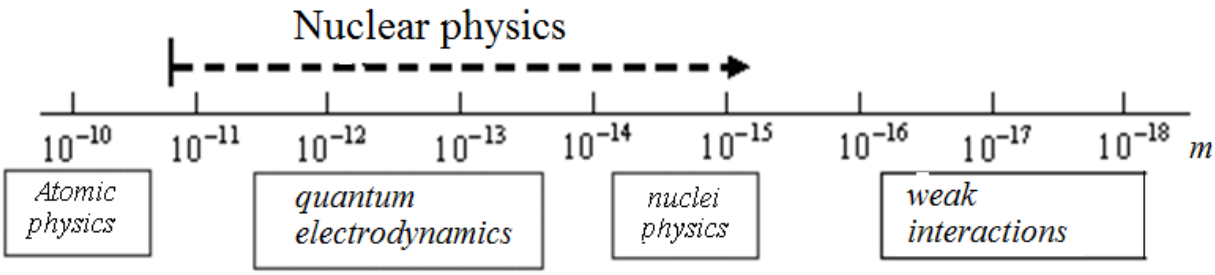


Figure 11.1

11.1 Structure and characteristics of an atomic nucleus

The nucleus represents a system of particles - *nucleons*, which interact very strongly. They are hold together by nuclear attractive forces and move with nonrelativistic speeds inside a nucleus. A nucleon is a general name of particles forming a nucleus – *protons and neutrons.* Main characteristics of these particles are given in table 11.1.

Table 11.1 – Characteristics of nucleons

Particle (designation) Physical quantity	Proton (p)	Neutron (n)
Mass, kg Mass, MeV	$1{,}672648 \cdot 10^{-27}$ $938{,}28$	$1{,}674954 \cdot 10^{-27}$ $939{,}57$
Electric charge	$+e$	0
Magnetic moment	$+2{,}79\mu_N$	$-1{,}913\mu_N$
Spin	1/2	1/2
$\mu_N = e\hbar / 2m_p c = 0{,}505 \cdot 10^{-26}\, J/T$ - the nuclear magneton – unit of magnetic moments of nucleons		

A neutron in a free state is unstable and spontaneously breaks up; it turns into a proton emitting an electron and an antineutrino.

Proton in a free state is a stable particle. Inside a nucleus a proton can turn into a neutron emitting out a positron and a neutrino.

The proximity of many properties of a proton and neutron allows to consider them as *two conditions of one particle – a nucleon.*

The main characteristics of stable nucleus are *the charge, mass, binding energy, radius, energy spectrum of states.*

Radioactive (unstable) *nuclei* are characterized by a number of additional parameters: *time of life (half-life period), type of radioactive transformations, energy spectrum of emitting particles*, etc.

Charging number Z coincides with the number of protons in a nucleus and defines the nucleus charge equal to $+Ze$.

Mass number A defines total quantity of nucleons in a nucleus, as well as the number of neutrons $N = A - Z$.

The considered characteristics of a nucleus are expressed in symbolical designation ${}^{A}_{Z}X$.

Sizes of nuclei. Forms and sizes of nuclei are conditional enough because the behavior of particles forming a nucleus is conformed to quantum laws. It is possible to speak about *average distribution of density of nuclear substance,* and there are experimental methods to measure it.

In the first approximation a nucleus can be considered as a sphere, which radius is

$$r = r_0 A^{1/3}, \tag{11.1}$$

where $r_0 = (1{,}2 \div 1{,}3) \cdot 10^{-15}\, m$.

As seen in from (11.1) nucleus mass is proportional to its volume. Therefore, substance density in all nuclei is approximately identical and approximately equals $\rho_N \approx 10^{17}\, kg/m^3$.

A nucleus spin *(full mechanical moment)* consists of impulse moments of protons and neutrons being parts of nucleus.

11.2 Mass and binding energy of a nucleus

Exact measurements of masses showed that *the nucleus mass* m_N *is always less than a sum of nucleons mass within a nucleus*

$$m_N = Zm_p + (A - Z)\, m_n - \Delta m\,. \qquad (11.2)$$

Difference Δm between the sum of nucleons mass and mass of a nucleus is called *the mass defect* and characterizes the *binding energy of nucleons* in a nucleus. It is the minimum energy W_b, which should be spent to split the nucleus into parts. It is obviously that W_b is one of the main characteristics of *nucleus* strength. Knowing binding energy of a *nucleus* it is possible to calculate an energy exit for any processes of disintegrations and mutual transformations of nuclei

$$W_b = \Delta m \cdot c^2 = c^2 \{ [Zm_p + (A - Z)m_n] - m_N \}. \qquad (11.3)$$

For practical calculations it is more convenient to use a formula

$$W_b = c^2 \{ [Zm_H + (A - Z)m_n - m_a] \}, \qquad (11.4)$$

where m_a – mass of an atom;

m_H – mass of a hydrogen atom.

The relation of binding energy W_b to full number of nucleons in a nucleus A is called *specific binding energy* (or binding energy on for one nucleon). The analysis of dependence of specific binding energy on mass number (figure 11.2) for stable nuclei gives interesting information about properties of nuclei and about a character of nuclei forces.

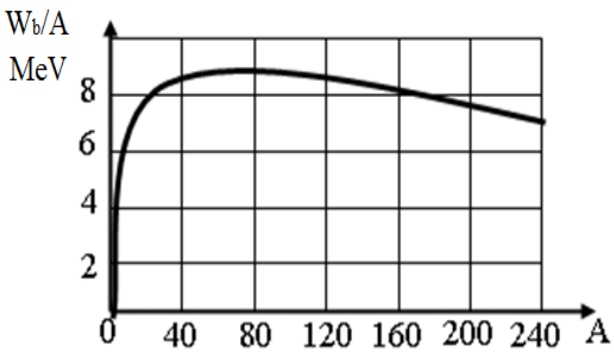


Figure 11.2

The graphs in (figure 11.2) shows that: the nuclear attractive forces are short-range - about the size of a nucleon; nucleons in a nucleus are connected most strongly with values of mass number from 50 to 60 (binding energy for these nuclei is 8,7 MeV/nucleon), synthesis of light nuclei and division of heavy ones are energetically favorable ones.

Process *of nuclear fission of uranium or plutonium* under the influence of neutrons is the base of nuclear reactor operation as well as a usual atomic bomb.

Process of synthesis of light nuclei can be realized only at very high temperatures (*thermonuclear reaction).* It proceeds in a subsoil of the Sun and stars. Uncontrolled thermonuclear reactions as explosions of hydrogen bombs were realized in 1952 in USA and in Russia in 1953. Now scientists work hard on finding controlled thermonuclear synthesis.

11.3. Nuclear forces

Huge binding energy of nucleons in a nucleus indicates that there is very intensive interaction holding nucleons at very small distances from each other despite strong Coulomb pushing away. Nuclear interaction between nucleons is called *strong interaction.* It can be described by means of *a field of nuclear forces.* They have distinctive features.

Short-acting. Radius of action of nuclear forces is an order ~ 10^{-15} m.

Charging independence of nuclear forces. Strong interaction does not depend on a charge of nucleons.

Dependence on mutual orientation of nucleons spins. For example, a nucleus of heavy hydrogen (deuteron) is formed of a proton and a neutron only if their spins are parallel.

Nuclear forces are not central. They are not directed along the straight line connecting the centers of the interacting nucleons.

Nuclear forces have a property of saturation. Each nucleon in a nucleus interacts with limited number of other nucleons.

Nuclear forces depend on the speed of relative movement of nucleons.

Exchange character of nuclear forces. Nucleons virtually exchange particles which are named *pi- mesons* (π). They are called pions.

There are two charging conditions of a pions with a positive and negative charge $\pm e$, and one condition is electrically neutral. These particles are unstable and they don't have a spin. Main properties of pions are given in appendix 17.

11.4 Radioactive processes

11.4.1 α – disintegration.

Radioactive transformation with emission of α– particles is described by the equation:

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He . \qquad (11.5)$$

The α – particle is a helium atomic nucleus and is designated ${}_{2}^{4}He$. Owing to emission α – particles the charge of a maternal nucleus X decreases on 2 units and mass number – on 4 units.

Energy of α – particles emitting by radioactive material is strictly particular (a discrete power range). In most cases radioactive material emits some groups of α – particles of a various energy. The scheme explaining appearance of α – particles of various groups is given in figure 11.3. The dotted line shows possible transitions with emission γ – quantum.

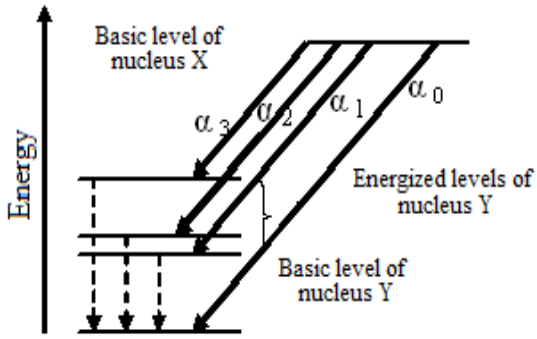


Figure 11.3

Energy of α – particles of various radioactive sources has values from 4 to 9 MeV. Passing through substance α – particles lose the energy owing to ionization of substance molecules and eventually they stop. Distance, passed by a α – particle in substance to a full stop, is called *the run of* α – particles. For the majority the α – particles the run in air is several centimeters.

11.5 β – disintegration

There are three types of β – disintegration. In one case the nucleus *emits an electron*, in another – *a positron*, the third case is called *electronic capture* (e^- or K – capture), when the nucleus absorbs one of electrons of K – cover. Let's introduce symbolical designation for an electron ${}_{-1}^{0}e$ (the charge is equal to 1, mass number is 0), for a positron $-{}_{-1}^{0}e$. There are schemes of β –according to its three types respectively:

$$ {}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + \tilde{\nu}\,; \qquad (11.6) $$

$$ {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + {}_{-1}^{0}e + \nu\,; \qquad (11.7) $$

$$ {}_{Z}^{A}X + {}_{-1}^{0}e \rightarrow {}_{Z-1}^{A}Y + \nu\,. \qquad (11.8) $$

An antineutrino also emitted $(\tilde{\nu})$ with an electron at electronic disintegration; as to positron disintegration and e-capture – *a neutrino* (ν) is emitted.

β –disintegration can be followed by gamma radiation. The mechanism of its emergence is similar to α – disintegration.

At β^- – disintegration two particles are emitted(an electron and an antineutrino) which *are not present* inside a nucleus. They occur during disintegration due to *transformation of a neutron into a proton:*

$$ {}_{0}^{1}n \rightarrow {}_{1}^{1}p + {}_{-1}^{0}e + \tilde{\nu} . \qquad (11.9) $$

Process *of positron disintegration* (equation 11.7) occurs with emission of a positron and a neutrino:

$$ {}_{1}^{1}p \rightarrow {}_{0}^{1}n + {}_{1}^{0}e + \nu . \qquad (11.10) $$

The third type of $\beta-$ disintegration, connected with capture an electron by nucleus from near to it $K-$ covers, leads to that one proton turns into a neutron:

$$ {}_{1}^{1}p + {}_{-1}^{0}e \rightarrow {}_{0}^{1}n + \nu . \qquad (11.11) $$

Passing through substance $\beta-$ particles lose their energy and at a certain distance they are completely absorbed. Maximum run of $\beta-$ particles in air can reach 1 – 3 meters. $\beta-$ particles possessing large energy can penetrate thin layers of light metals. Lead of some millimeters can completely absorb all $\beta-$ particles.

11.6 Scale of radiation

Gamma radiation *($\gamma-$ radiation) represents short-wave electromagnetic radiation with wavelength not more than* 10^{-10} m. This radiation is generated while nuclei transfer from one excited state into another or in to the main state. In this case $\gamma-$ quantum is radiated. Energy spectrum of $\gamma-$ radiations is discrete. It has much bigger penetration than α - and $\beta-$ particles. Passing $\gamma-$ radiations through substance are followed by three main processes: *photo-electric absorption, classical and Compton's dispersion, formation of electron and positron couples.*

Appendix 1

Studying a phenomenon of electromagnetic induction it was found that in a contour in rest a variable magnetic field occurs an induction current. Action of third-party forces is the reason of emergence of the latter. These are not magnetic forces since they cannot set the being in rest charges in motion ($\vec{\upsilon} = 0$). There are forces of an electric field $\vec{F} = q\vec{E}$. This field is responsible for emergence of EMF of induction in a motionless contour when a magnetic field changes in time. Maxwell assumed that a conductor is only an indicator of induced electric field. The field acts a movement of free electrons in the conductor and detects itself by that, but it exist and without a conductor possessing an energy stock.

Unlike an electrostatic field an induced electric field is unpotential and *vortex* as the work completed in this field when moving a single positive charge over the closed contour is equal to induction EMF, but it is not zero,

$$\oint_V \vec{E}_V \, d\vec{l} = \varepsilon ,$$

where $\vec{E}_V$ - intensity of the electric field induced by a variable magnetic field.

Appendix 2

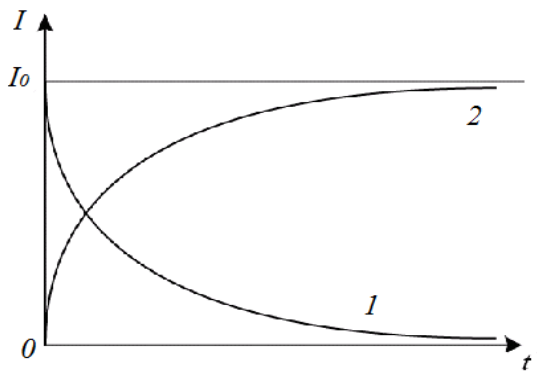


Figure 1.1- The graphs of dependence of current *I* from time *t*

There are the graphs I(t): curve 1 shows the decrease of current when a circuit is disconnected; curve 2 is the increase of current at its short circuit, so, current $I_0 = \frac{\varepsilon}{R}$ represents the settling current (at $t \to \infty$).

The speed of a current change (decreasing or setting) is characterized by constant $\tau = \frac{L}{R}$, it is called *time of relaxation and has* the dimension of time.

Appendix 3

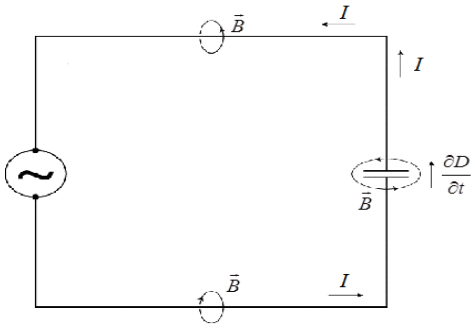


Figure 1.2 - Circuit of alternating current

Appendix 4

Oscillations are called free oscillations if they occur in the system given to itself after being brought out of an equilibrium state.

Oscillations occurring in any system under the influence of external variable influence are called *compelled* fluctuations.

Self-oscillations as compelled are induced by impact of an oscillatory system of external forces, but time points of implementation of these influences are set by the system. The system controls external influence itself.

At *parametrical oscillations* periodic change of any parameter of a system occurs due to external influence. Oscillations are *periodic* if values of all physical quantities characterizing an *oscillatory* system and changing at its fluctuations repeat in regular intervals.

Appendix 5

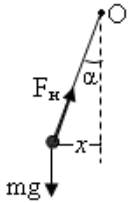
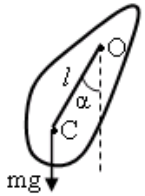
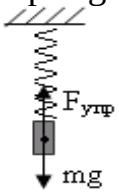
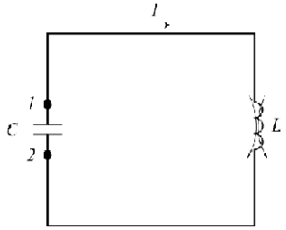
As seen in table 5.1 own cyclic frequency of the oscillator depends on its parameters. In table 5.1 value $\dot{S}$ is a speed (linear υ or angular ω) of mechanical oscillator and of current I in an oscillatory contour; $\ddot{S}$ is an acceleration (linear a or angular ε) of mechanical oscillator; U_C is a voltage on the condenser. Amplitudes of values $\dot{S}$ are respectively equal $\omega_0 x_m$, $\omega_0 \alpha_m$, $\omega_0 q_m$, and $\ddot{S}$ is $\omega_0^2 x_m$, $\omega_0^2 \alpha_m$. Their phases differ; a phase of value S differs from a phase of value $\dot{S}$ on $\pi/2$ (it lags behind), and the phase of value $\ddot{S}$ from a phase S on π (it advances).

In case of mathematical and spring pendulums a returning force is equal to $F = -m\omega^2 x$. For a physical pendulum there is a value $L = \frac{I}{m\ell}$, called specified length of a physical pendulum . It is the length of such a mathematical pendulum which period is equal to the period of this physical pendulum. So, the period of a physical pendulum is written as:

$$T = 2\pi\sqrt{\frac{L}{g}} .$$

Appendix 5

Table 5.1

Oscillator / Characteristics, equations of fluctuations	Pendulums			Ideal oscillatory contour
	mathematical 	physical 	spring 	
Main equation of a system	$m\ddot{x} = -mg\sin\alpha$	$I\ddot{\alpha} = -mg\ell\sin\alpha$	$m\ddot{x} = -kx$	$IR = (\varphi_1 - \varphi_2) + \varepsilon_S$ $R = 0;\ \varphi_1 - \varphi_2 = -\frac{q}{C};\ \varepsilon_S = -L\frac{dI}{dt}$
Diff.equation	$\ddot{x} + \frac{g}{l}x = 0$	$\ddot{\alpha} + \frac{mg\ell}{I}\alpha = 0$	$\ddot{x} + \frac{k}{m}x = 0$	$\ddot{q} + \frac{1}{LC}q = 0$
S in (3.1)	x	α	x	q
Equation of fluctuations	$x = x_m\cos(\omega_0 t + \varphi)$	$\alpha = \alpha_m\cos(\omega_0 t + \varphi)$	$x = x_m\cos(\omega_0 t + \varphi)$	$q = q_m\cos(\omega_0 t + \varphi)$ $U_C = \frac{q}{C} = \frac{q_m}{C}\cos(\omega_0 t + \varphi)$
Cycle.frequency ω_0, T period	$\omega_0 = \sqrt{g/\ell}$ $T = 2\pi\sqrt{\ell/g}$	$\omega_0 = \sqrt{(mg\ell)/I} = \sqrt{g/L}$ $T = 2\pi\sqrt{I/(mg\ell)} = 2\pi\sqrt{L/g}$	$\omega_0 = \sqrt{k/m}$ $T = 2\pi\sqrt{m/k}$	$\omega_0 = 1/\sqrt{LC}$ $T = 2\pi\sqrt{LC}$
$\dot{S} = \frac{dS}{dt}$	$\upsilon = -\omega_0 x_m\sin(\omega_0 t + \varphi) =$ $= \omega_0 x_m\cos\left(\omega_0 t + \varphi + \frac{\pi}{2}\right)$	$\dot{\alpha} = -\omega_0\alpha_m\sin(\omega_0 t + \varphi) =$ $= \omega_0\alpha_m\cos\left(\omega_0 t + \varphi + \frac{\pi}{2}\right)$	$\upsilon = -\omega_0 x_m\sin(\omega_0 t + \varphi) =$ $= \omega_0 x_m\cos\left(\omega_0 t + \varphi + \frac{\pi}{2}\right)$	$I = -\omega_0 q_m\sin(\omega_0 t + \varphi) =$ $= \omega_0 q_m\cos\left(\omega_0 t + \varphi + \frac{\pi}{2}\right)$
$\ddot{S} = \frac{d^2S}{dt^2}$	$a = -\omega_0^2 x_m\cos(\omega_0 t + \varphi) =$ $= \omega_0^2 x_m\cos(\omega_0 t + \varphi + \pi)$	$\ddot{\alpha} = -\omega_0^2\alpha_m\cos(\omega_0 t + \varphi) =$ $= \omega_0^2\alpha_m\cos(\omega_0 t + \varphi + \pi)$	$a = -\omega_0^2 x_m\cos(\omega_0 t + \varphi) =$ $= \omega_0^2 x_m\cos(\omega_0 t + \varphi + \pi)$	

Appendix 6

As seen in (figure 1.3) kinetic energy of a material point $W_k(t)$ changes from 0 to $\frac{m\omega_0^2 A^2}{2}$, making harmonic oscillations with frequency $2\omega_0$ and amplitude $\frac{1}{4}m\omega_0^2 A^2$ at average value equal to $\frac{1}{4}m\omega_0^2 A^2$.

Oscillations of potential energy $W_p(t)$ occur as previous, but with a shift of phases is equal to π. Total energy W remains constant.

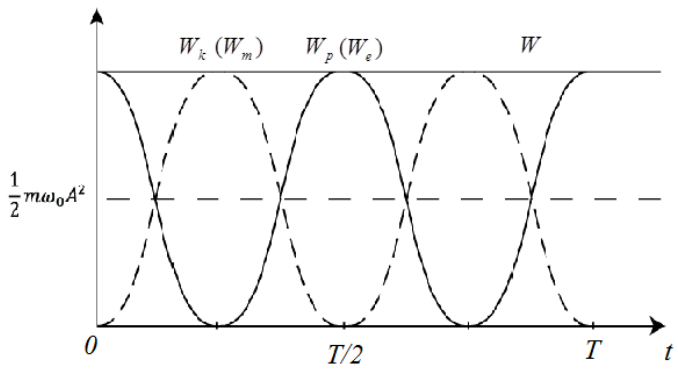


Figure 1.3 - graphs of dependence W_k, W_p and W from time *t*

Appendix 7

Energy of a magnetic field W_m is equal to

$$W_m = \frac{1}{2}LI^2 = \frac{1}{4}LI_m^2\left[1-\cos(2\omega_0 t + 2\varphi)\right].$$

Energy of an electric field W_e is equal to

$$W_e = \frac{1}{2}\frac{q^2}{C} = \frac{1}{4}LI_m^2\left[1+\cos(2\omega t + 2\varphi)\right].$$

Appendix 8

Let's consider an oscillation occurring along axis *x* under the law $x = A\cos(\omega_0 t + \varphi_0)$. As a basic axis is axis *x*.

Harmonic oscillation can be set by means of a vector which length is equal to amplitude *A* of oscillations, and the direction of the vector forms an angle with axis *x* and it is equal to initial phase of oscillation. Vector projection $\vec{A}$ on axis *x* is written as: $x_0 = A\cos\varphi_0$

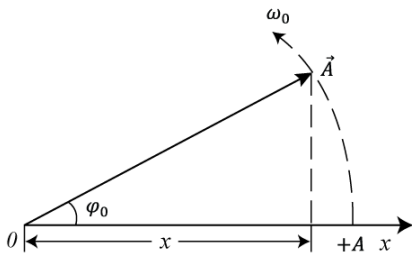


Figure 1.4 - The rotating amplitude vector

If this vector is made to rotate with an angular speed ω_0, a projection of vector $\vec{A}$ on *x* axis will make the harmonic oscillation described by equation (2.9) ranging from + *A* to – *A*. Cyclic frequency of these fluctuations is equal to angular speed of rotation, and initial phase φ_0 is equal to angle which is formed by vector $\vec{A}$ with a basic axis at an initial timepoint.

Appendix 9

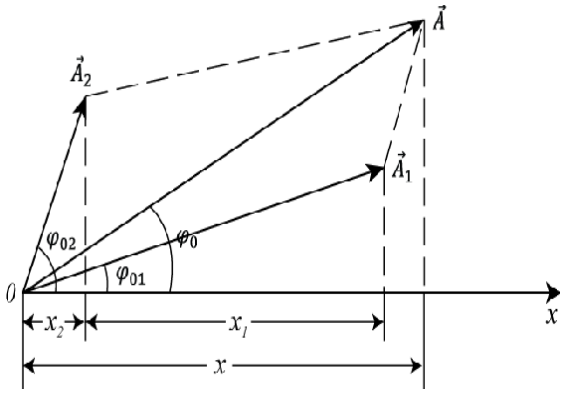


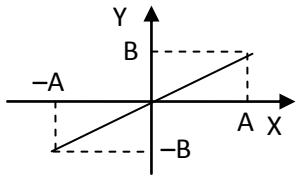
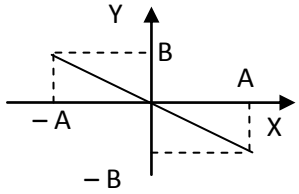
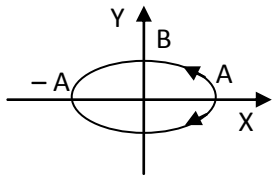
Figure 1.5 - Method of vector diagram

Appendix 10

At addition of fluctuations of identical frequency and shift of phases $(\varphi_{02} - \varphi_{01}) = \pm 2k\pi$ $(k = 0,1,2...)$ the resulting amplitude is equal to $A = A_1 + A_2$. Oscillations are in one phase (sinphase). If the shift of phases $(\varphi_{02} - \varphi_{01}) = \pm(2k+1)\pi$ and $A = |A_1 - A_2|$ oscillations are in an antiphase. In both cases amplitude of the resulting fluctuation does not change over time. *Two oscillatory processes are called coherent fluctuations if they are conformed in time so, a difference of their phases remains constant.*

Appendix 11

Table 11.1 –Addition of mutually perpendicular periodic motions

Difference of phases	Trajectory equation	Graphical representation
$\varphi_0 = m\pi$ $(m = 0, \pm 2, ...)$	$y = \frac{B}{A}x$	
$\varphi_0 = m\pi$ $(m = \pm 1, \pm 3...)$	$y = -\frac{B}{A}x$	
$\varphi_0 = (2m+1)\frac{\pi}{2}$ $(m = 0, \pm 1, ...)$	$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$	 $\varphi_0 = -\frac{\pi}{2}$ $\varphi_0 = \frac{\pi}{2}$

Appendix 12

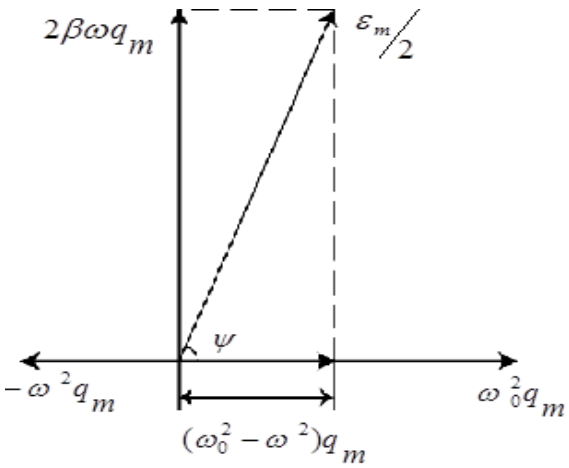


Figure 1.6 - Vector diagram for a real oscillatory contour

Differentiating expression (2.20) on t once again let's write down:

$$\ddot{q} = \omega^2 q_m \cos(\omega t - \psi) = \omega_0^2 q_m \cos(\omega t - \psi + \pi).$$

Using the last formula and expressions (2.20) and (2.18) it is possible to be convinced that $\frac{\varepsilon_m}{L}\cos\omega t$ is the sum of three harmonic oscillations of the same frequency, the phases which are carried out by the shift. The vector diagram of addition of these fluctuations is represented in drawing. As seen in the drawing:

$$q_m = \frac{\varepsilon_m}{L\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2}};$$

$$tg\psi = \frac{2\beta\omega}{\omega_0^2 - \omega^2}.$$

If we consider that $\beta = \frac{R}{2L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$ the value of charge is $q_m = \frac{\varepsilon_m}{\omega\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$.

Appendix 13

Resonant frequencies for a charge (voltage on the condenser) and current are defined by the following formulas:

$$\omega_{q\,res} = \omega_{Uc\,res} = \sqrt{\omega_0^2 - 2\beta^2};$$

$$\omega_I = \omega_0.$$

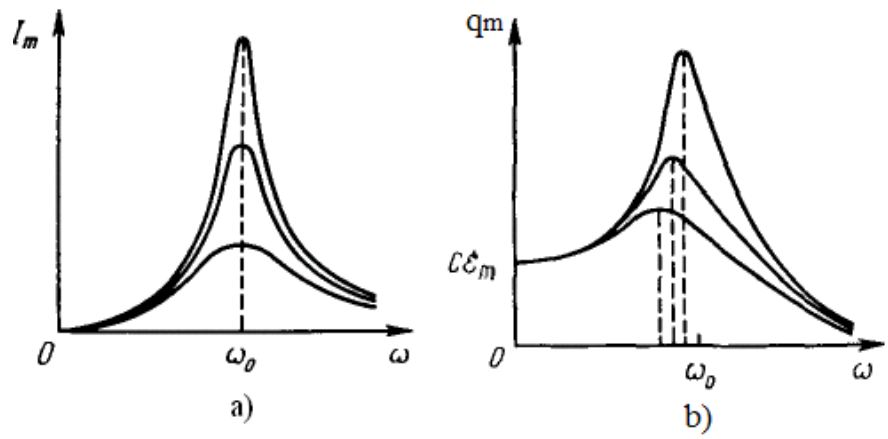


Figure 1.7 - Resonant curves for current *I* and charge *q*

Resonant curves for voltage U_C and current I are given in drawings a) and b). The maximum at a resonance of curves is higher and sharper, the β is less, i.e. the active resistance is less. Unlike resonant curves for current at $\omega \to 0$ the resonant curves for voltage U_C converge in point $U_C = U_m$. Value U_m is a voltage occurring on the condenser when connected it to a source of constant voltage.

Amplitude value of current is achieved at $\omega = \omega_0$, amplitude value U_C is reached at $\omega \le \omega_0$.

Appendix 14

Let's mentally allocate small volume ΔV where in all points speed of movement and deformation of a medium can be considered identical and equal respectively to $\frac{\partial \xi}{\partial t}$ and $\frac{\partial \xi}{\partial x}$. The allocated volume possesses kinetic energy

$$\Delta W_k = \frac{\Delta m}{2}\left(\frac{\partial \xi}{\partial t}\right)^2 = \frac{\rho}{2}\left(\frac{\partial \xi}{\partial t}\right)^2 \Delta V ,$$

where $\Delta m = \rho \cdot \Delta V$ is the mass of substance in volume ΔV,

$\frac{\partial \xi}{\partial t} = -\omega A \sin(\omega t - kx)$ is the speed of a point.

The potential energy in this case is equal to:

$$\Delta W_p = \frac{E\varepsilon^2}{2}\Delta V ,$$

where E - Jung's module;

$\varepsilon = \frac{\partial \xi}{\partial x}$ - relative lengthening or compression.

Considering that the speed of longitudinal waves is equal to $\upsilon = \sqrt{E/\rho}$, the relative compression is: $\frac{\partial \xi}{\partial x} = kA\sin(\omega t - kx)$.

Appendix 15

For explanation of Compton is effect we will consider elastic collision of a x-ray photon with a quasi free electron in rest (binding energy of an electron in atom is much less than energy that a photon can transfer to an electron).

Let's write down conservation laws of energy and an impulse:

$$\hbar\omega + m_0 c^2 = \hbar\omega' + c\sqrt{p^2 + m_0 c^2} ; \qquad (15.1)$$

$$\hbar\vec{k} = \vec{p} + \hbar\vec{k}', \qquad (15.2)$$

where $\hbar\omega$ and $\hbar\omega'$ – energy of a x-ray photon before collision respectively;

m_0c^2 – energy of an electron before collision;

$c\sqrt{p^2 + m_0c^2}$ - energy of an electron after collision;

$\vec{p}$ – an electron impulse after collision;

Expressions $\hbar\vec{k}$ and $\hbar\vec{k}'$ are a photon impulse before and after collisions respectively.

The vector diagram of equation 1 is given in drawing.

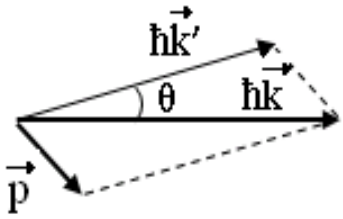


Figure 1.8 - Vector diagram

By means of this diagram we will pass to a scalar type of equation (15.2) and, solving it together with equation (15.1), we will receive:

$$\lambda'(\theta) - \lambda = \frac{2\pi\hbar}{m_0c}(1 - \cos\theta), \qquad (15.3)$$

where value $\frac{2\pi\hbar}{m_0c} = 2{,}426 \cdot 10^{-12}\, m = \lambda_C$ is Compton's length.

Formula (15.3) is coordinated perfectly with experimental results of Compton. It proves correctness of ideas of corpuscular properties of electromagnetic radiation.

Appendix 16

The tunnel effect is passing of microparticles through space areas forbidden by classical physic laws. Let a particle fly on an elementary rectangular potential barrier from the left side (figure 1.9). If total energy of particle W is less than the height of a potential barrier U_0 in point x_1, it will be reflected , i.e. it will turn back. There is another situation in quantum mechanics. From particle of Schrödinger's equation follows that there is a probability of passing *different of zero,* into area $x \rangle x_1$. We have the falling and reflected waves from the left side of the barrier, and there is only a passing wave from the right side of the barrier. Into the barrier ψ - function has not a wave character therefore the probability decreases almost exponentially.

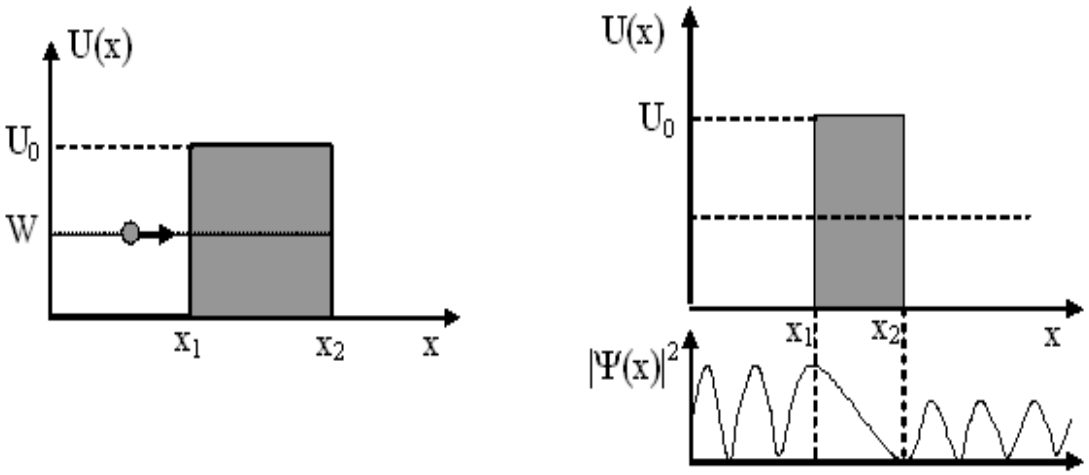


Figure 1.9 - Passing of microparticles through space areas forbidden by laws of classical physics

Tunnel effect is a specifically quantum phenomenon. Cold electron's emission from metals, alpha- decay, spontaneous division of nuclei are explained by this effect.

A type of potential energy of a quantum harmonious oscillator is the same as classical one. If the movement of a particle with mass m and frequency of oscillations ω occurs on axis *x:*

$$U(x) = \frac{m\omega^2 x^2}{2}.$$

The solution of Schrödinger's equation for a quantum oscillator is a complex mathematical problem. Let's consider only a power range of the quantum harmonious oscillator:

$$W_n = \left(n + \frac{1}{2}\right)\hbar\omega,$$

where $n = 0,1,2...$ – any whole positive number.

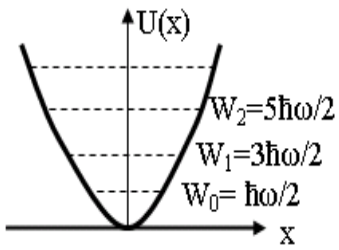


Figure 1.10 – "A potential pit"

From the last formula it follows that the power range of the oscillator is discrete and limited below by energy of value $W_0 = \hbar\omega/2$. It is the main level of the

quantum oscillator. Intervals $W = \hbar\omega$ between the next levels do not depend on quantum number n, i.e. they are identical (figure 1.10).

As the main level $W_0 \rangle 0$ the quantum oscillator cannot be stopped. For example, because of it fluctuations of atoms in crystal lattices do not stop even at the temperature of absolute zero. Quantum fluctuations with the minimum energy are called *zero oscillations.*

The concept about the quantum oscillator plays an important role in physics of those phenomena that is caused by fluctuations of microscopic particles.

Appendix 17

Table 17.1 – Characteristics of pions

Designation of a pion	Mass, MeV	Electric charge, e	Lifetime, s
$\pi^{\pm}$	140	± 1	10^{8}
π^{0}	135	0	10^{-16}

Let's consider the interaction between nucleons. A nucleon can emit a pion if its energy has uncertainty no less, than $m_{\pi}c^2$. In this case violation of the law of energy conservation cannot be found. According to a ratio of uncertainty “energy-time” the emitted pion can exist during time $\tau = \hbar / m_{\pi}c^2$, then it is absorbed by the same nucleon, or another, being nearby the first one.

Let's emphasize that the exchange of particles is the cornerstone of all interactions and the fundamental quantum property of the nature. Particles, which emission and absorption happen with the seeming violation of energy conservation law, are called virtual.

As a result of virtual processes $p \leftrightarrow n + \pi^{+}$, $n \leftrightarrow p + \pi^{-}$, $p \leftrightarrow p + \pi^{0}$, $n \leftrightarrow n + \pi^{0}$ a single nucleon is surrounded by virtual π - mezon cloud (a mezon fur coat) forming a field of nuclear forces.

At approaching of two nucleons before contact of their mezon fur coats the conditions for an exchange of pions are created. Thus, *the mechanism of strong interaction between nucleons consists that nucleons, exchanging among themselves virtual pions, keep each other*. Strong interaction can happen according to several schemes, for example $p + n \leftrightarrow n + \pi^{+} + n \leftrightarrow n + p$, $n + p \leftrightarrow p + \pi^{-} + p \leftrightarrow p + n$, $n + n \leftrightarrow n + \pi^{0} + n \leftrightarrow n + n$.

Exchange character of nuclear forces allows to explain existence of the magnetic moment of a neutron. The neutron spends a part of life time in a virtual state $(p + \pi^{-})$. Orbital movement of π^{-}– mezon also leads to emergence of the observed negative magnetic moment of a neutron.

Let's note that the satisfactory *quantitative* theory of interaction of nucleons by means of exchange of pions is not created. There were serious mathematical difficulties at its development. The main reason is that nuclear forces are very powerful.

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